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## GENERALIZED INTEGRAL OPERATOR AND MULTIVALENT FUNCTIONS

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Abstract

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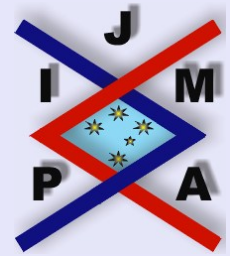


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## Abstract

Let  $\mathcal{A}(p)$  be the class of functions  $f : f(z) = z^p + \sum_{j=1}^{\infty} a_j z^{p+j}$  analytic in the open unit disc  $E$ . Let, for any integer  $n > -p$ ,  $f_{n+p-1}(z) = \frac{z^p}{(1-z)^{n+p}}$ . We define  $f_{n+p-1}^{(-1)}(z)$  by using convolution  $\star$  as  $f_{n+p-1}(z) \star f_{n+p-1}^{(-1)}(z) = \frac{z^p}{(1-z)^{n+p}}$ . A function  $p$ , analytic in  $E$  with  $p(0) = 1$ , is in the class  $P_k(\rho)$  if  $\int_0^{2\pi} \left| \frac{\operatorname{Re} p(z) - \rho}{p - \rho} \right| d\theta \leq k\pi$ , where  $z = re^{i\theta}$ ,  $k \geq 2$  and  $0 \leq \rho < p$ . We use the class  $P_k(\rho)$  to introduce a new class of multivalent analytic functions and define an integral operator  $I_{n+p-1}(f) = f_{n+p-1}^{(-1)} \star f(z)$  for  $f(z)$  belonging to this class. We derive some interesting properties of this generalized integral operator which include inclusion results and radius problems.

**2000 Mathematics Subject Classification:** Primary 30C45, 30C50.

**Key words:** Convolution (Hadamard product), Integral operator, Functions with positive real part, Convex functions.

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# 1. Introduction

Let  $\mathcal{A}(p)$  denote the class of functions  $f$  given by

$$f(z) = z^p + \sum_{j=1}^{\infty} a_j z^{p+j}, \quad p \in N = \{1, 2, \dots\}$$

which are analytic in the unit disk  $E = \{z : |z| < 1\}$ . The Hadamard product or convolution ( $f \star g$ ) of two functions with

$$f(z) = z^p + \sum_{j=1}^{\infty} a_{j,1} z^{p+j} \quad \text{and} \quad g(z) = z^p + \sum_{j=1}^{\infty} z^{p+j}$$

is given by

$$(f \star g)(z) = z^p + \sum_{j=1}^{\infty} a_{j,1} a_{j,2} z^{p+j}.$$

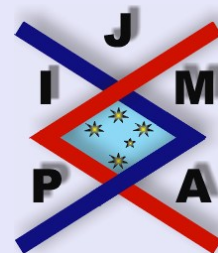
The integral operator  $I_{n+p-1} : \mathcal{A}(p) \longrightarrow \mathcal{A}(p)$  is defined as follows, see [2].

For any integer  $n$  greater than  $-p$ , let  $f_{n+p-1}(z) = \frac{z^p}{(1-z)^{n+p}}$  and let  $f_{n+p-1}^{(-1)}(z)$  be defined such that

$$(1.1) \quad f_{n+p-1}(z) \star f_{n+p-1}^{(-1)}(z) = \frac{z^p}{(1-z)^{p+1}}.$$

Then

$$(1.2) \quad I_{n+p-1} f(z) = f_{n+p-1}^{(-1)}(z) \star f(z) = \left[ \frac{z^p}{(1-z)^{n+p}} \right]^{(-1)} \star f(z).$$



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From (1.1) and (1.2) and a well known identity for the Ruscheweyh derivative [1, 8], it follows that

$$(1.3) \quad z (I_{n+p}f(z))' = (n+p)I_{n+p-1}f(z) - nI_{n+p}f(z).$$

For  $p = 1$ , the identity (1.3) is given by Noor and Noor [3].

