

Journal of Inequalities in Pure and Applied Mathematics

THE DZIOK-SRIVASTAVA OPERATOR AND k -UNIFORMLY STARLIKE FUNCTIONS

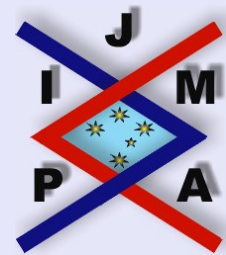
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Abstract

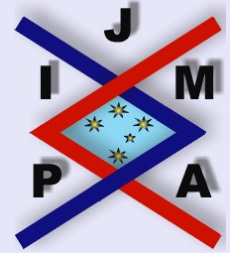
Inclusion relations for k -uniformly starlike functions under the Dziok-Srivastava operator are established. These results are also extended to k -uniformly convex functions, close-to-convex, and quasi-convex functions.

2000 Mathematics Subject Classification: Primary 30C45; Secondary 30C50.

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1. Introduction

Let A denote the class of functions of the form $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ which are analytic in the open unit disc $U = \{z : |z| < 1\}$. A function $f \in A$ is said to be in $UST(k, \gamma)$, the class of k -uniformly starlike functions of order γ , $0 \leq \gamma < 1$, if f satisfies the condition

$$(1.1) \quad \Re \left(\frac{zf'(z)}{f(z)} \right) > k \left| \frac{zf'(z)}{f(z)} - 1 \right| + \gamma, \quad k \geq 0.$$

Replacing f in (1.1) by zf' we obtain the condition

$$(1.2) \quad \Re \left\{ 1 + \frac{zf''(z)}{f'(z)} \right\} > k \left| \frac{zf''(z)}{f'(z)} \right| + \gamma, \quad k \geq 0$$

required for the function f to be in the subclass $UCV(k, \gamma)$ of k -uniformly convex functions of order γ .

Uniformly starlike and convex functions were first introduced by Goodman [5] and then studied by various authors. For a wealth of references, see Ronning [13].

Setting

$$\Omega_{k,\gamma} = \left\{ u + iv; u > k\sqrt{(u-1)^2 + v^2} + \gamma \right\},$$

with $p(z) = \frac{zf'(z)}{f(z)}$ or $p(z) = 1 + \frac{zf''(z)}{f'(z)}$ and considering the functions which map U on to the conic domain $\Omega_{k,\gamma}$, such that $1 \in \Omega_{k,\gamma}$, we may rewrite the conditions (1.1) or (1.2) in the form

$$(1.3) \quad p(z) \prec q_{k,\gamma}(z).$$



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We note that the explicit forms of function $q_{k,\gamma}$ for $k = 0$ and $k = 1$ are

$$q_{0,\gamma}(z) = \frac{1 + (1 - 2\gamma)z}{1 - z}, \text{ and } q_{1,\gamma}(z) = 1 + \frac{2(1 - \gamma)}{\pi^2} \left(\log \frac{1 + \sqrt{z}}{1 - \sqrt{z}} \right)^2.$$

For $0 < k < 1$ we obtain

$$q_{k,\gamma}(z) = \frac{1 - \gamma}{1 - k^2} \cos \left\{ \frac{2}{\pi} (\arccos k) i \log \frac{1 + \sqrt{z}}{1 - \sqrt{z}} \right\} - \frac{k^2 - \gamma}{1 - k^2},$$

and if $k > 1$, then $q_{k,\gamma}$ has the form

$$q_{k,\gamma}(z) = \frac{1 - \gamma}{k^2 - 1} \sin \left(\frac{\pi}{2K(k)} \int_0^{\frac{u(z)}{\sqrt{k}}} \frac{dt}{\sqrt{1 - t^2} \sqrt{1 - k^2 t^2}} \right) + \frac{k^2 - \gamma}{k^2 - 1},$$

where $u(z) = \frac{z - \sqrt{k}}{1 - \sqrt{kz}}$ and K is such that $k = \cosh \frac{\pi K'(z)}{4K(z)}$.

