



A GEOMETRIC INEQUALITY INVOLVING A MOBILE POINT IN THE PLACE OF THE TRIANGLE

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Abstract: By using **Bottema's inequality** and several identities in triangles, we prove a weighted inequality concerning the distances between a mobile point P and three vertexes A, B, C of $\triangle ABC$. As an application, a conjecture with regard to Fermat's sum $PA + PB + PC$ is proved.

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Dedicatory: Dedicated to Professor Lu Yang on the occasion of his 73rd birthday.

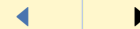
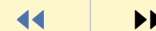
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1. Introduction and Main Results

For $\triangle ABC$, let a, b, c denote the side-lengths, A, B, C the angles, Δ the area, p the semi-perimeter, R the circumradius and r the inradius, respectively. In addition, supposing that P is a mobile point in the plane containing $\triangle ABC$, let PA, PB, PC denote the distances between P and A, B, C , respectively. We will customarily use the cyclic symbol, that is: $\sum f(a) = f(a) + f(b) + f(c)$, $\sum f(a, b) = f(a, b) + f(b, c) + f(c, a)$, $\prod f(a) = f(a)f(b)f(c)$, etc.

The following inequality can be easily proved by making use of **Bottema's inequality**:

$$(1.1) \quad (PB + PC) \cos \frac{A}{2} + (PC + PA) \cos \frac{B}{2} + (PA + PB) \cos \frac{C}{2} \\ \geq p \cdot \frac{p^2 + 2Rr + r^2}{4R^2}.$$

Here we choose to omit the details. From inequality (1.1) and the following known inequality (1.2) and identity (1.3) (see [3, 4, 6]):

$$(1.2) \quad PA \cos \frac{A}{2} + PB \cos \frac{B}{2} + PC \cos \frac{C}{2} \geq p,$$

and

$$(1.3) \quad q_1 = \cos^2 \frac{B-C}{2} + \cos^2 \frac{C-A}{2} + \cos^2 \frac{A-B}{2} = \frac{p^2 + 4R^2 + 2Rr + r^2}{4R^2},$$

we easily get

$$(1.4) \quad PA + PB + PC \geq p \cdot \frac{\cos^2 \frac{B-C}{2} + \cos^2 \frac{C-A}{2} + \cos^2 \frac{A-B}{2}}{\cos \frac{A}{2} + \cos \frac{B}{2} + \cos \frac{C}{2}}.$$



Considering the refinement of inequality (1.4), Chu [2] posed a conjecture as follows.

Corollary 1.1. *For any $\triangle ABC$,*

$$(1.5) \quad PA + PB + PC \geq p \cdot \frac{\cos \frac{B-C}{2} + \cos \frac{C-A}{2} + \cos \frac{A-B}{2}}{\cos \frac{A}{2} + \cos \frac{B}{2} + \cos \frac{C}{2}}.$$

The main object of this paper is to prove Conjecture 1.1, which is easily seen to follow from the following stronger result.

Theorem 1.2. *In $\triangle ABC$, we have*

$$(1.6) \quad (PB + PC) \cos \frac{A}{2} + (PC + PA) \cos \frac{B}{2} + (PA + PB) \cos \frac{C}{2} \\ \geq p \cdot \left[\cos \frac{B-C}{2} + \cos \frac{C-A}{2} + \cos \frac{A-B}{2} - 1 \right].$$

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2. Preliminary Results

In order to prove our main result, we shall require the following four lemmas.

Lemma 2.1. *In $\triangle ABC$, we have that*

$$(2.1) \quad q_2 = \cos \frac{B-C}{2} \cdot \cos \frac{C-A}{2} \cdot \cos \frac{A-B}{2} = \frac{p^2 + 2Rr + r^2}{8R^2},$$

$$(2.2) \quad \cos \frac{A}{2} \cdot \cos \frac{B}{2} \cdot \cos \frac{C}{2} = \frac{p}{4R},$$

$$(2.3) \quad q_3 = \sum \cos \frac{B}{2} \cos \frac{C}{2} \cos \frac{B-C}{2} (b^2 + c^2 - a^2) \\ = \frac{p^4 + 2Rrp^2 - r(2R+r)(4R+r)^2}{4R^2},$$