Let  $P_k(\rho)$  be the class of functions  $p(z)$  analytic in  $E$  satisfying the properties  $p(0) = 1$  and

$$(1.4) \quad \int_0^{2\pi} \left| \frac{\operatorname{Re} p(z) - \rho}{p - \rho} \right| d\theta \leq k\pi,$$

where  $z = re^{i\theta}$ ,  $k \geq 2$  and  $0 \leq \rho < p$ . For  $p = 1$ , this class was introduced in [5] and for  $\rho = 0$ , see [6]. For  $\rho = 0$ ,  $k = 2$ , we have the well known class  $P$  of functions with positive real part and the class  $k = 2$  gives us the class  $P(\rho)$  of functions with positive real part greater than  $\rho$ . Also from (1.4), we note that  $p \in P_k(\rho)$  if and only if there exist  $p_1, p_2 \in P_k(\rho)$  such that

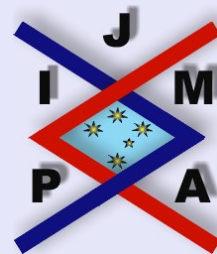
$$(1.5) \quad p(z) = \left( \frac{k}{4} + \frac{1}{2} \right) p_1(z) - \left( \frac{k}{4} - \frac{1}{2} \right) p_2(z).$$

It is known [4] that the class  $P_k(\rho)$  is a convex set.

**Definition 1.1.** Let  $f \in \mathcal{A}(p)$ . Then  $f \in T_k(\alpha, p, n, \rho)$  if and only if

$$\left[ (1 - \alpha) \frac{I_{n+p-1}f(z)}{z^p} + \alpha \frac{I_{n+p}f(z)}{z^p} \right] \in P_k(\rho),$$

for  $\alpha \geq 0, n > -p, 0 \leq \rho < p, k \geq 2$  and  $z \in E$ .



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## 2. Preliminary Results

**Lemma 2.1.** Let  $p(z) = 1 + b_1z + b_2z^2 + \dots \in P(\rho)$ . Then

$$\operatorname{Re} p(z) \geq 2\rho - 1 + \frac{2(1 - \rho)}{1 + |z|}.$$

*This result is well known.*

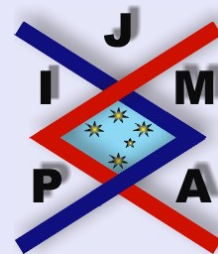
**Lemma 2.2 ([7]).** If  $p(z)$  is analytic in  $E$  with  $p(0) = 1$  and if  $\lambda_1$  is a complex number satisfying  $\operatorname{Re} \lambda_1 \geq 0$ , ( $\lambda_1 \neq 0$ ), then  $\operatorname{Re}\{p(z) + \lambda_1 zp'(z)\} > \beta$  ( $0 \leq \beta < p$ ) implies

$$\operatorname{Re} p(z) > \beta + (1 - \beta)(2\gamma_1 - 1),$$

where  $\gamma_1$  is given by

$$\gamma_1 = \int_0^1 (1 + t^{\operatorname{Re} \lambda_1})^{-1} dt.$$

**Lemma 2.3 ([9]).** If  $p(z)$  is analytic in  $E$ ,  $p(0) = 1$  and  $\operatorname{Re} p(z) > \frac{1}{2}$ ,  $z \in E$ , then for any function  $F$  analytic in  $E$ , the function  $p \star F$  takes values in the convex hull of the image  $E$  under  $F$ .



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### 3. Main Results

**Theorem 3.1.** Let  $f \in T_k(\alpha, p, n, \rho_1)$  and  $g \in T_k(\alpha, p, n, \rho_2)$ , and let  $F = f \star g$ . Then  $F \in T_k(\alpha, p, n, \rho_3)$  where

$$(3.1) \quad \rho_3 = 1 - 4(1 - \rho_1)(1 - \rho_2) \left[ 1 - \frac{n+p}{1-\alpha} \int_0^1 \frac{u^{\left(\frac{n+p}{1-\alpha}\right)-1}}{1+u} du \right].$$

This result is sharp.

