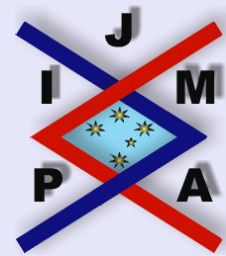


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Abstract

In this paper we consider trigonometric series with the coefficients from $R_0^+ BVS$ class. We prove the theorems on belonging to these series to Orlicz space.

2000 Mathematics Subject Classification: 42A32, 46E30.

Key words: Trigonometric series, Integrability, Orlicz space.

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1. Introduction

We will study the problems of integrability of formal sine and cosine series

$$(1.1) \quad g(x) = \sum_{n=1}^{\infty} \lambda_n \sin nx,$$

$$(1.2) \quad f(x) = \sum_{n=1}^{\infty} \lambda_n \cos nx.$$

First, we will rewrite the classical result of Young, Boas and Heywood for series (1.1) and (1.2) with monotone coefficients.

Theorem 1.1 ([1], [2], [11]). *Let $\lambda_n \downarrow 0$.*

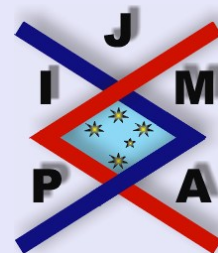
If $0 \leq \alpha < 2$, then

$$\frac{g(x)}{x^\alpha} \in L(0, \pi) \iff \sum_{n=1}^{\infty} n^{\alpha-1} \lambda_n < \infty.$$

If $0 < \alpha < 1$, then

$$\frac{f(x)}{x^\alpha} \in L(0, \pi) \iff \sum_{n=1}^{\infty} n^{\alpha-1} \lambda_n < \infty.$$

Several generalizations of this theorem have been obtained in the following directions: more general weighted functions $\gamma(x)$ have been considered; also,



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integrability of $g(x)\gamma(x)$ and $f(x)\gamma(x)$ of order p have been examined for different values of p ; finally, more general conditions on coefficients $\{\lambda_n\}$ have been considered.

Igari ([3]) obtained the generalization of Boas-Heywood's results. The author used the notation of a slowly oscillating function.

A positive measurable function $S(t)$ defined on $[D; +\infty)$, $D > 0$ is said to be slowly oscillating if $\lim_{t \rightarrow \infty} \frac{S(\lambda t)}{S(t)} = 1$ holds for all $x > 0$.

Theorem 1.2 ([3]). *Let $\lambda_n \downarrow 0$, $p \geq 1$, and let $S(t)$ be a slowly oscillating function.*

If $-1 < \theta < 1$, then

$$\frac{g^p(x)S\left(\frac{1}{x}\right)}{x^{p\theta+1}} \in L(0, \pi) \iff \sum_{n=1}^{\infty} n^{p\theta+p-1} S(n)\lambda_n^p < \infty.$$

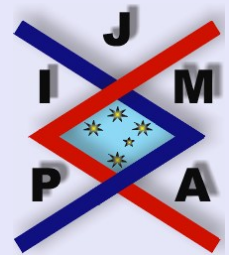
If $-1 < \theta < 0$, then

$$\frac{f^p(x)S\left(\frac{1}{x}\right)}{x^{p\theta+1}} \in L(0, \pi) \iff \sum_{n=1}^{\infty} n^{p\theta+p-1} S(n)\lambda_n^p < \infty.$$

Vukolova and Dyachenko in [10], considering the Hardy-Littlewood type theorem found the sufficient conditions of belonging of series (1.1) and (1.2) to the classes L_p for $p > 0$.

Theorem 1.3 ([10]). *Let $\lambda_n \downarrow 0$, and $p > 0$. Then*

$$\sum_{n=1}^{\infty} n^{p-2}\lambda_n^p < \infty \implies \psi(x) \in L^p(0, \pi),$$



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where a function $\psi(x)$ is either a $f(x)$ or a $g(x)$.

In the same work it is shown that the converse result does not hold for cosine series.