$$(2.4) \quad q_4 = \frac{1}{2} \sum \left(\cos^2 \frac{B}{2} + \cos^2 \frac{C}{2} \right) (b^2 + c^2 - a^2) \\ = \frac{(2R+3r)p^2 - r(4R+r)^2}{2R},$$

and

$$(2.5) \quad Q = \sum \left(\cos \frac{B-C}{2} - \cos \frac{C-A}{2} \cos \frac{A-B}{2} \right) \\ = q_1 - 3q_2 - \frac{p\Delta^4 \prod (b-c)^2}{a^2 b^2 c^2 \prod (X+x)},$$



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where

$$(2.6) \quad X = a\sqrt{bc(p-b)(p-c)}, \quad Y = b\sqrt{ca(p-c)(p-a)},$$

$$Z = c\sqrt{ab(p-a)(p-b)},$$

and

$$(2.7) \quad x = (b+c)(p-b)(p-c), \quad y = (c+a)(p-c)(p-a),$$

$$z = (a+b)(p-a)(p-b).$$

Proof. The proofs of identities (2.1) and (2.2) were given in [6]. Now, we present the proofs of identities (2.3) – (2.5). By utilizing the formulas

$$\cos \frac{A}{2} = \sqrt{\frac{p(p-a)}{bc}}, \quad \sin \frac{A}{2} = \sqrt{\frac{(p-b)(p-c)}{bc}},$$

$$\cos \frac{B-C}{2} = \frac{b+c}{a} \sqrt{\frac{(p-b)(p-c)}{bc}},$$

and (see [5, pp.52])

$$\prod a = 4Rrp, \quad \sum a = 2p \quad \text{and} \quad \sum bc = p^2 + 4Rr + r^2,$$

we get that

$$q_3 = p \sum \frac{(b+c)(p-b)(p-c)}{a^2bc} (b^2 + c^2 - a^2)$$

$$= \frac{p}{4a^2b^2c^2} \sum bc(b+c)(c+a-b)(a+b-c)(b^2 + c^2 - a^2)$$

$$\begin{aligned}
&= \frac{p}{4a^2b^2c^2} [6(ab + bc + ca)^2(a + b + c)^3 - 8(ab + bc + ca)^3(a + b + c) \\
&\quad - (ab + bc + ca)(a + b + c)^5 - 2abc(ab + bc + ca)(a + b + c)^2 \\
&\quad + 8abc(ab + bc + ca)^2 - abc(a + b + c)^4 - 4(a + b + c)a^2b^2c^2] \\
&= \frac{p^4 + 2Rrp^2 - r(2R + r)(4R + r)^2}{4R^2}
\end{aligned}$$

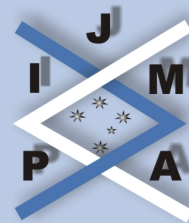
and

$$\begin{aligned}
&\frac{1}{2} \sum \left(\cos^2 \frac{B}{2} + \cos^2 \frac{C}{2} \right) (b^2 + c^2 - a^2) \\
&= \sum a^2 \cos^2 \frac{A}{2} \\
&= \frac{p}{abc} \left(p \sum a^3 - \sum a^4 \right) \\
&= \frac{p}{abc} \left[\frac{5}{2}(ab + bc + ca)(a + b + c)^2 - 2(ab + bc + ca)^2 \right. \\
&\quad \left. - \frac{1}{2}(a + b + c)^4 - \frac{5}{2}(a + b + c)abc \right] \\
&= \frac{(2R + 3r)p^2 - r(4R + r)^2}{2R}.
\end{aligned}$$

Thus, identities (2.3) and (2.4) hold true.