*Proof.* Since  $f \in T_k(\alpha, p, n, \rho_1)$ , it follows that

$$H(z) = \left[ (1-\alpha) \frac{I_{n+p-1}f(z)}{z^p} + \alpha \frac{I_{n+p}f(z)}{z^p} \right] \in P_k(\rho_1),$$

and so using (1.3), we have

$$(3.2) \quad I_{n+p}f(z) = \frac{n+p}{1-\alpha} z^{-\left(\frac{n+p}{1-\alpha}\right)} \int_0^z t^{\frac{n+p}{1-\alpha}-1} H(t) dt.$$

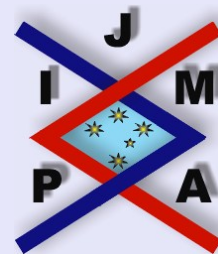
Similarly

$$(3.3) \quad I_{n+p}g(z) = \frac{n+p}{1-\alpha} z^{-\left(\frac{n+p}{1-\alpha}\right)} \int_0^z t^{\frac{n+p}{1-\alpha}-1} H^*(t) dt,$$

where  $H^* \in P_k(\rho_2)$ .

Using (3.1) and (3.2), we have

$$(3.4) \quad I_{n+p}F(z) = \frac{n+p}{1-\alpha} z^{-\left(\frac{n+p}{1-\alpha}\right)} \int_0^z t^{\frac{n+p}{1-\alpha}-1} Q(t) dt,$$



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where

$$\begin{aligned}
 Q(z) &= \left(\frac{k}{4} + \frac{1}{2}\right) q_1(z) - \left(\frac{k}{4} - \frac{1}{2}\right) q_2(z) \\
 (3.5) \quad &= \frac{n+p}{1-\alpha} z^{-\left(\frac{n+p}{1-\alpha}\right)} \int_0^z t^{\frac{n+p}{1-\alpha}-1} (H \star H^*)(t) dt.
 \end{aligned}$$

Now

$$\begin{aligned}
 H(z) &= \left(\frac{k}{4} + \frac{1}{2}\right) h_1(z) - \left(\frac{k}{4} - \frac{1}{2}\right) h_2(z) \\
 (3.6) \quad H(z)^* &= \left(\frac{k}{4} + \frac{1}{2}\right) h_1^*(z) - \left(\frac{k}{4} - \frac{1}{2}\right) h_2^*(z),
 \end{aligned}$$

where  $h_i \in P(\rho_1)$  and  $h_i^* \in P_k(\rho_2)$ ,  $i = 1, 2$ .

Since

$$p_i^*(z) = \frac{h_i^*(z) - \rho_2}{2(1 - \rho_2)} + \frac{1}{2} \in P\left(\frac{1}{2}\right), \quad i = 1, 2,$$

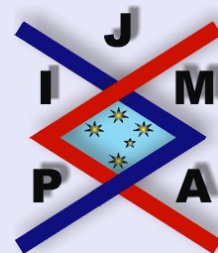
we obtain that  $(h_i \star p_i^*)(z) \in P(\rho_1)$ , by using the Herglotz formula.

Thus

$$(h_i \star h_i^*)(z) \in P(\rho_3)$$

with

$$(3.7) \quad \rho_3 = 1 - 2(1 - \rho_1)(1 - \rho_2).$$



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Using (3.4), (3.5), (3.6), (3.7) and Lemma 2.1, we have

$$\begin{aligned} \operatorname{Re} q_i(z) &= \frac{n+p}{1-\alpha} \int_0^1 u^{\frac{n+p}{1-\alpha}-1} \operatorname{Re}\{(h_i \star h_i^*)(uz)\} du \\ &\geq \frac{n+p}{1-\alpha} \int_0^1 u^{\frac{n+p}{1-\alpha}-1} \left(2\rho_3 - 1 + \frac{2(1-\rho_3)}{1+u|z|}\right) du \\ &> \frac{n+p}{1-\alpha} \int_0^1 u^{\frac{n+p}{1-\alpha}-1} \left(2\rho_3 - 1 + \frac{2(1-\rho_3)}{1+u}\right) du \\ &= 1 - 4(1-\rho_1)(1-\rho_2) \left[1 - \frac{n+p}{1-\alpha} \int_0^1 \frac{u^{\frac{n+p}{1-\alpha}-1}}{1+u} du\right]. \end{aligned}$$

From this we conclude that  $F \in T_k(\alpha, p, n, \rho_3)$ , where  $\rho_3$  is given by (3.1).

