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INEQUALITIES FOR LATTICE CONSTRAINED PLANAR CONVEX SETS

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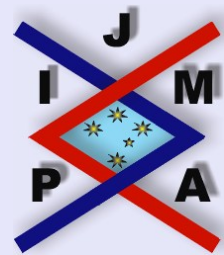
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Abstract

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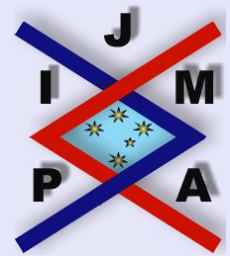


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Abstract

Every convex set in the plane gives rise to geometric functionals such as the area, perimeter, diameter, width, inradius and circumradius. In this paper, we prove new inequalities involving these geometric functionals for planar convex sets containing zero or one interior lattice point. We also conjecture two results concerning sets containing one interior lattice point. Finally, we summarize known inequalities for sets containing zero or one interior lattice point.

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Key words: Planar Convex Set, Lattice, Lattice Point Enumerator, Lattice-Point-Free, Sublattice, Area, Perimeter, Diameter, Width, Inradius, Circumradius.

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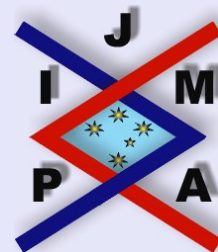
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1. Introduction

Let \mathcal{K}^2 denote the set of all planar, compact, convex sets. Let K be a set in \mathcal{K}^2 with area $A = A(K)$, perimeter $p = p(K)$, diameter $d = d(K)$, width $w = w(K)$, inradius $r = r(K)$ and circumradius $R = R(K)$. Let K° denote the interior of K . Let Γ denote the integer lattice. The lattice point enumerator $G(K^\circ, \Gamma)$ is defined to be the number of points of Γ contained in K° . In the case where $G(K^\circ, \Gamma) = 0$, we say that K is lattice-point-free.

In this article, we prove new inequalities involving the geometric functionals A, p, d, w, r and R for sets $K \in \mathcal{K}^2$ with $G(K^\circ, \Gamma) = 0$ and $G(K^\circ, \Gamma) = 1$. These may be found in Sections 2 and 3 respectively. In Section 4, we conjecture two results concerning sets $K \in \mathcal{K}^2$ with $G(K^\circ, \Gamma) = 1$. Finally, in Sections 5 and 6, we summarize known inequalities in one and two functionals for sets $K \in \mathcal{K}^2$ with $G(K^\circ, \Gamma) = 0$ and $G(K^\circ, \Gamma) = 1$ respectively (see [26] for a summary of inequalities involving two and three functionals for sets $K \in \mathcal{K}^2$ without lattice constraints). Although there are extensive bibliographies for lattice constrained convex sets [8, 10, 11, 12, 24], this article attempts to organise the numerous results for sets $K \in \mathcal{K}^2$ with $G(K^\circ, \Gamma) = 0$ and $G(K^\circ, \Gamma) = 1$. Although these results are rather special, they are a natural starting point for problems in the area and have in fact served as a springboard for many new and interesting problems.

In the statements of the theorems and the conjecture, each inequality is followed by a set for which the inequality is sharp.



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2. Some Elementary Results for Lattice-Point-Free Sets

Theorem 2.1. Let $K \in \mathcal{K}^2$ with $G(K^\circ, \Gamma) = 0$. Let $\lambda = 2\sqrt{2} \sin \phi/2$, ϕ being the unique solution of the equation $\sin \theta = \pi/2 - \theta$, ($\phi \approx 0.832 \approx 47.4^\circ$). Then

$$(2.1) \quad r \leq \frac{\sqrt{2}}{2}, \quad \mathcal{C}_0 \text{ (Figure 2a),}$$

$$(2.2) \quad \frac{A}{R} \leq 2\lambda \approx 2.288, \quad \mathcal{H}_0 \text{ (Figure 2c),}$$

$$(2.3) \quad \frac{A}{w^3} \geq \frac{1}{\sqrt{3}} \left(1 + \frac{\sqrt{3}}{2}\right)^{-1} \approx 0.309, \quad \mathcal{E}_0 \text{ (Figure 2b),}$$

$$(2.4) \quad (2r - 1)p \leq \frac{4}{r}(\sqrt{2} - 1), \quad \mathcal{S}_0 \text{ (Figure 2e).}$$