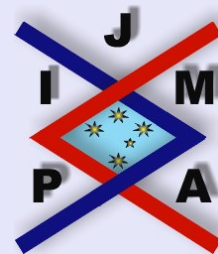
By virtue of (1.3) and the properties of the domains $\Omega_{k,\gamma}$ we have

$$(1.4) \quad \Re(p(z)) > \Re(q_{k,\gamma}(z)) > \frac{k + \gamma}{k + 1}.$$

Define $UCC(k, \gamma, \beta)$ to be the family of functions $f \in A$ such that

$$\Re \left(\frac{zf'(z)}{g(z)} \right) \geq k \left| \frac{zf'(z)}{g(z)} - 1 \right| + \gamma, \quad k \geq 0, 0 \leq \gamma < 1$$

for some $g \in UST(k, \beta)$.



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Similarly, we define $UQC(k, \gamma, \beta)$ to be the family of functions $f \in A$ such that

$$\Re \left(\frac{(zf'(z))'}{g'(z)} \right) \geq k \left| \frac{(zf'(z))'}{g'(z)} - 1 \right| + \gamma, \quad k \geq 0, \quad 0 \leq \gamma < 1$$

for some $g \in UCV(k, \beta)$.

We note that $UCC(0, \gamma, \beta)$ is the class of close-to-convex functions of order γ and type β and $UQC(0, \gamma, \beta)$ is the class of quasi-convex functions of order γ and type β .

The aim of this paper is to study the inclusion properties of the above mentioned classes under the following linear operator which is defined by Dziok and Srivastava [3].

For $\alpha_j \in C$ ($j = 1, 2, 3, \dots, l$) and $\beta_j \in C - \{0, -1, -2, \dots\}$ ($j = 1, 2, \dots, m$), the generalized hypergeometric function is defined by

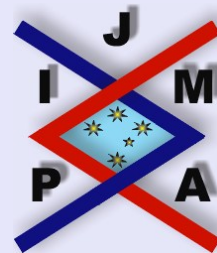
$${}_lF_m(\alpha_1, \dots, \alpha_l; \beta_1, \dots, \beta_m) = \sum_{n=0}^{\infty} \frac{(\alpha_1)_n \cdots (\alpha_l)_n}{(\beta_1)_n \cdots (\beta_m)_n} \cdot \frac{z^n}{n!},$$

$$(l \leq m + 1; l, m \in N_0 = \{0, 1, 2, \dots\}),$$

where $(a)_n$ is the Pochhammer symbol defined by $(a)_n = \frac{\Gamma(a+n)}{\Gamma(a)} = a(a+1) \cdots (a+n-1)$ for $n \in \mathbb{N} = \{1, 2, \dots\}$ and 1 when $n = 0$.

Corresponding to the function

$$h(\alpha_1, \dots, \alpha_l; \beta_1, \dots, \beta_m; z) = z {}_lF_m(\alpha_1, \dots, \alpha_l; \beta_1, \dots, \beta_m)$$



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the Dziok-Srivastava operator [3], $H_m^l(\alpha_1, \dots, \alpha_l; \beta_1, \dots, \beta_m)$ is defined by

$$\begin{aligned} H_m^l(\alpha_1, \dots, \alpha_l; \beta_1, \dots, \beta_m)f(z) &= h(\alpha_1, \dots, \alpha_l; \beta_1, \dots, \beta_m; z) * f(z) \\ &= z + \sum_{n=2}^{\infty} \frac{(\alpha_1)_{n-1} \cdots (\alpha_l)_{n-1}}{(\beta_1)_{n-1} \cdots (\beta_m)_{n-1}} \cdot \frac{a_n z^n}{(n-1)!}. \end{aligned}$$

where “*” stands for convolution.