Leindler ([5]) introduced the following definition. A sequence $\mathbf{c} := \{c_n\}$ of positive numbers tending to zero is of rest bounded variation, or briefly $R_0^+ BVS$, if it possesses the property

$$\sum_{n=m}^{\infty} |c_n - c_{n+1}| \leq K(\mathbf{c}) c_m$$

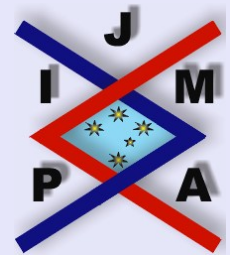
for all natural numbers m , where $K(\mathbf{c})$ is a constant depending only on \mathbf{c} . In [5] it was shown that the class $R_0^+ BVS$ was not comparable to the class of quasi-monotone sequences, that is, to the class of sequences $\mathbf{c} = \{c_n\}$ such that $n^{-\alpha} c_n \downarrow 0$ for some $\alpha \geq 0$. Also, in [5] it was proved that the series (1.1) and (1.2) are uniformly convergent over $\delta \leq x \leq \pi - \delta$ for any $0 < \delta < \pi$. In the same paper the following was proved.

Theorem 1.4 ([5]). Let $\{\lambda_n\} \in R_0^+ BVS$, $p \geq 1$, and $\frac{1}{p} - 1 < \theta < \frac{1}{p}$. Then

$$\frac{\psi^p(x)}{x^{p\theta}} \in L(0, \pi) \iff \sum_{n=1}^{\infty} n^{p\theta+p-2} \lambda_n^p < \infty,$$

where a function $\psi(x)$ is either a $f(x)$ or a $g(x)$.

Very recently Nemeth [8] has found the sufficient condition of integrability of series (1.1) with the sequence of coefficients $\{\lambda_n\} \in R_0^+ BVS$ and with quite



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general conditions on a weight function. The author has used the notation of almost monotonic sequences.

A sequence $\gamma := \{\gamma_n\}$ of positive terms will be called almost increasing (decreasing) if there exists constant $C := C(\gamma) \geq 1$ such that

$$C\gamma_n \geq \gamma_m \quad (\gamma_n \leq C\gamma_m)$$

holds for any $n \geq m$.

Here and further, C, C_i denote positive constants that are not necessarily the same at each occurrence.

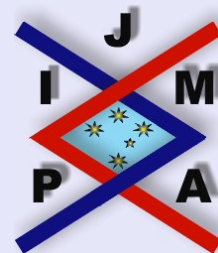
Theorem 1.5 ([8]). *If $\{\lambda_n\} \in R_0^+ BVS$, and the sequence $\gamma := \{\gamma_n\}$ such that $\{\gamma_n n^{-2+\varepsilon}\}$ is almost decreasing for some $\varepsilon > 0$, then*

$$\sum_{n=1}^{\infty} \frac{\gamma_n}{n} \lambda_n < \infty \implies \gamma(x)g(x) \in L(0, \pi).$$

Here and in the sequel, a function $\gamma(x)$ is defined by the sequence γ in the following way: $\gamma\left(\frac{\pi}{n}\right) := \gamma_n$, $n \in \mathbf{N}$ and there exist positive constants A and B such that $A\gamma_{n+1} \leq \gamma(x) \leq B\gamma_n$ for $x \in \left(\frac{\pi}{n+1}, \frac{\pi}{n}\right)$.

We will solve the problem of finding of sufficient conditions, for which series (1.1) and (1.2) belong to the weighted Orlicz space $L(\Phi, \gamma)$. In particular, we will obtain sufficient conditions for series (1.1) and (1.2) to belong to weighted space L_γ^p .