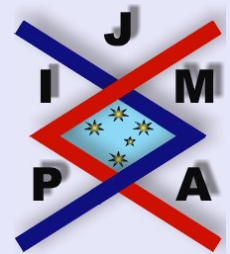
Proof. To prove (2.1), we use the following lemma from [3]:

Lemma 2.2. Suppose that $K \in \mathcal{K}^2$ and $G(K^\circ, \Gamma) = 0$. Then there is a set $K_* \in \mathcal{K}^2$ with $G(K_*^\circ, \Gamma) = 0$. satisfying the following conditions:

(a) $r(K) \leq r(K_*)$,

(b) K_* is symmetric about the lines $x = \frac{1}{2}, y = \frac{1}{2}$.

From the lemma, it suffices to prove (2.1) for sets K which are symmetric about the lines $x = \frac{1}{2}$ and $y = \frac{1}{2}$. To fully utilise the symmetry of K about the lines $x = \frac{1}{2}$ and $y = \frac{1}{2}$, we move the origin to the point $(\frac{1}{2}, \frac{1}{2})$. If $r \leq \frac{1}{2}$, then (2.1) is trivially true. Hence we may assume that $r > \frac{1}{2}$. Since K° does not contain the points $P_1(\frac{1}{2}, \frac{1}{2}), P_2(-\frac{1}{2}, \frac{1}{2}), P_3(-\frac{1}{2}, -\frac{1}{2})$ and $P_4(\frac{1}{2}, -\frac{1}{2})$, it follows



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by the convexity of K that for each $i = 1, \dots, 4$, K is bounded by a line l_i through the point P_i with l_1 and l_3 having negative slope and l_2 and l_4 having positive slope. Furthermore, since K is symmetric about the coordinate axes, K is contained in a rhombus Q determined by the lines $l_i, i = 1, \dots, 4$. Since $K \subseteq Q$, $r(K) \leq r(Q)$. Clearly $r(Q) \leq \sqrt{2}/2$. Hence $r(K) \leq \sqrt{2}/2$ and (2.1) is proved. An example of a set for which the inequality is sharp is the circle \mathcal{C}_0 (Figure 2a).

(2.2) follows easily from a result by Scott [18], that if $K \in \mathcal{K}^2$ with $G(K^\circ, \Gamma) = 0$, then

$$(2.5) \quad \frac{A}{d} \leq \lambda \approx 1.144,$$

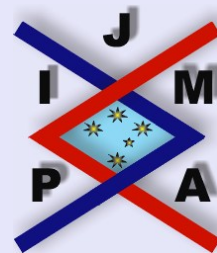
where λ is as defined in Theorem 2.1. The result is best possible with equality when and only when $K \cong \mathcal{H}_0$ (Figure 2c). Using $d \leq 2R$ and (2.5), it follows immediately that

$$\frac{A}{R} \leq 2\lambda \approx 2.288,$$

with equality when and only when $K \cong \mathcal{H}_0$ (Figure 2c).

The proof of (2.3) follows easily by combining two known results. The first is that of all sets in \mathcal{K}^2 with a given width, the equilateral triangle has the least area [27, p. 68]. Hence $A \geq (1/\sqrt{3})w^2$. We also recall from [17] that if $K \in \mathcal{K}^2$ with $G(K^\circ, \Gamma) = 0$, then

$$w \leq 1 + \frac{\sqrt{3}}{2},$$



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with equality when and only when $K \cong \mathcal{E}_0$ (Figure 2b). Hence

$$\frac{A}{w^3} = \left(\frac{A}{w^2}\right) \frac{1}{w} \geq \frac{1}{\sqrt{3}} \left(1 + \frac{\sqrt{3}}{2}\right)^{-1} \approx 0.309.$$

Equality holds when and only when $K \cong \mathcal{E}_0$ (Figure 2b).