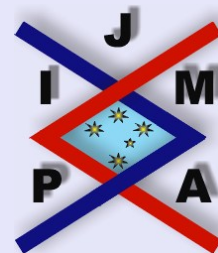
It is well known [3] that

$$\begin{aligned} (1.5) \quad \alpha_1 H_m^l(\alpha_1 + 1, \dots, \alpha_l; \beta_1, \dots, \beta_m)f(z) \\ &= z[H_m^l(\alpha_1, \dots, \alpha_l; \beta_1, \dots, \beta_m; z)f(z)]' \\ &\quad + (\alpha_1 - 1)H_m^l(\alpha_1, \dots, \alpha_l; \beta_1, \dots, \beta_m)f(z). \end{aligned}$$

To make the notation simple, we write,

$$H_m^l[\alpha_1]f(z) = H_m^l(\alpha_1, \dots, \alpha_l; \beta_1, \dots, \beta_m; z)f(z).$$

We note that many subclasses of analytic functions, associated with the Dziok-Srivastava operator $H_m^l[\alpha_1]$ and many special cases, were investigated recently by Dziok-Srivastava [3], Liu [7], Liu and Srivastava [9], [10] and others. Also we note that special cases of the Dziok-Srivastava linear operator include the Hohlov linear operator [6], the Carlson-Shaffer operator [2], the Ruscheweyh derivative operator [14], the generalized Bernardi-Libera-Livingston linear operator (cf. [1]) and the Srivastava-Owa fractional derivative operators (cf. [11], [12]).



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2. Main Results

In this section we prove some results on the linear operator $H_m^l[\alpha_1]$. First is the inclusion theorem.

Theorem 2.1. *Let $\Re\alpha_1 > \frac{1-\gamma}{k+1}$, and $f \in A$. If $H_m^l[\alpha_1 + 1]f \in UST(k, \gamma)$ then $H_m^l[\alpha_1]f \in UST(k, \gamma)$.*

In order to prove the above theorem we shall need the following lemma which is due to Eenigenburg, Miller, Mocanu, and Read [4].

Lemma A. *Let β, γ be complex constants and h be univalently convex in the unit disk U with $h(0) = c$ and $\Re(\beta h(z) + \gamma) > 0$. Let $g(z) = c + \sum_{n=1}^{\infty} p_n z^n$ be analytic in U . Then*

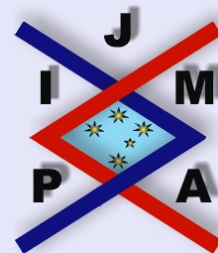
$$g(z) + \frac{z g'(z)}{\beta g(z) + \gamma} \prec h(z) \Rightarrow g(z) \prec h(z).$$

Proof of Theorem 2.1. Setting $p(z) = z(H_m^l[\alpha_1]f(z))' / (H_m^l[\alpha_1]f(z))$ in (1.5) we can write

$$(2.1) \quad \alpha_1 \frac{H_m^l[\alpha_1 + 1]f(z)}{H_m^l[\alpha_1]f(z)} = \frac{z(H_m^l[\alpha_1]f(z))'}{H_m^l[\alpha_1]f(z)} + (\alpha_1 - 1) = p(z) + (\alpha_1 - 1).$$

Differentiating (2.1) yields

$$(2.2) \quad \frac{z(H_m^l[\alpha_1 + 1]f(z))'}{H_m^l[\alpha_1 + 1]} = p(z) + \frac{z p'(z)}{p(z) + (\alpha_1 - 1)}.$$



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From this and the argument given in Section 1 we may write

$$p(z) + \frac{zp'(z)}{p(z) + (\alpha_1 - 1)} \prec q_{k,\gamma}(z).$$

Therefore the theorem follows by Lemma A and the condition (1.4) since $q_{k,\gamma}$ is univalent and convex in U and $\Re(q_{k,\gamma}) > \frac{k+\gamma}{k+1}$. \square

Theorem 2.2. Let $\Re\alpha_1 > \frac{1-\gamma}{k+1}$, and $f \in A$. If $H_m^l[\alpha_1 + 1]f \in UCV(k, \gamma)$ then $H_m^l[\alpha_1]f \in UCV(k, \gamma)$.