Definition 1.1. *A locally integrable almost everywhere positive function $\gamma(x) : [0, \pi] \rightarrow [0, \infty)$ is said to be a weight function. Let $\Phi(t)$ be a nondecreasing*



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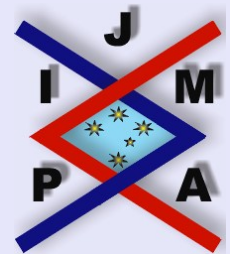
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continuous function defined on $[0, \infty)$ such that $\Phi(0) = 0$ and $\lim_{t \rightarrow \infty} \Phi(t) = +\infty$.
 For a weight $\gamma(x)$ the weighted Orlicz space $L(\Phi, \gamma)$ is defined by (see [9], [12])

$$(1.3) \quad L(\Phi, \gamma) = \left\{ h : \int_0^\pi \gamma(x) \Phi(\varepsilon |h(x)|) dx < \infty \text{ for some } \varepsilon > 0 \right\}.$$

If $\Phi(x) = x^p$ for $1 \leq p < \infty$, when the weighted Orlicz space $L(\Phi, \gamma)$ defined by (1.3) is the usual weighted space $L_\gamma^p(0, \pi)$.

We will denote (see [6]) by $\Delta(p, q)$ ($0 \leq q \leq p$) the set of all nonnegative functions $\Phi(x)$ defined on $[0, \infty)$ such that $\Phi(0) = 0$ and $\Phi(x)/x^p$ is nonincreasing and $\Phi(x)/x^q$ is nondecreasing. It is clear that $\Delta(p, q) \subset \Delta(p, 0)$ ($0 < q \leq p$). As an example, $\Delta(p, 0)$ contains the function $\Phi(x) = \log(1 + x)$.



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2. Results

The following theorems provide the sufficient conditions of belonging of $f(x)$ and $g(x)$ to Orlicz spaces.

Theorem 2.1. Let $\Phi(x) \in \Delta(p, 0)$ ($0 \leq p$). If $\{\lambda_n\} \in R_0^+ BVS$, and sequence $\{\gamma_n\}$ is such that $\{\gamma_n n^{-1+\varepsilon}\}$ is almost decreasing for some $\varepsilon > 0$, then

$$\sum_{n=1}^{\infty} \frac{\gamma_n}{n^2} \Phi(n\lambda_n) < \infty \implies \psi(x) \in L(\Phi, \gamma),$$

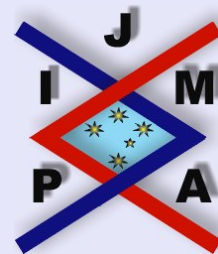
where a function $\psi(x)$ is either a sine or cosine series.

For the sine series it is possible to obtain the sufficient condition of its belonging to Orlicz space with more general conditions on the sequence $\{\gamma_n\}$ but with stronger restrictions on the function $\Phi(x)$.

Theorem 2.2. Let $\Phi(x) \in \Delta(p, q)$ ($0 \leq q \leq p$). If $\{\lambda_n\} \in R_0^+ BVS$, and sequence $\{\gamma_n\}$ is such that $\{\gamma_n n^{-(1+q)+\varepsilon}\}$ is almost decreasing for some $\varepsilon > 0$, then

$$\sum_{n=1}^{\infty} \frac{\gamma_n}{n^{2+q}} \Phi(n^2\lambda_n) < \infty \implies g(x) \in L(\Phi, \gamma).$$

Remark 2.1. If $\Phi(t) = t$, then Theorem 2.2 implies Theorem 1.5, and if $\Phi(t) = t^p$ with $0 < p$ and $\{\gamma_n = 1, n \in \mathbf{N}\}$, then Theorem 2.1 is a generalization of Theorem 1.3. Also, if $\Phi(t) = t^p$ with $1 \leq p$ and $\{\gamma_n = n^\alpha S(n), n \in \mathbf{N}\}$ with corresponding conditions on α and $S(n)$, then Theorems 2.1 and 2.2 imply the sufficiency parts (\Leftarrow) of Theorems 1.2 and 1.4.