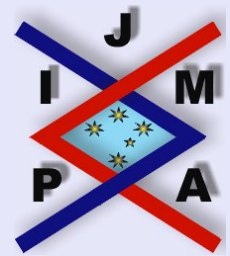
To prove (2.4), we use a result from [3]: If $K \in \mathcal{K}^2$ with $G(K^o, \Gamma) = 0$, then

$$(2.6) \quad (2r - 1)A \leq 2(\sqrt{2} - 1),$$

with equality when and only when $K \cong \mathcal{S}_0$ (Figure 2e). We also note from the same paper, that if K is a convex polygon, K may be partitioned into triangles by joining each vertex of K to an in-centre of K . Summing the areas of these triangles gives

$$A \geq \frac{1}{2}pr,$$

with equality when and only when every edge of K touches the unique incircle. Since any set in \mathcal{K}^2 is either a convex polygon, or may be approximated by a convex polygon, this inequality is valid for all sets in \mathcal{K}^2 . By combining this inequality with (2.6), we have (2.4), with equality when and only when $K \cong \mathcal{S}_0$ (Figure 2e). \square



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3. Some Elementary Results for Sets Containing One Interior Lattice Point

Theorem 3.1. *Let $K \in \mathcal{K}^2$ with $G(K^\circ, \Gamma) = 1$, . Let λ be as defined in Theorem 2.1. Then*

$$(3.1) \quad r \leq 1, \quad \mathcal{C}_1 \text{ (Figure 3a),}$$

$$(3.2) \quad \frac{A}{R} \leq 2\sqrt{2}\lambda \approx 3.232, \quad \mathcal{H}_1 \text{ (Figure 3d),}$$

$$(3.3) \quad A(w - \sqrt{2}) \leq \frac{1}{2}w^2, \quad \mathcal{T}_1 \text{ (Figure 3e),}$$

$$(3.4) \quad (2r - \sqrt{2})p \leq \frac{8}{r}(2 - \sqrt{2}), \quad \mathcal{S}_1 \text{ (Figure 3g).}$$

We note that (3.1), (3.2) and (3.4) are the results for sets $K \in \mathcal{K}^2$ having $G(K^\circ, \Gamma) = 1$ corresponding to (2.1), (2.2) and (2.4) respectively. Furthermore, we recall from [22] that if $K \in \mathcal{K}^2$ with $G(K^\circ, \Gamma) = 0$, then

$$(3.5) \quad A(w - 1) \leq \frac{1}{2}w^2,$$

with equality when and only when $K \cong \mathcal{T}_0$ (Figure 2f). We observe that (3.3) is the result corresponding to (3.5) for sets $K \in \mathcal{K}^2$ having $G(K^\circ, \Gamma) = 1$.

In fact, (3.3) has been proved in [14], where the method of proof is an adaptation of the method in [22]. In this paper we present a short and different proof for (3.3). We will see that all the inequalities of Theorem 3.1 follow immediately from their corresponding inequalities for lattice-point-free sets by using a simple sublattice argument.



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Proof. Let

$$\Gamma' = \{(x, y) : x + y \equiv 1 \pmod{2}\}.$$

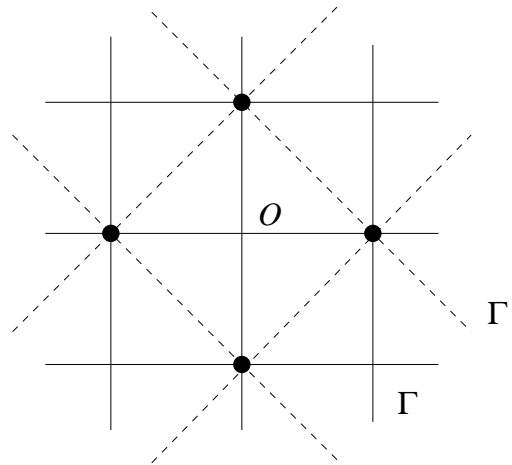
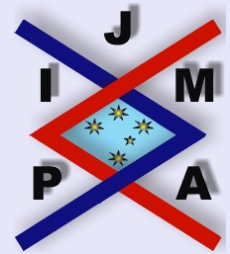


Figure 1: The lattice Γ' .