Proof. By virtue of (1.1), (1.2) and Theorem 2.1 we have

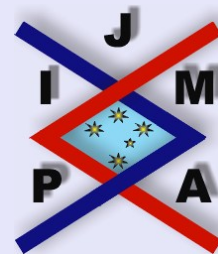
$$\begin{aligned} H_m^l[\alpha_1 + 1]f \in UCV(k, \gamma) &\Leftrightarrow z (H_m^l[\alpha_1 + 1]f)' \in UST(k, \gamma) \\ &\Leftrightarrow H_m^l[\alpha_1 + 1]zf' \in UST(k, \gamma) \\ &\Rightarrow H_m^l[\alpha_1]zf' \in UST(k, \gamma) \\ &\Leftrightarrow H_m^l[\alpha_1]f \in UCV(k, \gamma). \end{aligned}$$

and the proof is complete. \square

We next prove

Theorem 2.3. Let $\Re\alpha_1 > \frac{1-\gamma}{k+1}$, and $f \in A$. If $H_m^l[\alpha_1 + 1]f \in UCC(k, \gamma, \beta)$ then $H_m^l[\alpha_1]f \in UCC(k, \gamma, \beta)$.

To prove the above theorem, we shall need the following lemma which is due to Miller and Mocanu [10].



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Lemma B. Let h be convex in the unit disk U and let $E \geq 0$. Suppose $B(z)$ is analytic in U with $\Re B(z) \geq E$. If g is analytic in U and $g(0) = h(0)$. Then

$$Ez^2g''(z) + B(z)zg'(z) + g(z) \prec h(z) \Rightarrow g(z) \prec h(z).$$

Proof of Theorem 2.3. Since $H_m^l[\alpha_1 + 1]f \in UCC(k, \gamma, \beta)$, by definition, we can write

$$\frac{z(H_m^l[\alpha_1 + 1]f)'(z)}{k(z)} \prec q_{k,\gamma}(z)$$

for some $k(z) \in UST(k, \beta)$. For g such that $H_m^l[\alpha_1 + 1]g(z) = k(z)$, we have

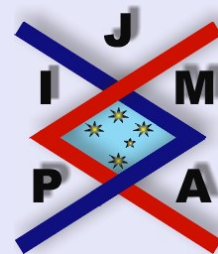
$$(2.3) \quad \frac{z(H_m^l[\alpha_1 + 1]f)'(z)}{H_m^l[\alpha_1 + 1]g(z)} \prec q_{k,\gamma}(z).$$

Letting $h(z) = \frac{z(H_m^l[\alpha_1]f)'(z)}{(H_m^l[\alpha_1]g)(z)}$ and $H(z) = \frac{z(H_m^l[\alpha_1]g)'(z)}{H_m^l[\alpha_1]g(z)}$ we observe that h and H are analytic in U and $h(0) = H(0) = 1$. Now, by Theorem 2.1, $H_m^l[\alpha_1]g \in UST(k, \beta)$ and so $\Re H(z) > \frac{k+\beta}{k+1}$. Also, note that

$$(2.4) \quad z(H_m^l[\alpha_1]f)'(z) = (H_m^l[\alpha_1]g(z))h(z).$$

Differentiating both sides of (2.4) yields

$$\frac{z(H_m^l[\alpha_1](zf'))'(z)}{H_m^l[\alpha_1]g(z)} = \frac{z(H_m^l[\alpha_1]g)'(z)}{H_m^l[\alpha_1]g(z)}h(z) + zh'(z) = H(z)h(z) + zh'(z).$$



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Now using the identity (1.5) we obtain

$$\begin{aligned}
 (2.5) \quad & \frac{z(H_m^l[\alpha_1 + 1]f)'(z)}{H_m^l[\alpha_1 + 1]g(z)} \\
 &= \frac{H_m^l[\alpha_1 + 1](zf')(z)}{H_m^l[\alpha_1 + 1]g(z)} \\
 &= \frac{z(H_m^l[\alpha_1](zf'))'(z) + (\alpha_1 - 1)H_m^l[\alpha_1](zf')(z)}{z(H_m^l[\alpha_1]g)'(z) + (\alpha_1 - 1)H_m^l[\alpha_1]g(z)} \\
 &= \frac{\frac{z(H_m^l[\alpha_1](zf'))'(z)}{H_m^l[\alpha_1]g(z)} + (\alpha_1 - 1)\frac{H_m^l[\alpha_1](zf')(z)}{H_m^l[\alpha_1]g(z)}}{\frac{z(H_m^l[\alpha_1]g)'(z)}{H_m^l[\alpha_1]g(z)} + (\alpha_1 - 1)} \\
 &= \frac{H(z)h(z) + zh'(z) + (\alpha_1 - 1)h(z)}{H(z) + (\alpha_1 - 1)} \\
 &= h(z) + \frac{1}{H(z) + (\alpha_1 - 1)}zh'(z).
 \end{aligned}$$

