Liminary on Guilbaud's 1952 paper

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We take advantage of this issue –partly– devoted to the history of the theory of social choice to provide the first complete English translation of the famous Guilbaud's 1952 paper *Les théories de l'intérêt général et le problème logique de l'agrégation* (*Theories of the general interest and the logical problem of aggregation*)[2]. In fact, in the anglo-saxon world the paper was certainly mainly quoted because it contained the formula $1 - \frac{3}{\pi} \arccos\left(\frac{1}{\sqrt{3}}\right) = 0.0877\cdots$ giving the limit value of the probability of the "effet Condorcet" (the "voting paradox") when there are three options and as *n* –the number of voters– approaches infinity. Moreover, the only explanation given for the obtainment of this formula was the rather puzzling "by the means of the usual methods". But in fact, Guilbaud's paper contained many other significant mathematical results intertwined with historical perspectives: Condorcet's *Essai*[3] was recovered from the deep oblivion into which it had fallen, –partially– rehabilitated and embedded in a long line of works from Rousseau to Arrow (and on the way Lhuilier, Laplace, Quetelet, Cournot, Nanson, Dom Quentin, Pareto, Walras, Pigou, Marshall, Hicks, Bergson, Samuelson, Fréchet, Black or Kendall)[4]. As Arrow wrote in the second

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[2] *Économie appliquée* 5(4): 501-584 (October-December 1952).

[3] *Essai sur l'application de l'analyse à la probabilité des décisions rendues à la pluralité des voix*, Paris, 1785.

[4] For an analysis of the mathematical contents of the paper see B. Monjardet, Social choice theory and the "Centre de Mathématique Sociale". Some historical notes, *Social choice and Welfare*, 25, 2006, 433-456, and D.Eckert and B.

edition of *Social Choice and Individual Values* (1963): "it is a remarkable exposition of the theory of collective choice and the general problem of aggregation".

Before giving editorial details on the present translation of Guilbaud's paper, let us recall the context in which it was published. Arrow's book *Social Choice and Individual Values* appeared in 1951 and immediately caused a considerable interest as well as critical discussions. In May 1952, Arrow went to the *Conference on Risk* held at Marseille and then to Paris where François Perroux head of the "Institut des Sciences Économiques Appliquées (ISEA)" had asked him to make a presentation of his work. The lecture entitled *The rationality principle in collective decisions* was given on June 9th 1952 at the Institute. Translated into French under the title *Le principe de rationalité dans les décisions collectives* it was published in a special issue of *Économie Appliquée*[5] devoted to welfare economics and called *L'avantage collectif*. Guilbaud, a mathematician was then one of the two assistant heads of the ISEA. In the same issue of *Économie appliquée* he published his paper *Les théories de l'intérêt général et le problème logique de l'agrégation*, where "il espère ne pas diminuer la valeur et l'originalité de ces belles recherches (i.e. "les profondes études de K.J. Arrow") que d'évoquer à leur occasion Rousseau ou Condorcet…" (he hopes not to diminish the value and the originality of this fine research (i.e., the deep studies of K.J. Arrow) if he mentions in relation to them Rousseau or Condorcet…).

Guilbaud's paper contained 83 pages[6]. An English translation of about 45 pages was published in 1966 in the book *Readings in Mathematical Social Sciences*[7]. The first thirteen and the last twenty-five pages were replaced by (very short) summaries by the editors. Moreover, and without any notice, the editors deleted some parts of the translated text[8]. This is why one will find here the first complete English translation of Guilbaud's paper.[9] [10]

Monjardet, Guilbaud's Theorem: An early contribution to judgment aggregation, forthcoming. Guilbaud's paper contains in particular a "dictatorial" Arrovian theorem on the aggregation of consistent sets of propositions, proved by – implicitly – using the notion of ultrafilter, and the fact that the majority rule can be applied to the domain of the "unimodal" linear orders without producing Condorcet effects, because this domain can be endowed with a structure of distributive lattice (an early contribution to general results found in Galambos and Reiner's paper: Acyclic Sets of Linear Orders via the Bruhat Order, *Social Choice and Welfare*, 30(2), 2008, 245-264.)

[5] *Économie apliquée* 5(4): 469-484 (October-December 1952). By the way, Arrow's proof of his "dictator" theorem given in this paper was right, contrary – as it is well-known – to the proof given in *Social Choice and individual Values* (see the second edition of this book).

[6] It was also published again with two other Guilbaud's papers in the following book: G.Th; Guilbaud, *Eléments de la théorie des Jeux*, Paris, Dunod, 1968,

[7] Theories of the general interest and the logical problem of aggregation. In P.F. Lazarsfeld and N.W. Henry (eds) *Readings in Mathematical Social Sciences*, Science Research Association, Inc., Chicago, 1966, pp. 262-307.

[8] In particular in the section *The logical problem* they deleted two figures and the comments concerning the case where the "borders (corresponding to individual opinions) cross". They also added some elements to exemplify the first case (where the borders do not cross). In the present translation we keep these elements but we reintroduce the omitted text and we add similar elements.

[9] Guilbaud's text contains many quotations in French. In the translated text they are followed by an English translation into brackets and in a less size.