With (1.3), (2.1) and the formulas of half-angles, we obtain that

$$\begin{aligned}
q_1 - 3q_2 &= \frac{-p^2 + 8R^2 - 2Rr - r^2}{8R^2} \\
&= \frac{1}{a^2b^2c^2} \sum x [bc(b + c) - (c + a)(a + b)(s - a)],
\end{aligned}$$



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and

$$Q = \sum \left(\cos \frac{B-C}{2} - \cos \frac{C-A}{2} \cos \frac{A-B}{2} \right) \\ = \frac{\sum X [bc(b+c) - (c+a)(a+b)(s-a)]}{a^2b^2c^2}.$$

It is easy to see that

$$X - x = \frac{\Delta^2 (b-c)^2}{X+x}, \quad Y - y = \frac{\Delta^2 (c-a)^2}{Y+y}, \quad \text{and} \quad Z - z = \frac{\Delta^2 (a-b)^2}{Z+z}.$$

Then

$$a^2b^2c^2[Q - (q_1 - 3q_2)] \\ = \sum [bc(b+c) - (c+a)(a+b)(p-a)](X-x) \\ = \sum [bc(b+c) - (c+a)(a+b)(p-a)] \frac{\Delta^2 (b-c)^2}{X+x} \\ = \sum p\Delta^2 \frac{(a-b)(a-c)(b-c)^2}{(X+x)}.$$

Therefore,

$$Q - (q_1 - 3q_2) = \sum p\Delta^2 \frac{(a-b)(a-c)(b-c)^2}{a^2b^2c^2(X+x)} \\ = \frac{p\Delta^2(a-b)(a-c)(b-c)}{a^2b^2c^2} \sum \frac{b-c}{X+x},$$



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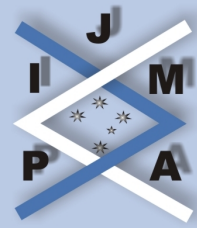
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where

$$\begin{aligned}
 & \frac{b-c}{X+x} + \frac{c-a}{Y+y} + \frac{a-b}{Z+z} \\
 &= \frac{(b-c)(Y-X+y-x)}{(X+x)(Y+y)} + \frac{(a-b)(Y-Z+y-z)}{(Z+z)(Y+y)} \\
 &= \frac{p(p-c)(b-c)(b-a)}{(X+x)(Y+y)} + \frac{pabc(p-c)(b-c)(b-a)}{(X+x)(Y+y)(X+Y)} \\
 & \quad + \frac{p(p-a)(a-b)(b-c)}{(Z+z)(Y+y)} + \frac{pabc(p-a)(a-b)(b-c)}{(Z+z)(Y+y)(Z+Y)} \\
 &= \frac{p(b-c)(a-b)}{\prod(X+x)} [(p-a)(X+x) - (p-c)(Z+z)] \\
 & \quad + \frac{pabc(b-c)(a-b)}{(X+Y)(Y+Z)\prod(X+x)} \\
 & \quad \times [(p-a)(X+x)(X+Y) - (p-c)(Z+z)(Y+Z)] \\
 &= \frac{-\Delta^2(b-c)(a-b)(a-c)}{\prod(X+x)} \\
 & \quad + \frac{p(b-c)(a-b)(a-c)}{(Z+X)\prod(X+x)} \left[abc \prod(p-a) - ca(p-b)Y \right] \\
 & \quad + \frac{pabc(b-c)(a-b)(a-c)}{(X+Y)(Z+Y)\prod(X+x)} \left[abc \prod(p-a) - Y \prod(p-a) \right] \\
 & \quad + \frac{abc(Y-abc)\prod(p-a)}{Z+X} \\
 & \quad + \frac{abc(pb+ca)(p-b)\prod(p-a) - ca(p-b)Y\prod(p-a)}{Z+X} \Big]
 \end{aligned}$$



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$$\begin{aligned}
 &= \frac{-\Delta^2(b-c)(a-b)(a-c)}{\prod(X+x)} + \frac{pabc(b-c)(a-b)(a-c)}{(X+Y)(Z+Y)\prod(X+x)} \\
 &\quad \cdot \left\{ \left[\prod(p-a) - (p-b)\sqrt{ca(p-c)(p-a)} \right] (X+Y)(Y+Z) \right. \\
 &\quad \left. + abc \prod(p-a) \left[Y - abc + pb(p-b) + ca(p-b) - (p-b)\sqrt{ca(p-c)(p-a)} \right] \right. \\
 &\quad \left. + (Z+X)(abc-Y) \prod(p-a) \right\} \\
 &= \frac{-\Delta^2(b-c)(a-b)(a-c)}{\prod(X+x)},
 \end{aligned}$$

which implies the assertion (2.5). □

Lemma 2.2. For any $\triangle ABC$,

$$(2.8) \quad \sqrt{\cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2} \left(\cos \frac{A}{2} + \cos \frac{B}{2} + \cos \frac{C}{2} \right)} \geq \frac{p}{2R}.$$