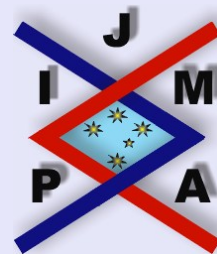
From (2.3), (2.4), and (2.5) we conclude that

$$h(z) + \frac{1}{H(z) + (\alpha_1 - 1)}zh'(z) \prec q_{k,\gamma}(z).$$

On letting $E = 0$ and $B(z) = \frac{1}{H(z) + (\alpha_1 - 1)}$, we obtain

$$\Re(B(z)) = \frac{1}{|(\alpha_1 - 1) + H(z)|^2} \Re((\alpha_1 - 1) + H(z)) > 0.$$

The above inequality satisfies the conditions required by Lemma B. Hence $h(z) \prec q_{k,\gamma}(z)$ and so the proof is complete. \square



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Using a similar argument to that in Theorem 2.2 we can prove

Theorem 2.4. Let $\Re\alpha_1 > \frac{1-\gamma}{k+1}$, and $f \in A$. If $H_m^l[\alpha_1 + 1]f \in UQC(k, \gamma, \beta)$, then $H_m^l[\alpha_1]f \in UQC(k, \gamma, \beta)$.

Finally, we examine the closure properties of the above classes of functions under the generalized Bernardi-Libera-Livingston integral operator $L_c(f)$ which is defined by

$$L_c(f) = \frac{c+1}{z^c} \int_0^z t^{c-1} f(t) dt, \quad c > -1.$$

Theorem 2.5. Let $c > \frac{-(k+\gamma)}{k+1}$. If $H_m^l[\alpha_1]f \in UST(k, \gamma)$ so is $L_c(H_m^l[\alpha_1]f)$.

Proof. From definition of $L_c(f)$ and the linearity of operator $H_m^l[\alpha_1]$ we have

$$(2.6) \quad z(H_m^l[\alpha_1]L_c(f))'(z) = (c+1)H_m^l[\alpha_1]f(z) - c(H_m^l[\alpha_1]L_c(f))(z).$$

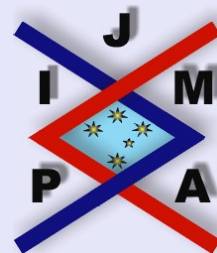
Substituting $\frac{z(H_m^l[\alpha_1]L_c(f))'(z)}{H_m^l[\alpha_1]L_c(f)(z)} = p(z)$ in (2.6) we may write

$$(2.7) \quad p(z) = (c+1) \frac{H_m^l[\alpha_1]f(z)}{(H_m^l[\alpha_1]L_c(f))(z)} - c.$$

Differentiating (2.7) gives

$$\frac{z(H_m^l[\alpha_1]f)'(z)}{(H_m^l[\alpha_1]f)(z)} = p(z) + \frac{zp'(z)}{p(z) + c}.$$

Now, the theorem follows by Lemma A, since $\Re(q_{k,\gamma}(z) + c) > 0$. □



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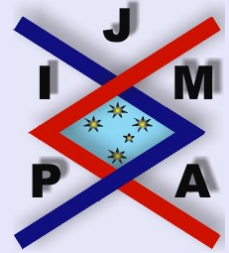


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A similar argument leads to

Theorem 2.6. Let $c > \frac{-(k+\gamma)}{k+1}$. If $H_m^l[\alpha_1]f \in UCV(k, \gamma)$ so is $L_c(H_m^l[\alpha_1]f)$.