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3. Auxiliary Results

Lemma 3.1 ([4]). If $a_n \geq 0$, $\lambda_n > 0$, and if $p \geq 1$, then

$$\sum_{n=1}^{\infty} \lambda_n \left(\sum_{\nu=1}^n a_{\nu} \right)^p \leq C \sum_{n=1}^{\infty} \lambda_n^{1-p} a_n^p \left(\sum_{\nu=n}^{\infty} \lambda_{\nu} \right)^p.$$

Lemma 3.2 ([6]). Let $\Phi \in \Delta(p, q)$ ($0 \leq q \leq p$) and $t_j \geq 0$, $j = 1, 2, \dots, n$, $n \in \mathbf{N}$. Then

$$(1) \theta^p \Phi(t) \leq \Phi(\theta t) \leq \theta^q \Phi(t), \quad 0 \leq \theta \leq 1, t \geq 0,$$

$$(2) \Phi \left(\sum_{j=1}^n t_j \right) \leq \left(\sum_{j=1}^n \Phi_{p^*}^{\frac{1}{p^*}}(t_j) \right)^{p^*}, \quad p^* = \max(1, p).$$

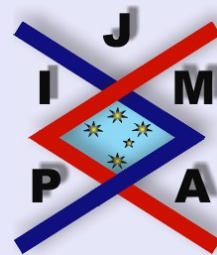
Lemma 3.3. Let $\Phi \in \Delta(p, q)$ ($0 \leq q \leq p$). If $\lambda_n > 0$, $a_n \geq 0$, and if there exists a constant K such that $a_{\nu+j} \leq K a_{\nu}$ holds for all $j, \nu \in \mathbf{N}$, $j \leq \nu$, then

$$\sum_{k=1}^{\infty} \lambda_k \Phi \left(\sum_{\nu=1}^k a_{\nu} \right) \leq C \sum_{k=1}^{\infty} \Phi(k a_k) \lambda_k \left(\frac{\sum_{\nu=k}^{\infty} \lambda_{\nu}}{k \lambda_k} \right)^{p^*},$$

where $p^* = \max(1, p)$.

Proof. Let ξ be an integer such that $2^{\xi} \leq k < 2^{\xi+1}$. Then

$$\begin{aligned} \sum_{\nu=1}^k a_{\nu} &\leq \sum_{m=0}^{\xi-1} \sum_{\nu=2^m}^{2^{m+1}-1} a_{\nu} + \sum_{\nu=2^{\xi}}^k a_{\nu} \\ &\leq C_1 \left(\sum_{m=0}^{\xi-1} 2^m a_{2^m} + 2^{\xi} a_{2^{\xi}} \right) \leq C_1 \sum_{m=0}^{\xi} 2^m a_{2^m}. \end{aligned}$$



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Lemma 3.2 implies

$$\begin{aligned}
 \Phi \left(\sum_{\nu=1}^k a_{\nu} \right) &\leq \Phi \left(C_1 \sum_{m=0}^{\xi} 2^m a_{2^m} \right) \\
 &\leq C_1^p \Phi \left(\sum_{m=0}^{\xi} 2^m a_{2^m} \right) \\
 &\leq C \left(\sum_{m=0}^{\xi} \Phi^{\frac{1}{p^*}} (2^m a_{2^m}) \right)^{p^*} \\
 &\leq C \left(\sum_{m=1}^k \frac{\Phi^{\frac{1}{p^*}} (m a_m)}{m} \right)^{p^*}.
 \end{aligned}$$

By Lemma 3.1, we have

$$\begin{aligned}
 \sum_{k=1}^{\infty} \lambda_k \Phi \left(\sum_{\nu=1}^k a_{\nu} \right) &\leq C \sum_{k=1}^{\infty} \lambda_k \left(\sum_{m=1}^k \frac{\Phi^{\frac{1}{p^*}} (m a_m)}{m} \right)^{p^*} \\
 &\leq C \sum_{k=1}^{\infty} \Phi (k a_k) \lambda_k \left(\frac{\sum_{\nu=k}^{\infty} \lambda_{\nu}}{k \lambda_k} \right)^{p^*}.
 \end{aligned}$$

□

Note that this Lemma was proved in [7] for the case $0 < p \leq 1$.