Proof. From **Euler's inequality** $R \geq 2r$, $abc = 4Rrp$, $a + b + c = 2p$ and the law of sines, we obtain that

$$(2.9) \quad 2R^2p \geq 4Rrp \iff R^2(a+b+c) \geq abc \\ \iff \sin A + \sin B + \sin C \geq 4 \sin A \sin B \sin C.$$

Taking

$$A \rightarrow \frac{\pi - A}{2}, \quad B \rightarrow \frac{\pi - B}{2}, \quad \text{and} \quad C \rightarrow \frac{\pi - C}{2},$$

we easily get

$$(2.10) \quad \cos \frac{A}{2} + \cos \frac{B}{2} + \cos \frac{C}{2} \geq 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}.$$

Inequality (2.8) follows immediately in view of (2.10) and (2.2). □

Lemma 2.3. *In $\triangle ABC$, we have*

$$(2.11) \quad \sum \cos \frac{B}{2} \cos \frac{C}{2} (b^2 + c^2 - a^2) \geq \frac{p^4 + 2Rrp^2 - r(2R+r)(4R+r)^2}{4R^2}.$$

Proof. By employing (2.3) and the formulas of half-angles, inequality (2.11) is equivalent to

$$(2.12) \quad \begin{aligned} \sum \cos \frac{B}{2} \cos \frac{C}{2} (b^2 + c^2 - a^2) \\ \geq \sum \cos \frac{B}{2} \cos \frac{C}{2} \cos \frac{B-C}{2} (b^2 + c^2 - a^2), \end{aligned}$$

or

$$(2.13) \quad \sum \frac{(b^2 + c^2 - a^2)}{a^2bc} \left[a\sqrt{bc(s-b)(s-c)} - (b+c)(s-b)(s-c) \right] \geq 0,$$

that is

$$(2.14) \quad \sum \frac{\Delta^2}{abc} \cdot \frac{(b^2 + c^2 - a^2)}{a(X+x)} (b-c)^2 \geq 0,$$

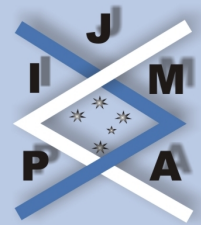
where X, Y, Z and x, y, z are given, just as in the proof of Lemma 2.1, by (2.6) and (2.7), respectively.

Without loss of generality, we can assume that $a \geq b \geq c$ to obtain

$$a(X+x) \geq b(Y+y) \geq c(Z+z),$$

and

$$(a-c)^2 \geq (b-c)^2,$$



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and thus

$$\frac{(b-c)^2}{a(X+x)} \leq \frac{(c-a)^2}{b(Y+y)}.$$

Hence, in order to prove inequality (2.14), we only need to prove that

$$(2.15) \quad \frac{\Delta^2}{abc} \left[\frac{(b^2 + c^2 - a^2)}{a(X+x)}(b-c)^2 + \frac{(c^2 + a^2 - b^2)}{b(Y+y)}(c-a)^2 \right] \geq 0.$$

We readily arrive at the following result for $a \geq b \geq c$,

$$\begin{aligned} & \frac{\Delta^2}{abc} \left[\frac{(b^2 + c^2 - a^2)}{a(X+x)}(b-c)^2 + \frac{(c^2 + a^2 - b^2)}{b(Y+y)}(c-a)^2 \right] \\ & \geq \frac{\Delta^2}{abc} (b^2 + c^2 - a^2 + c^2 + a^2 - b^2) \frac{(c-a)^2}{b(Y+y)} \\ & = 2c^2 \cdot \frac{\Delta^2}{abc} \cdot \frac{(c-a)^2}{b(Y+y)} \geq 0. \end{aligned}$$

This shows that the inequality (2.15) or (2.11) holds true. The proof of Lemma 2.3 is thus complete. \square

