

ON STARLIKENESS AND CONVEXITY OF ANALYTIC FUNCTIONS SATISFYING A DIFFERENTIAL INEQUALITY

SUKHWINDER SINGH

Department of Applied Sciences
Baba Banda Singh Bahadur Engineering College
Fatehgarh Sahib -140407 (Punjab), INDIA.

EEmail: ss_billing@yahoo.co.in

SUSHMA GUPTA AND SUKHJIT SINGH

Department of Mathematics
Sant Longowal Institute of Engineering & Technology
Longowal-148106 (Punjab), INDIA.

EEmail: sushmagupta1@yahoo.com sukhjiti_d@yahoo.com

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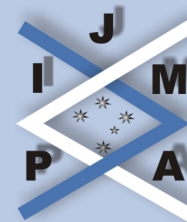
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Abstract: In the present paper, the authors investigate a differential inequality defined by multiplier transformation in the open unit disk $E = \{z : |z| < 1\}$. As consequences, sufficient conditions for starlikeness and convexity of analytic functions are obtained.



**Starlikeness and Convexity
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Sukhwinder Singh, Sushma Gupta
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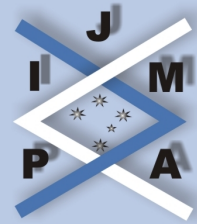
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1. Introduction

Let \mathcal{A}_p denote the class of functions of the form $f(z) = z^p + \sum_{k=p+1}^{\infty} a_k z^k$, $p \in \mathbb{N} = \{1, 2, \dots\}$, which are analytic in the open unit disc $E = \{z : |z| < 1\}$. We write $\mathcal{A}_1 = \mathcal{A}$. A function $f \in \mathcal{A}_p$ is said to be p -valent starlike of order α ($0 \leq \alpha < p$) in E if

$$\Re \left(\frac{z f'(z)}{f(z)} \right) > \alpha, \quad z \in E.$$

We denote by $S_p^*(\alpha)$, the class of all such functions. A function $f \in \mathcal{A}_p$ is said to be p -valent convex of order α ($0 \leq \alpha < p$) in E if

$$\Re \left(1 + \frac{z f''(z)}{f'(z)} \right) > \alpha, \quad z \in E.$$

Let $K_p(\alpha)$ denote the class of all those functions $f \in \mathcal{A}_p$ which are multivalently convex of order α in E . Note that $S_1^*(\alpha)$ and $K_1(\alpha)$ are, respectively, the usual classes of univalent starlike functions of order α and univalent convex functions of order α , $0 \leq \alpha < 1$, and will be denoted here by $S^*(\alpha)$ and $K(\alpha)$, respectively. We shall use S^* and K to denote $S^*(0)$ and $K(0)$, respectively which are the classes of univalent starlike (w.r.t. the origin) and univalent convex functions.

For $f \in \mathcal{A}_p$, we define the multiplier transformation $I_p(n, \lambda)$ as

$$(1.1) \quad I_p(n, \lambda) f(z) = z^p + \sum_{k=p+1}^{\infty} \left(\frac{k + \lambda}{p + \lambda} \right)^n a_k z^k, \quad (\lambda \geq 0, n \in \mathbb{Z}).$$

The operator $I_p(n, \lambda)$ has recently been studied by Aghalary et.al. [1]. Earlier, the operator $I_1(n, \lambda)$ was investigated by Cho and Srivastava [3] and Cho and Kim [2], whereas the operator $I_1(n, 1)$ was studied by Uralegaddi and Somanatha [11].

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$I_1(n, 0)$ is the well-known Sălăgean [10] derivative operator D^n , defined as: $D^n f(z) = z + \sum_{k=2}^{\infty} k^n a_k z^k$, $n \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$ and $f \in \mathcal{A}$.

A function $f \in \mathcal{A}_p$ is said to be in the class $S_n(p, \lambda, \alpha)$ for all z in E if it satisfies

$$(1.2) \quad \Re \left(\frac{I_p(n+1, \lambda) f(z)}{I_p(n, \lambda) f(z)} \right) > \frac{\alpha}{p},$$

for some α ($0 \leq \alpha < p, p \in \mathbb{N}$). We note that $S_0(1, 0, \alpha)$ and $S_1(1, 0, \alpha)$ are the usual classes $S^*(\alpha)$ and $K(\alpha)$ of starlike functions of order α and convex functions of order α , respectively.