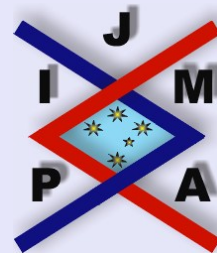
We discuss the sharpness as follows:

We take

$$\begin{aligned} H(z) &= \left(\frac{k}{4} + \frac{1}{2}\right) \frac{1 + (1-2\rho_1)z}{1-z} - \left(\frac{k}{4} - \frac{1}{2}\right) \frac{1 - (1-2\rho_1)z}{1+z}, \\ H^*(z) &= \left(\frac{k}{4} + \frac{1}{2}\right) \frac{1 + (1-2\rho_2)z}{1-z} - \left(\frac{k}{4} - \frac{1}{2}\right) \frac{1 - (1-2\rho_2)z}{1+z}. \end{aligned}$$

Since

$$\begin{aligned} \left(\frac{1 + (1-2\rho_1)z}{1-z}\right) \star \left(\frac{1 + (1-2\rho_2)z}{1-z}\right) \\ = 1 - 4(1-\rho_1)(1-\rho_2) + \frac{4(1-\rho_1)(1-\rho_2)}{1-z}, \end{aligned}$$



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it follows from (3.5) that

$$q_i(z) = \frac{n+p}{1-\alpha} \int_0^1 u^{\frac{n+p}{1-\alpha}-1} \left\{ 1 - 4(1-\rho_1)(1-\rho_2) + \frac{4(1-\rho_1)(1-\rho_2)}{1-uz} \right\} du$$

$$\longrightarrow 1 - 4(1-\rho_1)(1-\rho_2) \left\{ 1 - \frac{n+p}{1-\alpha} \int_0^1 \frac{u^{\frac{n+p}{1-\alpha}-1}}{1+u} du \right\} \text{ as } z \longrightarrow 1.$$

This completes the proof. □

We define  $J_c : \mathcal{A}(p) \longrightarrow \mathcal{A}(p)$  as follows:

$$(3.8) \quad J_c(f) = \frac{c+p}{z^c} \int_0^z t^{c-1} f(t) dt,$$

where  $c$  is real and  $c > -p$ .

**Theorem 3.2.** Let  $f \in T_k(\alpha, p, n, \rho)$  and  $J_c(f)$  be given by (3.8). If

$$(3.9) \quad \left[ (1-\alpha) \frac{I_{n+p} f(z)}{z^p} + \alpha \frac{I_{n+p} J_c(f)}{z^p} \right] \in P_k(\rho),$$

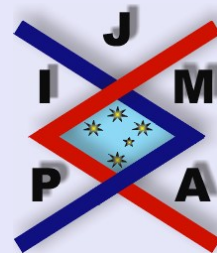
then

$$\left\{ \frac{I_{n+p} J_c(f)}{z^p} \right\} \in P_k(\gamma), \quad z \in E$$

and

$$(3.10) \quad \gamma = \rho(1-\rho)(2\sigma-1)$$

$$\sigma = \int_0^1 \left[ 1 + t^{\operatorname{Re} \frac{1-\alpha}{\lambda+p}} \right]^{-1} dt.$$



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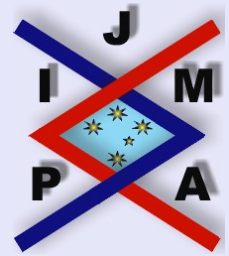


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*Proof.* From (3.8), we have

$$(c + p)I_{n+p}f(z) = cI_{n+p}J_c(f) + z(I_{n+p}J_c(f))'$$

Let

$$(3.11) \quad H_c(z) = \left(\frac{k}{4} + \frac{1}{2}\right) s_1(z) - \left(\frac{k}{4} - \frac{1}{2}\right) s_2(z) = \frac{I_{n+p}J_c(f)}{z^p}.$$