Suppose that $K \in \mathcal{K}^2$, with $G(K^o, \Gamma) = 1$. Then clearly $G(K^o, \Gamma') = 0$ (Figure 1). We also observe that Γ' is essentially an anticlockwise rotation of Γ about O through an angle $\pi/4$ and scaled by a factor of $\sqrt{2}$. Now let A' , p' , d' , w' , r' , and R' be the area, perimeter, diameter, width, inradius and circumradius respectively of K measured in the scale of Γ' . Then since $G(K^o, \Gamma') = 0$, the



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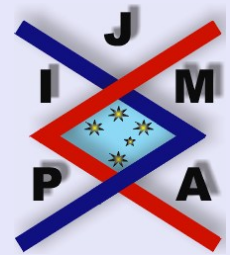
inequalities (2.1), (2.2), (3.5), and (2.4) apply, from which we have

$$\begin{aligned} r' &\leq \frac{\sqrt{2}}{2}, & \mathcal{C}_0' \\ \frac{A'}{R'} &\leq 2\lambda, & \mathcal{H}_0' \\ A'(w' - 1) &\leq \frac{1}{2}(w')^2, & \mathcal{T}_0' \\ (2r' - 1)p' &\leq \frac{4}{r'}(\sqrt{2} - 1), & \mathcal{S}_0', \end{aligned}$$

where \mathcal{C}_0' , \mathcal{H}_0' , \mathcal{T}_0' , and \mathcal{S}_0' are the sets \mathcal{C}_0 , \mathcal{H}_0 , \mathcal{T}_0 and \mathcal{S}_0 respectively rotated anticlockwise about O through $\pi/4$ and scaled by a factor of $\sqrt{2}$. Hence $\mathcal{C}_0' = \mathcal{C}_1$ (Figure 3a), $\mathcal{H}_0' = \mathcal{H}_1$ (Figure 3d), $\mathcal{T}_0' = \mathcal{T}_1$ (Figure 3e), and $\mathcal{S}_0' = \mathcal{S}_1$ (Figure 3g). Furthermore, since Γ' is a rotation of Γ scaled by a factor of $\sqrt{2}$, we have

$$A' = \left(\frac{1}{\sqrt{2}}\right)^2 A, \quad p' = \frac{1}{\sqrt{2}}p, \quad w' = \frac{1}{\sqrt{2}}w, \quad r' = \frac{1}{\sqrt{2}}r, \quad R' = \frac{1}{\sqrt{2}}R.$$

Substituting these into the above inequalities, we obtain (3.1), (3.2), (3.3), and (3.4) respectively. \square



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4. Conjectures for Sets Containing One Interior Lattice Point

Conjecture 4.1. Let $K \in \mathcal{K}^2$ with $G(K^\circ, \Gamma') = 1$. Let O be the circumcentre of K in (4.2). Then

$$(4.1) \quad \frac{A}{w^3} \geq \frac{1}{\sqrt{3}} \cdot \frac{4}{\sqrt{2}(5 + \sqrt{3})} \approx 0.243, \quad \mathcal{E}_1 \text{ (Figure 3b),}$$

$$(4.2) \quad A \leq \alpha \approx 4.05, \quad \mathcal{Q}_1 \text{ (Figure 3f).}$$

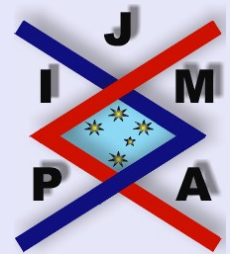
The problem which occurs in (4.1) is that for a set $K \in \mathcal{K}^2$ with $G(K^\circ, \Gamma) = 1$, $w \leq 1 + \sqrt{2} \approx 2.414$, with equality when and only when $K \cong \mathcal{I}_1$ (Figure 3e) [23]. Since this set of largest width is not an equilateral triangle, the method used to prove (2.3) cannot be applied.