In 1989, Owa, Shen and Obradović [8] obtained a sufficient condition for a function $f \in \mathcal{A}$ to belong to the class $S_n(1, 0, \alpha) = S_n(\alpha)$.

Recently, Li and Owa [4] studied the operator $I_1(n, 0)$.

In the present paper, we investigate the differential inequality

$$\Re \left(\frac{(1-\alpha)I_p(n+1, \lambda)f(z) + \alpha I_p(n+2, \lambda)f(z)}{(1-\beta)I_p(n, \lambda)f(z) + \beta I_p(n+1, \lambda)f(z)} \right) > M(\alpha, \beta, \gamma, \lambda, p)$$

where α and β are real numbers and $M(\alpha, \beta, \gamma, \lambda, p)$ is a certain real number given in Section 2, for starlikeness and convexity of $f \in \mathcal{A}_p$. We obtain sufficient conditions for $f \in \mathcal{A}_p$ to be a member of $S_n(p, \lambda, \gamma)$, for some γ ($0 \leq \gamma < p, p \in \mathbb{N}$). Many known results for starlikeness appear as corollaries to our main result and some new results regarding convexity of analytic functions are obtained.

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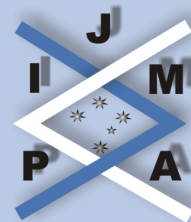
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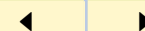
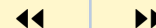
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2. Main Result

We shall make use of the following lemma of Miller and Mocanu to prove our result.

Lemma 2.1 ([6, 7]). *Let Ω be a set in the complex plane \mathbb{C} and let $\psi : \mathbb{C}^2 \times E \rightarrow \mathbb{C}$. For $u = u_1 + iu_2$, $v = v_1 + iv_2$, assume that ψ satisfies the condition $\psi(iu_2, v_1; z) \notin \Omega$, for all $u_2, v_1 \in \mathbb{R}$, with $v_1 \leq -(1 + u_2^2)/2$ and for all $z \in E$. If the function p , $p(z) = 1 + p_1z + p_2z^2 + \dots$, is analytic in E and if $\psi(p(z), zp'(z); z) \in \Omega$, then $\Re p(z) > 0$ in E .*

We, now, state and prove our main theorem.

Theorem 2.2. *Let $\alpha \geq 0$, $\beta \leq 1$, $\lambda \geq 0$ and $0 \leq \gamma < p$ be real numbers such that $\beta(1 - \frac{\gamma}{p}) < \frac{1}{2}$ and $\beta \leq \alpha$. If $f \in \mathcal{A}_p$ satisfies the condition*

$$(2.1) \quad \Re \left(\frac{(1 - \alpha)I_p(n + 1, \lambda)f(z) + \alpha I_p(n + 2, \lambda)f(z)}{(1 - \beta)I_p(n, \lambda)f(z) + \beta I_p(n + 1, \lambda)f(z)} \right) > M(\alpha, \beta, \gamma, \lambda, p),$$

then

$$\Re \left(\frac{I_p(n + 1, \lambda)f(z)}{I_p(n, \lambda)f(z)} \right) > \frac{\gamma}{p}$$

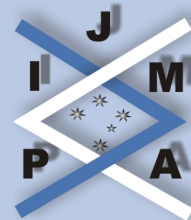
i.e., $f(z) \in S_n(p, \lambda, \gamma)$ where,

$$M(\alpha, \beta, \gamma, \lambda, p) = \frac{\frac{(1-\alpha)\gamma}{p} + \frac{\alpha\gamma^2}{p^2} - \frac{\alpha(1-\frac{\gamma}{p})}{2(p+\lambda)}}{1 - \beta \left(1 - \frac{\gamma}{p}\right)}.$$

Proof. Since $0 \leq \gamma < p$, let us write $\mu = \frac{\gamma}{p}$. Thus, we have $0 \leq \mu < 1$.

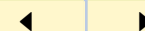
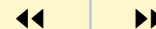
Now we define,

$$(2.2) \quad \frac{I_p(n + 1, \lambda)f(z)}{I_p(n, \lambda)f(z)} = \mu + (1 - \mu)r(z), \quad z \in E.$$



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Therefore $r(z)$ is analytic in E and $r(0) = 1$.