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4. Proofs of Theorems

Proof of Theorem 2.1. Let $x \in (\frac{\pi}{n+1}, \frac{\pi}{n}]$. Applying Abel's transformation we obtain

$$|f(x)| \leq \sum_{k=1}^n \lambda_k + \left| \sum_{k=n+1}^{\infty} \lambda_k \cos kx \right| \leq \sum_{k=1}^n \lambda_k + \sum_{k=n}^{\infty} |(\lambda_k - \lambda_{k+1}) D_k(x)|,$$

where $D_k(x)$ are the Dirichlet kernels, i.e.

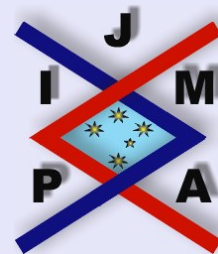
$$D_k(x) = \frac{1}{2} + \sum_{n=1}^k \cos nx, \quad k \in \mathbf{N}.$$

Since $|D_k(x)| = O(\frac{1}{x})$ and $\lambda_n \in R_0^+ BVS$, we see that

$$|f(x)| \leq C \left(\sum_{k=1}^n \lambda_k + n \sum_{k=n}^{\infty} |\lambda_k - \lambda_{k+1}| \right) \leq C \left(\sum_{k=1}^n \lambda_k + n\lambda_n \right).$$

The following estimates for series (1.2) can be obtained in the same way:

$$\begin{aligned} |g(x)| &\leq \sum_{k=1}^n \lambda_k + \left| \sum_{k=n+1}^{\infty} \lambda_k \sin kx \right| \\ &\leq \sum_{k=1}^n \lambda_k + \sum_{k=n}^{\infty} |(\lambda_k - \lambda_{k+1}) \tilde{D}_k(x)| \\ &\leq C \left(\sum_{k=1}^n \lambda_k + n \sum_{k=n}^{\infty} |\lambda_k - \lambda_{k+1}| \right) \leq C \left(\sum_{k=1}^n \lambda_k + n\lambda_n \right), \end{aligned}$$



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where $\tilde{D}_k(x)$ are the conjugate Dirichlet kernels, i.e. $\tilde{D}_k(x) := \sum_{n=1}^k \sin nx$, $k \in \mathbb{N}$.

Therefore,

$$|\psi(x)| \leq C \left(\sum_{k=1}^n \lambda_k + n\lambda_n \right),$$

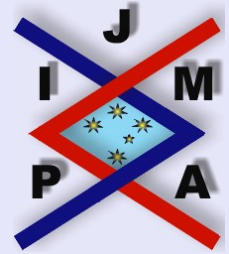
where a function $\psi(x)$ is either a $f(x)$ or a $g(x)$.

One can see that if $\{\lambda_n\} \in R_0^+ BVS$, then $\{\lambda_n\}$ is almost decreasing sequence, i.e. there exists a constant $K \geq 1$ such that $\lambda_n \leq K\lambda_k$ holds for any $k \leq n$. Then

$$(4.1) \quad |\psi(x)| \leq C \left(\sum_{k=1}^n \lambda_k + \lambda_n \sum_{k=1}^n 1 \right) \leq C \sum_{k=1}^n \lambda_k.$$

We will use (4.1) and the fact that $\{\lambda_k\}$ is almost decreasing sequence; also, we will use Lemmas 3.2 and 3.3:

$$\begin{aligned} \int_0^\pi \gamma(x) \Phi(|\psi(x)|) dx &\leq \sum_{n=1}^\infty \Phi \left(C_1 \sum_{k=1}^n \lambda_k \right) \int_{\pi/(n+1)}^{\pi/n} \gamma(x) dx \\ &\leq C_1^p \pi B \sum_{n=1}^\infty \frac{\gamma_n}{n^2} \Phi \left(\sum_{k=1}^n \lambda_k \right) \\ &\leq C \sum_{k=1}^\infty \Phi(k\lambda_k) \frac{\gamma_k}{k^2} \left(\frac{k}{\nu} \sum_{\nu=k}^\infty \frac{\gamma_\nu}{\nu^2} \right)^{p^*}, \end{aligned}$$