A simple calculation shows that the width of \mathcal{E}_1 (Figure 3b) is $\frac{1}{4}\sqrt{2}(5 + \sqrt{3}) \approx 2.38$. Hence if $0 < w \leq \frac{1}{4}\sqrt{2}(5 + \sqrt{3})$, an equilateral triangle containing one interior lattice point may be constructed. Since $A \geq (1/\sqrt{3})w^2$ with equality when and only when K is an equilateral triangle, for this range of w we have

$$\frac{A}{w^3} = \left(\frac{A}{w^2}\right) \frac{1}{w} \geq \frac{1}{\sqrt{3}} \cdot \frac{4}{\sqrt{2}(5 + \sqrt{3})} \approx 0.243,$$

with equality when and only when $K \cong \mathcal{E}_1$ (Figure 3b).

This leaves unresolved those cases for which $\frac{1}{4}\sqrt{2}(5 + \sqrt{3}) < w \leq 1 + \sqrt{2}$. We believe that the set for which A/w^3 is minimal is congruent to the equilateral triangle \mathcal{E}_1 (Figure 3b).



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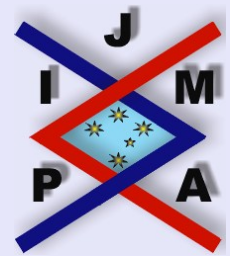
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In [21], Scott conjectures a result concerning the maximal area of a set $K \in \mathcal{K}^2$ with $G(K^o, \Gamma) = 1$ and having circumcentre O . Using a computer run, we discover that the conjecture is false. We revise the conjecture as stated in (4.2), with equality when and only when $K \cong Q_1$ (Figure 3f).



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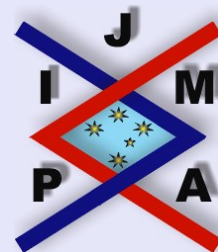
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5. Inequalities Involving One and Two Functionals for Lattice-Point-Free Sets

Tables 5.1 and 6.1 list the known inequalities (including conjectures) involving one and two functionals for lattice-point-free sets and sets containing one interior lattice point respectively. The extremal sets referred to in the tables may be found in Figures 2 and 3 respectively. Where a star (★) appears in the inequality column, no inequality is known for the corresponding functionals.

Parameters	Inequality	Extremal Set	Reference
A	unbounded		
p	unbounded		
d	unbounded		
w	$w \leq \frac{1}{2}(2 + \sqrt{3}) \approx 1.866$	\mathcal{E}_0	[17]
R	unbounded		
r	$r \leq \sqrt{2}/2$	\mathcal{C}_0	(2.1)
A, p	$A < \frac{1}{2}p$	\mathcal{P}_0	[6]
A, d	$A/d \leq \lambda, \lambda \approx 1.144$	\mathcal{H}_0	[18]
A, w	1. $(w - 1)A \leq \frac{1}{2}w^2$ 2. $\frac{A}{w^3} \geq \frac{1}{\sqrt{3}}(1 + \frac{\sqrt{3}}{2})^{-1} \approx 0.309$	\mathcal{T}_0 \mathcal{E}_0	[20] (2.3)
A, R	$A/R \leq 2\lambda, \lambda \approx 1.144$	\mathcal{H}_0	(2.2)
A, r	1. $(2r - 1)A \leq 2(\sqrt{2} - 1) \approx 0.828$ 2. $(2r - 1) A - 1 < \frac{1}{2}$	\mathcal{S}_0 \mathcal{P}_0	[3] [3]

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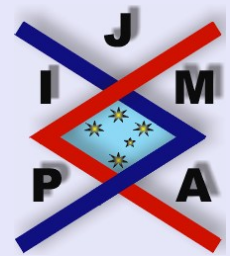