Lemma 2.4 (Bottema's inequality, see [1, pp. 118, Theorem 12.56]). *Let Δ' denote the area of $\triangle A'B'C'$, and a', b', c' the side-lengths of $\triangle A'B'C'$, respectively. Then*

$$(2.16) \quad (a'PA + b'PB + c'PC)^2 \geq \frac{1}{2}[a'^2(b^2 + c^2 - a^2) + b'^2(c^2 + a^2 - b^2) + c'^2(a^2 + b^2 - c^2)] + 8\Delta\Delta'.$$



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3. The Proof of Theorem 1.2

Proof. It is easy to show that

$$a' = \cos \frac{B}{2} + \cos \frac{C}{2}, \quad b' = \cos \frac{C}{2} + \cos \frac{A}{2}, \quad \text{and}$$

$$c' = \cos \frac{A}{2} + \cos \frac{B}{2}$$

are three side-lengths of a certain triangle. By using **Bottema's inequality** (2.16), in order to prove inequality (1.6), we only need to prove that

$$8\Delta \sqrt{\prod \cos \frac{A}{2} \sum \cos \frac{A}{2}} + \frac{1}{2} \sum \left(\cos \frac{B}{2} + \cos \frac{C}{2} \right)^2 (b^2 + c^2 - a^2)$$

$$\geq p^2 \left[\sum \cos \frac{B-C}{2} - 1 \right]^2$$

or

$$(3.1) \quad 8\Delta \sqrt{\prod \cos \frac{A}{2} \sum \cos \frac{A}{2}} + q_4$$

$$+ \sum \cos \frac{B}{2} \cos \frac{C}{2} (b^2 + c^2 - a^2) + 2p^2 Q \geq p^2 (q_1 + 1).$$

With identities (1.3), (2.4), (2.5), together with Lemma 2.2 and Lemma 2.3, in order to prove inequality (3.1), we only need to prove that

$$8\Delta \cdot \frac{p}{2R} + \frac{(2R+3r)p^2 - r(4R+r)^2}{2R} + \frac{p^4 + 2Rrp^2 - r(2R+r)(4R+r)^2}{4R^2}$$



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$$+ 2p^2 \left[\frac{-p^2 + 8R^2 - 2Rr - r^2}{8R^2} - \frac{p\Delta^4 \prod (b-c)^2}{a^2b^2c^2 \prod (X+x)} \right] \geq p^2 \left(\frac{p^2 + 4R^2 + 2Rr + r^2}{4R^2} + 1 \right)$$

or

$$(3.2) \quad \frac{-p^4 + (4R^2 + 20Rr - 2r^2)p^2 - r(4R+r)^3}{4R^2} \geq \frac{2p^3\Delta^4 \prod (b-c)^2}{a^2b^2c^2 \prod (X+x)}.$$

From the known identities (see [5])

$$\Delta = rp \text{ and}$$

$$(b-c)^2(c-a)^2(a-b)^2 = 4r^2[-p^4 + (4R^2 + 20Rr - 2r^2)p^2 - r(4R+r)^3],$$

inequality (3.2) is equivalent to

$$(3.3) \quad \prod (X+x) \geq 2r^4p^5.$$

For $X \geq x$, and with the following two known identities (see [5, pp.53])

$$\prod (b+c) = 2p(p^2 + 2Rr + r^2), \quad \prod (p-a) = r^2p,$$

we obtain

$$\begin{aligned} \prod (X+x) &\geq 8 \prod x = 8 \prod (b+c) \prod (p-a)^2 \\ &= 16r^4p^3(p^2 + 2Rr + r^2) > 16r^4p^5 > 2r^4p^5. \end{aligned}$$

Therefore, inequality (3.3) holds. This completes the proof of Theorem 1.2. \square



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4. Remarks

Remark 1. From inequalities (1.2) and (1.6), it is easy to see that inequality (1.5) holds.

Remark 2. In view of

$$\begin{aligned} \sum \cos \frac{B-C}{2} &\geq \sum \cos^2 \frac{B-C}{2} = \frac{p^2 + 4R^2 + 2Rr + r^2}{4R^2} \\ &\iff \sum \cos \frac{B-C}{2} - 1 \geq \frac{p^2 + 2Rr + r^2}{4R^2}, \end{aligned}$$

it follows that inequality (1.6) is a refinement of inequality (1.1).

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