Theorem 2.7. Let $c > \frac{-(k+\gamma)}{k+1}$. If $H_m^l[\alpha_1]f \in UCC(k, \gamma, \beta)$ so is $L_c(H_m^l[\alpha_1]f)$.

Proof. By definition, there exists a function $k(z) = (H_m^l[\alpha_1]g)(z) \in UST(k, \beta)$ such that

$$(2.8) \quad \frac{z(H_m^l[\alpha_1]f)'(z)}{(H_m^l[\alpha_1]g)(z)} \prec q_{k,\gamma}(z) \quad (z \in U).$$

Now from (2.6) we have

$$(2.9) \quad \begin{aligned} \frac{z(H_m^l[\alpha_1]f)'(z)}{(H_m^l[\alpha_1]g)(z)} &= \frac{z(H_m^l[\alpha_1]L_c(zf'))'(z) + cH_m^l[\alpha_1]L_c(zf')(z)}{z(H_m^l[\alpha_1]L_c(g(z)))'(z) + c(H_m^l[\alpha_1]L_c(g))(z)} \\ &= \frac{\frac{z(H_m^l[\alpha_1]L_c(zf'))'(z)}{(H_m^l[\alpha_1]L_c(g))(z)} + \frac{c(H_m^l[\alpha_1]L_c(zf'))(z)}{(H_m^l[\alpha_1]L_c(g))(z)}}{\frac{z(H_m^l[\alpha_1]L_c(g))'(z)}{(H_m^l[\alpha_1]L_c(g))(z)} + c}. \end{aligned}$$

Since $H_m^l[\alpha_1]g \in UST(k, \beta)$, by Theorem 2.5, we have $L_c(H_m^l[\alpha_1]g) \in UST(k, \beta)$.

Letting $\frac{z(H_m^l[\alpha_1]L_c(g))'}{H_m^l[\alpha_1]L_c(g)} = H(z)$, we note that $\Re(H(z)) > \frac{k+\beta}{k+1}$. Now, let h be defined by

$$(2.10) \quad z(H_m^l[\alpha_1]L_c(f))' = h(z)H_m^l[\alpha_1]L_c(g).$$

Differentiating both sides of (2.10) yields

$$(2.11) \quad \begin{aligned} \frac{z(H_m^l[\alpha_1](zL_c(f))')'(z)}{(H_m^l[\alpha_1]L_c(g))(z)} &= zh'(z) + h(z) \frac{z(H_m^l[\alpha_1]L_c(g))'(z)}{(H_m^l[\alpha_1]L_c(g))(z)} \\ &= zh'(z) + H(z)h(z). \end{aligned}$$

Therefore from (2.9) and (2.11) we obtain

$$\frac{z(H_m^l[\alpha_1]f)'(z)}{(H_m^l[\alpha_1]g)(z)} = \frac{zh'(z) + h(z)H(z) + ch(z)}{H(z) + c}.$$

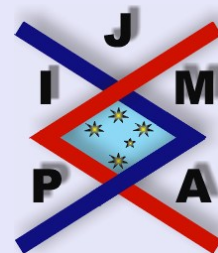
This in conjunction with (2.8) leads to

$$(2.12) \quad h(z) + \frac{zh'(z)}{H(z) + c} \prec q_{k,\gamma}(z).$$

Letting $B(z) = \frac{1}{H(z)+c}$ in (2.12) we note that $\Re(B(z)) > 0$ if $c > -\frac{k+\beta}{k+1}$. Now for $E = 0$ and B as described we conclude the proof since the required conditions of Lemma B are satisfied. \square

A similar argument yields

Theorem 2.8. Let $c > -\frac{(k+\gamma)}{k+1}$. If $H_m^l[\alpha_1]f \in UQC(k, \gamma, \beta)$ so is $L_c(H_m^l[\alpha_1]f)$.



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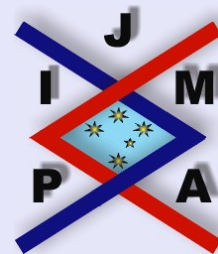
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