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Parameters	Inequality	Extremal Set	Reference
p, d	*		
p, w	$(w - 1)p \leq 3w$	\mathcal{E}_0	[20]
p, R	*		
p, r	1. $(2r - 1) p - 4 < 2$ 2. $(2r - 1)p \leq \frac{4}{r}(\sqrt{2} - 1)$	\mathcal{P}_0 \mathcal{S}_0	[3] (2.4)
d, w	$(w - 1)(d - 1) \leq 1$	\mathcal{T}_0	[19]
d, R	$2R - d \leq \frac{1}{3}$	\mathcal{E}_0	[4]
d, r	$(2r - 1)(d - 1) < 1$	\mathcal{P}_0	[3]
w, R	1. $(w - 1)R \leq \frac{1}{\sqrt{3}}w$ 2. $(w - 1)(2R - 1) \leq \frac{\sqrt{3}}{6} + 1 \approx 1.289$	\mathcal{E}_0 \mathcal{E}_0	[20] [25]
w, r	$w - 2r \leq \frac{1}{3} + \frac{1}{6}\sqrt{3} \approx 0.622$	\mathcal{E}_0	[4]
R, r	$(2r - 1)(2R - 1) \leq 1$	\mathcal{P}_0	[25]

Table 5.1: Inequalities for the case $G(K^\circ, \Gamma) = 0$.

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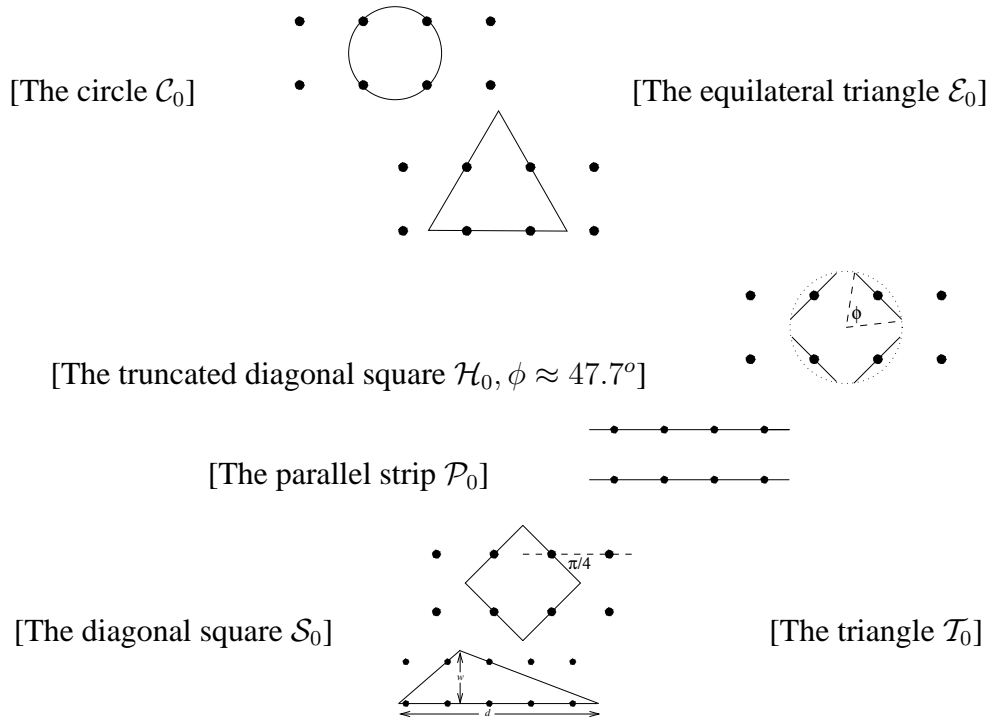
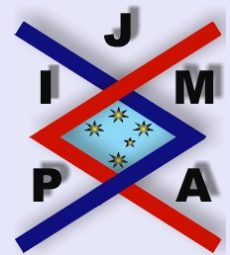


Figure 2: Extremal sets for the case $G(K^\circ, \Gamma) = 0$



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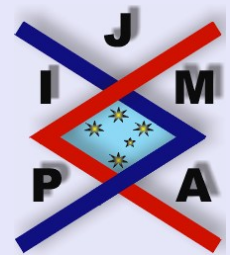
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6. Inequalities Involving One and Two Functionals for Sets Containing One Interior Lattice Point



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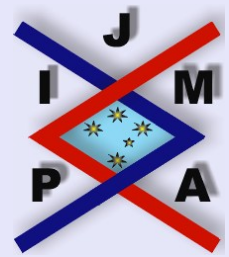