Differentiating (2.2) logarithmically, we obtain

$$(2.3) \quad \frac{zI_p'(n+1, \lambda)f(z)}{I_p(n+1, \lambda)f(z)} - \frac{zI_p'(n, \lambda)f(z)}{I_p(n, \lambda)f(z)} = \frac{(1-\mu)zr'(z)}{\mu + (1-\mu)r(z)}, \quad z \in E.$$

Using the fact that

$$zI_p'(n, \lambda)f(z) = (p + \lambda)I_p(n+1, \lambda)f(z) - \lambda I_p(n, \lambda)f(z).$$

Thus (2.3) reduces to

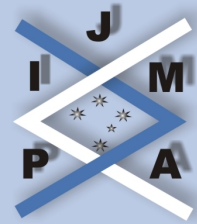
$$\frac{I_p(n+2, \lambda)f(z)}{I_p(n+1, \lambda)f(z)} = \mu + (1-\mu)r(z) + \frac{(1-\mu)zr'(z)}{(\lambda+p)[\mu + (1-\mu)r(z)]}.$$

Now, a simple calculation yields

$$\begin{aligned} & \frac{(1-\alpha)I_p(n+1, \lambda)f(z) + \alpha I_p(n+2, \lambda)f(z)}{(1-\beta)I_p(n, \lambda)f(z) + \beta I_p(n+1, \lambda)f(z)} \\ &= \frac{(1-\alpha) + \alpha \left(\mu + (1-\mu)r(z) + \frac{(1-\mu)zr'(z)}{(\lambda+p)[\mu + (1-\mu)r(z)]} \right)}{(1-\beta) + \beta[\mu + (1-\mu)r(z)]} [\mu + (1-\mu)r(z)] \\ &= \frac{(1-\alpha)[\mu + (1-\mu)r(z)] + \alpha \left([\mu + (1-\mu)r(z)]^2 + \frac{(1-\mu)zr'(z)}{(\lambda+p)} \right)}{(1-\beta) + \beta[\mu + (1-\mu)r(z)]} \\ (2.4) \quad &= \psi(r(z), zr'(z); z) \end{aligned}$$

where,

$$\psi(u, v; z) = \frac{(1-\alpha)[\mu + (1-\mu)u] + \alpha \left((\mu + (1-\mu)u)^2 + \frac{(1-\mu)v}{(\lambda+p)} \right)}{(1-\beta) + \beta[\mu + (1-\mu)u]}.$$



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Let $u = u_1 + iu_2$ and $v = v_1 + iv_2$, where u_1, u_2, v_1, v_2 are reals with $v_1 \leq -\frac{1+u_2^2}{2}$.
Then, we have

$$\begin{aligned}
 & \Re \psi(iu_2, v_1; z) \\
 &= \frac{[(1-\alpha)\mu + \alpha\mu^2][1 - \beta(1-\mu)]}{[1 - \beta(1-\mu)]^2 + \beta^2(1-\mu)^2u_2^2} \\
 &+ \frac{(1-\mu)^2[(1-\alpha)\beta - \alpha(1-\beta(1-\mu))] + 2\alpha\beta\mu u_2^2 + \frac{\alpha(1-\mu)[1-\beta(1-\mu)]v_1}{p+\lambda}}{[1 - \beta(1-\mu)]^2 + \beta^2(1-\mu)^2u_2^2} \\
 &\leq \frac{\left[(1-\alpha)\mu + \alpha\mu^2 - \frac{\alpha(1-\mu)}{2(\lambda+p)} \right] [1 - \beta(1-\mu)]}{[1 - \beta(1-\mu)]^2 + \beta^2(1-\mu)^2u_2^2} \\
 &+ \frac{\left[(1-\mu)^2[(1-\alpha)\beta - \alpha(1-\beta(1-\mu))] + 2\alpha\beta\mu \right] - \frac{\alpha(1-\mu)[1-\beta(1-\mu)]}{2(p+\lambda)}}{[1 - \beta(1-\mu)]^2 + \beta^2(1-\mu)^2u_2^2} u_2^2 \\
 &= \frac{A + Bu_2^2}{[1 - \beta(1-\mu)]^2 + \beta^2(1-\mu)^2u_2^2} \\
 &= \phi(u_2), \quad \text{say} \\
 (2.5) \quad & \leq \max \phi(u_2)
 \end{aligned}$$

where,

$$A = \left[(1-\alpha)\mu + \alpha\mu^2 - \frac{\alpha(1-\mu)}{2(\lambda+p)} \right] [1 - \beta(1-\mu)]$$

and

$$B = (1-\mu)^2[(1-\alpha)\beta - \alpha(1-\beta(1-\mu))] + 2\alpha\beta\mu - \frac{\alpha(1-\mu)[1 - \beta(1-\mu)]}{2(p+\lambda)}.$$