From (3.9), (3.10) and (3.11), we have

$$\left[ (1 - \alpha) \frac{I_{n+p}f(z)}{z^p} + \alpha \frac{I_{n+p}J_c(f)}{z^p} \right] = \left[ H_c(z) + \frac{1 - \alpha}{\lambda + p} zH'_c(z) \right]$$

and consequently

$$\left[ s_i(z) + \frac{1 - \alpha}{\lambda + p} z s'_i(z) \right] \in P(\rho), \quad i = 1, 2.$$

Using Lemma 2.2, we have  $\text{Re}\{s_i(z)\} > \gamma$  where  $\gamma$  is given by (3.10). Thus

$$H_c(z) = \frac{I_{n+p}J_c(f)}{z^p} \in P_k(\gamma)$$

and this completes the proof.  $\square$

Let

$$(3.12) \quad J_n(f(z)) := J_n(f) = \frac{n + p}{z^p} \int_0^z t^{n-1} f(t) dt.$$

Then

$$I_{n+p-1}J_n(f) = I_{n+p}(f),$$

and we have the following.

**Theorem 3.3.** Let  $f \in T_k(\alpha, p, n+1, \rho)$ . Then  $J_n(f) \in T_k(\alpha, p, n, \rho)$  for  $z \in E$ .

**Theorem 3.4.** Let  $\phi \in C_p$ , where  $C_p$  is the class of  $p$ -valent convex functions, and let  $f \in T_k(\alpha, p, n, \rho)$ . Then  $\phi \star f \in T_k(\alpha, p, n, \rho)$  for  $z \in E$ .

*Proof.* Let  $G = \phi \star f$ . Then

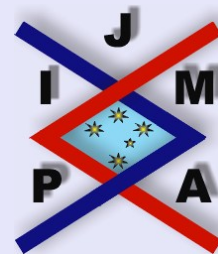
$$\begin{aligned} & (1 - \alpha) \frac{I_{n+p-1}G(z)}{z^p} + \alpha \frac{I_{n+p}G(z)}{z^p} \\ &= (1 - \alpha) \frac{I_{n+p-1}(\phi \star f)(z)}{z^p} + \alpha \frac{I_{n+p}(\phi \star f)(z)}{z^p} \\ &= \frac{\phi(z)}{z^p} \star \left[ (1 - \alpha) \frac{I_{n+p-1}f(z)}{z^p} + \alpha \frac{I_{n+p}f(z)}{z^p} \right] \\ &= \frac{\phi(z)}{z^p} \star H(z), \quad H \in P_k(\rho) \\ &= \left( \frac{k}{4} + \frac{1}{2} \right) \left\{ (p - \rho) \left( \frac{\phi(z)}{z^p} \star h_1(z) \right) + \rho \right\} \\ &\quad - \left( \frac{k}{4} - \frac{1}{2} \right) \left\{ (p - \rho) \left( \frac{\phi(z)}{z^p} \star h_2(z) \right) + \rho \right\}, \quad h_1, h_2 \in P. \end{aligned}$$

Since  $\phi \in C_p$ ,  $\operatorname{Re} \left\{ \frac{\phi(z)}{z^p} \right\} > \frac{1}{2}$ ,  $z \in E$  and so using Lemma 2.3, we conclude that  $G \in T_k(\alpha, p, n, \rho)$ .  $\square$

### 3.1. Applications

(1) We can write  $J_c(f)$  defined by (3.8) as

$$J_c(f) = \phi_c \star f,$$



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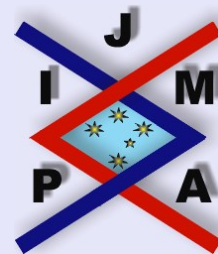


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where  $\phi_c$  is given by

$$\phi_c(z) = \sum_{m=p}^{\infty} \frac{p+c}{m+c} z^m, \quad (c > -p)$$

and  $\phi_c \in C_p$ . Therefore, from Theorem 3.4, it follows that  $J_c(f) \in T_k(\alpha, p, n, \rho)$ .