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where $p^* = \max(1, p)$. Since there exists a constant $\varepsilon > 0$ such that $\{\gamma_n n^{-1+\varepsilon}\}$ is almost decreasing, then

$$\sum_{\nu=k}^{\infty} \frac{\gamma_{\nu}}{\nu^2} \leq C \frac{\gamma_k}{k^{1-\varepsilon}} \sum_{\nu=k}^{\infty} \nu^{-\varepsilon-1} \leq C \frac{\gamma_k}{k}.$$

Then

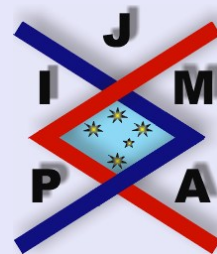
$$\int_0^{\pi} \gamma(x) \Phi(|\psi(x)|) dx \leq C \sum_{k=1}^{\infty} \frac{\gamma_k}{k^2} \Phi(k\lambda_k).$$

The proof of Theorem 2.1 is complete. \square

Proof of Theorem 2.2. While proving Theorem 2.2 we will follow the idea of the proof of Theorem 2.1.

Let $x \in (-\frac{\pi}{n+1}, \frac{\pi}{n}]$. Then

$$\begin{aligned} (4.2) \quad |g(x)| &\leq \sum_{k=1}^n kx\lambda_k + \left| \sum_{k=n+1}^{\infty} \lambda_k \sin kx \right| \\ &\leq \sum_{k=1}^n kx\lambda_k + \sum_{k=n}^{\infty} |(\lambda_k - \lambda_{k+1}) \tilde{D}_k(x)| \\ &\leq C \left(\frac{1}{n} \sum_{k=1}^n k\lambda_k + n\lambda_n \right) \\ &\leq C \left(\frac{1}{n} \sum_{k=1}^n k\lambda_k + \frac{1}{n} \lambda_n \sum_{k=1}^n k \right) \leq C_1 \frac{1}{n} \sum_{k=1}^n k\lambda_k. \end{aligned}$$



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Using Lemma 3.2, Lemma 3.3 and the estimate (4.2), we can write

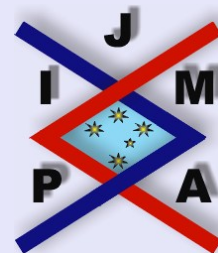
$$\begin{aligned} \int_0^\pi \gamma(x) \Phi(|g(x)|) dx &\leq \sum_{n=1}^{\infty} \Phi \left(C_1 \frac{1}{n} \sum_{k=1}^n k \lambda_k \right) \int_{\pi/(n+1)}^{\pi/n} \gamma(x) dx \\ &\leq C_1^p \pi B \sum_{n=1}^{\infty} \frac{\gamma_n}{n^{2+q}} \Phi \left(\sum_{k=1}^n k \lambda_k \right) \\ &\leq C_2 \sum_{k=1}^{\infty} \Phi(k^2 \lambda_k) \frac{\gamma_k}{k^{2+q}} \left(\frac{k^{1+q}}{\gamma_k} \sum_{\nu=k}^{\infty} \frac{\gamma_\nu}{\nu^{2+q}} \right)^{p^*}, \end{aligned}$$

where $p^* = \max(1, p)$.

By the assumption on $\{\gamma_n\}$,

$$\int_0^\pi \gamma(x) \Phi(|g(x)|) dx \leq C \sum_{k=1}^{\infty} \frac{\gamma_k}{k^{2+q}} \Phi(k^2 \lambda_k),$$

and the proof of Theorem 2.2 is complete. \square



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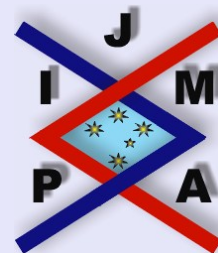
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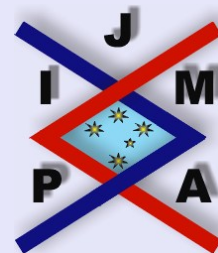
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