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It can be easily verified that $\phi'(u_2) = 0$ implies that $u_2 = 0$. Under the given conditions, we observe that $\phi''(0) < 0$. Therefore,

$$(2.6) \quad \max \phi(u_2) = \phi(0) = M(\alpha, \beta, \gamma, \lambda, p).$$

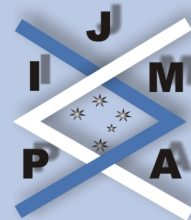
Let

$$\Omega = \{w : \Re w > M(\alpha, \beta, \gamma, \lambda, p)\}.$$

Then from (2.1) and (2.4), we have $\psi(r(z), zr'(z); z) \in \Omega$ for all $z \in E$, but $\psi(iu_2, v_1; z) \notin \Omega$, in view of (2.5) and (2.6). Therefore, by Lemma 2.1 and (2.2), we conclude that

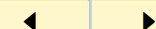
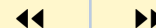
$$\Re \left(\frac{I_p(n+1, \lambda)f(z)}{I_p(n, \lambda)f(z)} \right) > \frac{\gamma}{p}.$$

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3. Corollaries

By taking $p = 1$ and $\lambda = 0$ in Theorem 2.2. We have the following corollary.

Corollary 3.1. Let $\alpha \geq 0$, $\beta \leq 1$ and $0 \leq \gamma < 1$ be real numbers such that $\beta(1 - \gamma) < \frac{1}{2}$ and $\beta \leq \alpha$. If $f \in \mathcal{A}$ satisfies the condition

$$\Re \left(\frac{(1 - \alpha)D^{n+1}f(z) + \alpha D^{n+2}f(z)}{(1 - \beta)D^n f(z) + \beta D^{n+1}f(z)} \right) > M(\alpha, \beta, \gamma, 0, 1),$$

then

$$\Re \frac{D^{n+1}f(z)}{D^n f(z)} > \gamma,$$

i.e. $f(z) \in S_n(\gamma)$, where,

$$M(\alpha, \beta, \gamma, 0, 1) = \frac{(1 - \alpha)\gamma + \alpha\gamma^2 - \frac{\alpha(1-\gamma)}{2}}{1 - \beta(1 - \gamma)}.$$

By taking $p = 1$, $n = 0$ and $\lambda = 0$ in Theorem 2.2. We have the following corollary.

Corollary 3.2. Let $\alpha \geq 0$, $\beta \leq 1$ and $0 \leq \gamma < 1$ be real numbers such that $\beta(1 - \gamma) < \frac{1}{2}$ and $\beta \leq \alpha$. If $f \in \mathcal{A}$ satisfies the condition

$$\Re \left(\frac{zf'(z) + \alpha z^2 f''(z)}{(1 - \beta)f(z) + \beta z f'(z)} \right) > M(\alpha, \beta, \gamma, 0, 1),$$

then

$$\Re \frac{zf'(z)}{f(z)} > \gamma,$$



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i.e. $f(z) \in S^*(\gamma)$, where,

$$M(\alpha, \beta, \gamma, 0, 1) = \frac{(1 - \alpha)\gamma + \alpha\gamma^2 - \frac{\alpha(1-\gamma)}{2}}{1 - \beta(1 - \gamma)}.$$

By taking $p = 1, n = 0, \lambda = 0$ and $\beta = 1$ in Theorem 2.2. We have the following corollary.

Corollary 3.3. Let $\alpha \geq 1$ and $\frac{1}{2} < \gamma < 1$ be real numbers. If $f \in \mathcal{A}$ satisfies the condition

$$\Re \left(1 + \alpha \frac{zf''(z)}{f'(z)} \right) > M(\alpha, 1, \gamma, 0, 1),$$

then

$$\Re \frac{zf'(z)}{f(z)} > \gamma,$$

i.e. $f(z) \in S^*(\gamma)$, where

$$M(\alpha, 1, \gamma, 0, 1) = 1 - \alpha(1 - \gamma) \left(1 + \frac{1}{2\gamma} \right)$$

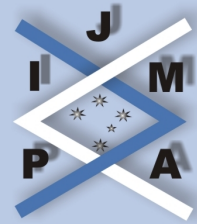
By taking $p = 1, n = 0, \lambda = 0$ and $\beta = 0$ in Theorem 2.2, we have the following result of Ravichandran et. al. [9].