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Parameters	Inequality	Extremal Set	Reference
A	1. $A \leq 4$ if O is centre of K 2. $A \leq 4.5$ if O is the C.G. 3. <i>Conjecture:</i> If O is the circumcentre then $A \approx 4.05$	e.g. \mathcal{S}_1 Ehrhart's Δ \mathcal{Q}_1	[16] [9] (4.2)
p	unbounded		
d	unbounded		
w	1. $w \leq 1 + \sqrt{2} \approx 2.414$ 2. If O is the C.G. then $w \leq 3\sqrt{2}/2$ for the family of triangles	\mathcal{I}_1 Ehrhart's Δ	[23] [13]
R	$R \leq \alpha \approx 1.685$ or R unbounded	\mathcal{T}	[2]
r	$r \leq 1$	\mathcal{C}_1	(3.1)
A, p	$A/p \leq 2(2 + \sqrt{\pi})^{-1} \approx 0.53$ (O is centre of K)	\mathcal{U}_1	[1, 7]
A, d	$A/d \leq \sqrt{2}\lambda, \lambda \approx 1.144$	\mathcal{H}_1	[15]
A, w	1. $A(w - \sqrt{2}) \leq \frac{1}{\sqrt{2}}w^2$ 2. <i>Conjecture:</i> $\frac{A}{w^3} \geq \frac{1}{\sqrt{3}} \cdot \frac{4}{\sqrt{2}(5+\sqrt{3})} \approx 0.243$	\mathcal{T}_1 \mathcal{E}_1	(3.3), [14] (4.1)
A, R	$A/R \leq 2\sqrt{2}\lambda$	\mathcal{H}_1	(3.2)
A, r	$A(2r - \sqrt{2}) \leq 4(2 - \sqrt{2}) \approx 2.343$	\mathcal{S}_1	[3]
p, d	*		
p, w	*		
p, R	*		
p, r	$p(2r - \sqrt{2}) \leq \frac{8}{r}(2 - \sqrt{2})$	\mathcal{S}_1	(3.4)
d, w	$(w - \sqrt{2})(d - \sqrt{2}) \leq 2$	\mathcal{T}_1	[23]

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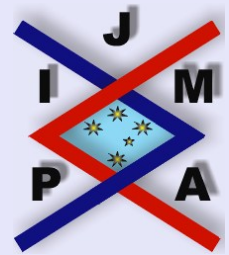
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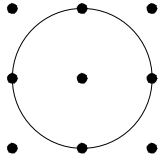


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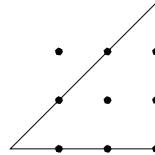
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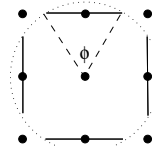
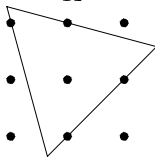
[The circle \mathcal{C}_1]



[The equilateral triangle \mathcal{E}_1]

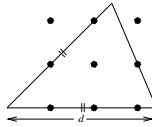


[Ehrhart's Δ]



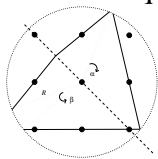
[The

[The truncated square \mathcal{H}_1 , $\phi \approx 47.7^\circ$]

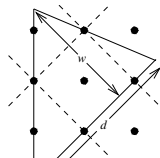
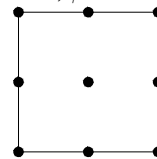


isosceles triangle \mathcal{I}_1]

[The truncated quadrilateral \mathcal{Q}_1 , $R \approx 1.593$, $\alpha \approx 5.47^\circ$, $\beta \approx 20.23^\circ$]



[The square \mathcal{S}_1]



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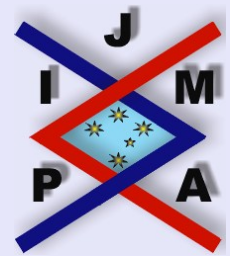
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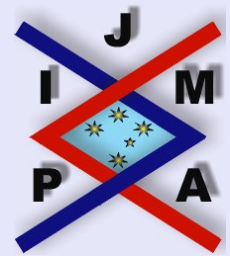
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