(2) Let  $J_n(f)$ , defined by (3.12), belong to  $T_k(\alpha, p, n, \rho)$ . Then  $f \in T_k(\alpha, p, n, \rho)$  for  $|z| < r_n = \frac{(1+n)}{2+\sqrt{3+n^2}}$ . In fact,  $J_n(f) = \Psi_n \star f$ , where

$$\begin{aligned} \Psi_n(z) &= z^p + \sum_{j=2}^{\infty} \frac{n+j-1}{n+1} z^{j+p-1} \\ &= \frac{n}{n+1} \cdot \frac{z^p}{1-z} + \frac{1}{n+1} \cdot \frac{z^p}{(1-z)^2} \end{aligned}$$

and  $\Psi_n \in C_p$  for

$$|z| < r_n = \frac{1+n}{2+\sqrt{3+n^2}}.$$

Now  $I_{n+p-1}J_n(f) = \Psi_n \star I_{n+p-1}f$ , and using Theorem 3.4, we obtain the result.

**Theorem 3.5.** For  $0 \leq \alpha_2 < \alpha_1$ ,  $T_k(\alpha_1, p, n, \rho) \subset T_k(\alpha_2, p, n, \rho)$ ,  $z \in E$ .

*Proof.* For  $\alpha_2 = 0$ , the proof is immediate. Let  $\alpha_2 > 0$  and let  $f \in T_k(\alpha_1, p, n, \rho)$ .

Then

$$\begin{aligned} & (1 - \alpha_2) \frac{I_{n+p-1}f(z)}{z^p} + \alpha_2 \frac{I_{n+p}f(z)}{z^p} \\ & + \frac{\alpha_2}{\alpha_1} \left[ \left( \frac{\alpha_1}{\alpha_2} - 1 \right) \frac{I_{n+p-1}f(z)}{z^p} + (1 - \alpha_1) \frac{I_{n+p-1}f(z)}{z^p} + \alpha_1 \frac{I_{n+p-1}f(z)}{z^p} \right] \\ & = \left( 1 - \frac{\alpha_2}{\alpha_1} \right) H_1(z) + \frac{\alpha_2}{\alpha_1} H_2(z), \quad H_1, H_2 \in P_k(\rho). \end{aligned}$$

Since  $P_k(\rho)$  is a convex set, we conclude that  $f \in T_k(\alpha_2, p, n, \rho)$  for  $z \in E$ .  $\square$

**Theorem 3.6.** Let  $f \in T_k(0, p, n, \rho)$ . Then  $f \in T_k(\alpha, p, n, \rho)$  for

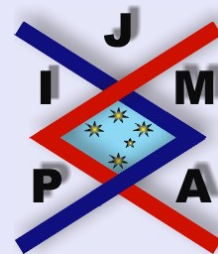
$$|z| < r_\alpha = \frac{1}{2\alpha + \sqrt{4\alpha^2 - 2\alpha + 1}}, \quad \alpha \neq \frac{1}{2}, \quad 0 < \alpha < 1.$$

*Proof.* Let

$$\begin{aligned} \Psi_\alpha(z) &= (1 - \alpha) \frac{z^p}{1 - z} + \alpha \frac{z^p}{(1 - z)^2} \\ &= z^p + \sum_{m=2}^{\infty} (1 + (m - 1)\alpha) z^{m+p-1}. \end{aligned}$$

$\Psi_\alpha \in C_p$  for

$$|z| < r_\alpha = \frac{1}{2\alpha + \sqrt{4\alpha^2 - 2\alpha + 1}} \quad \left( \alpha \neq \frac{1}{2}, \quad 0 < \alpha < 1 \right)$$



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We can write

$$\left[ (1 - \alpha) \frac{I_{n+p-1}f(z)}{z^p} + \alpha \frac{I_{n+p}f(z)}{z^p} \right] = \frac{\Psi_\alpha(z)}{z^p} \star \frac{I_{n+p-1}f(z)}{z^p}.$$

Applying Theorem 3.4, we see that  $f \in T_k(\alpha, p, n, \rho)$  for  $|z| < r_\alpha$ . □



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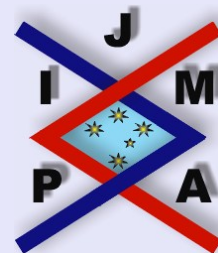
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