Corollary 3.4. Let $\alpha \geq 0$ and $0 \leq \gamma < 1$ be real numbers. If $f \in \mathcal{A}$ satisfies the condition

$$\Re \frac{zf'(z)}{f(z)} \left(1 + \alpha \frac{zf''(z)}{f'(z)} \right) > M(\alpha, 0, \gamma, 0, 1),$$

then

$$\Re \frac{zf'(z)}{f(z)} > \gamma,$$



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i.e. $f(z) \in S^*(\gamma)$, where,

$$M(\alpha, 0, \gamma, 0, 1) = (1 - \alpha)\gamma + \alpha\gamma^2 - \frac{\alpha(1 - \gamma)}{2}.$$

Remark 1. In the case when $\gamma = \frac{\alpha}{2}$, Corollary 3.4 reduces to the result of Li and Owa [5].

By taking $p = 1, n = 0$ and $\lambda = 1$ in Theorem 2.2, we have the following corollary.

Corollary 3.5. Let $\alpha \geq 0, \beta \leq 1$ and $0 \leq \gamma < 1$ be real numbers such that $\beta(1 - \gamma) < \frac{1}{2}$ and $\beta \leq \alpha$. If $f \in \mathcal{A}$ satisfies the condition

$$\Re \frac{1}{2} \left(\frac{(2 - \alpha)f(z) + (2 + \alpha)zf'(z) + \alpha z^2 f''(z)}{(2 - \beta)f(z) + \beta z f'(z)} \right) > M(\alpha, \beta, \gamma, 1, 1),$$

then

$$\Re \frac{1}{2} \left(1 + \frac{zf'(z)}{f(z)} \right) > \gamma,$$

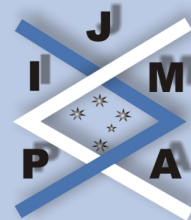
where,

$$M(\alpha, \beta, \gamma, 1, 1) = \frac{(1 - \alpha)\gamma + \alpha\gamma^2 - \frac{\alpha(1 - \gamma)}{4}}{1 - \beta(1 - \gamma)}.$$

By taking $p = 1, n = 1$ and $\lambda = 0$ in Theorem 2.2, we have the following corollary.

Corollary 3.6. Let $\alpha \geq 0, \beta \leq 1$ and $0 \leq \gamma < 1$ be real numbers such that $\beta(1 - \gamma) < \frac{1}{2}$ and $\beta \leq \alpha$. If $f \in \mathcal{A}$ satisfies the condition

$$\Re \left(\frac{zf'(z) + (2\alpha + 1)z^2 f''(z) + \alpha z^3 f'''(z)}{zf'(z) + \beta z^2 f''(z)} \right) > M(\alpha, \beta, \gamma, 0, 1),$$



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then

$$\Re \left(1 + \frac{zf''(z)}{f'(z)} \right) > \gamma,$$

i.e. $f(z) \in K(\gamma)$, where,

$$M(\alpha, \beta, \gamma, 0, 1) = \frac{(1 - \alpha)\gamma + \alpha\gamma^2 - \frac{\alpha(1-\gamma)}{2}}{1 - \beta(1 - \gamma)}.$$

By taking $p = 1, n = 1, \lambda = 0$ and $\beta = 0$ in Theorem 2.2, we have the following corollary.

Corollary 3.7. Let $\alpha \geq 0$ and $0 \leq \gamma < 1$ be real numbers. If $f \in \mathcal{A}$ satisfies the condition

$$\Re \left(1 + (2\alpha + 1) \frac{zf''(z)}{f'(z)} + \alpha \frac{z^2 f'''(z)}{f'(z)} \right) > M(\alpha, 0, \gamma, 0, 1),$$

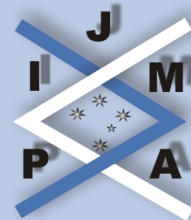
then

$$\Re \left(1 + \frac{zf''(z)}{f'(z)} \right) > \gamma,$$

i.e., $f(z) \in K(\gamma)$, where,

$$M(\alpha, 0, \gamma, 0, 1) = (1 - \alpha)\gamma + \alpha\gamma^2 - \frac{\alpha(1 - \gamma)}{2}.$$

Remark 2. In the main result, the real number $M(\alpha, \beta, \gamma, \lambda, p)$ may not be the best possible as authors have not obtained the extremal function for it. The problem is still open for the best possible real number $M(\alpha, \beta, \gamma, \lambda, p)$.



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