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A UNIFIED TREATMENT OF CERTAIN SUBCLASSES OF PRESTARLIKE FUNCTIONS

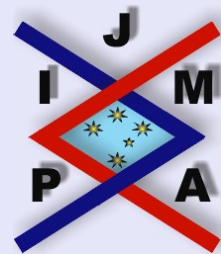
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Abstract

Contents



Home Page

Go Back

Close

Quit

Abstract

In this paper we introduce and study some properties of a unified class $\mathcal{U}[\mu, \alpha, \beta, \gamma, \lambda, \eta, A, B]$ of prestarlike functions with negative coefficients in a unit disk U . These properties include growth and distortion, radii of convexity, radii of starlikeness and radii of close-to-convexity.

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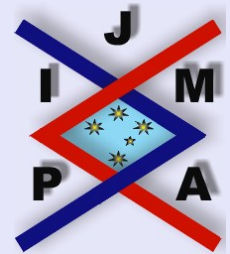
Key words: Analytic functions, Prestarlike functions, radii of starlikeness, convexity and close-to-convexity, Cauchy-Schwarz inequality.

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Contents

1	Introduction	3
2	Coefficient Inequality	7
3	Growth and Distortion Theorem	9
4	Radii Convexity and Starlikeness	12
	References	



A Unified Treatment of Certain Subclasses of Prestarlike Functions

Maslina Darus

Title Page

Contents



Go Back

Close

Quit

Page 2 of 16

1. Introduction

Let A denote the class of *normalized* analytic functions of the form:

$$(1.1) \quad f(z) = z + \sum_{n=2}^{\infty} a_n z^n,$$

in the unit disk $U = \{z : |z| < 1\}$. Further let S denote the subclass of A consisting of analytic and univalent functions f in the unit disk U . A function f in S is said to be starlike of order α if and only if

$$(1.2) \quad \operatorname{Re} \left(\frac{z f'(z)}{f(z)} \right) > \alpha$$

for some α ($0 \leq \alpha < 1$). We denote by $S^*(\alpha)$ the class of all starlike functions of order α . It is well-known that $S^*(\alpha) \subseteq S^*(0) \equiv S^*$.

Let the function

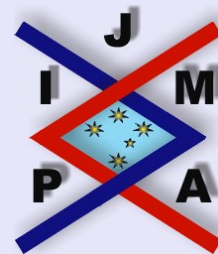
$$(1.3) \quad S_\alpha(z) = \frac{z}{(1-z)^{2(1-\alpha)}}, \quad (z \in U; \quad 0 \leq \alpha < 1)$$

which is the extremal function for the class $S^*(\alpha)$. We also note that $S_\alpha(z)$ can be written in the form:

$$(1.4) \quad S_\alpha(z) = z + \sum_{n=2}^{\infty} |c_n(\alpha)| z^n,$$

where

$$(1.5) \quad c_n(\alpha) = \frac{\prod_{j=2}^n (j - 2\alpha)}{(n-1)!} \quad (n \in \mathbf{N} \setminus \{1\}, \quad \mathbf{N} := \{1, 2, 3, \dots\}).$$



**A Unified Treatment of Certain
Subclasses of Prestarlike
Functions**

Maslina Darus

Title Page

Contents



Go Back

Close

Quit

Page 3 of 16

We note that $c_n(\alpha)$ is decreasing in α and satisfies

$$(1.6) \quad \lim_{n \rightarrow \infty} c_n(\alpha) = \begin{cases} \infty & \text{if } \alpha < \frac{1}{2}, \\ 1 & \text{if } \alpha = \frac{1}{2}, \\ 0 & \text{if } \alpha > \frac{1}{2}. \end{cases}$$

Also a function f in S is said to be convex of order α if and only if

$$(1.7) \quad \operatorname{Re} \left(1 + \frac{z f''(z)}{f'(z)} \right) > \alpha$$

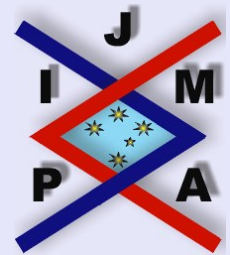
for some α ($0 \leq \alpha < 1$). We denote by $K(\alpha)$ the class of all convex functions of order α . It is a fact that $f \in K(\alpha)$ if and only if $z f'(z) \in S^*(\alpha)$.

The well-known Hadamard product (or convolution) of two functions $f(z)$ given by (1.1) and $g(z)$ given by $g(z) = z + \sum_{n=2}^{\infty} b_n z^n$ is defined by

$$(1.8) \quad (f * g)(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n, \quad (z \in U).$$

Let $\mathcal{R}[\mu, \alpha, \beta, \gamma, \lambda, A, B]$ denote the class of prestarlike functions satisfying the following condition

$$(1.9) \quad \left| \frac{\frac{z H'_\lambda(z)}{H_\lambda(z)} - 1}{2\gamma(B - A) \left(\frac{z H'_\lambda(z)}{H_\lambda(z)} - \mu \right) - B \left(\frac{z H'_\lambda(z)}{H_\lambda(z)} - 1 \right)} \right| < \beta,$$



A Unified Treatment of Certain Subclasses of Prestarlike Functions

Maslina Darus

Title Page

Contents



Go Back

Close

Quit

Page 4 of 16

where $H_\lambda(z) = (1 - \lambda)h(z) + \lambda zh'(z)$, $\lambda \geq 0$, $h = f * S_\alpha$, $0 < \beta \leq 1$, $0 \leq \mu < 1$, and

$$\frac{B}{2(B-A)} < \gamma \leq \begin{cases} \frac{B}{2(B-A)\mu}, & \mu \neq 0, \\ 1, & \mu = 0 \end{cases}$$

for fixed $-1 \leq A \leq B \leq 1$ and $0 < B \leq 1$.

We also note that a function f is a so-called α -prestarlike ($0 \leq \alpha < 1$) function if, and only if, $h = f * S_\alpha \in S^*(\alpha)$ which was first introduced by Ruscheweyh [3], and was rigorously studied by Silverman and Silvia [4], Owa and Ahuja [5] and Uralegaddi and Sarangi [6]. Further, a function $f \in \mathcal{A}$ is in the class $\mathcal{C}[\mu, \alpha, \beta, \gamma, \lambda, A, B]$ if and only if, $zf'(z) \in \mathcal{R}[\mu, \alpha, \beta, \gamma, \lambda, A, B]$.

Let T denote the subclass of A consisting of functions of the form

$$(1.10) \quad f(z) = z - \sum_{n=2}^{\infty} a_n z^n, \quad (a_n \geq 0).$$

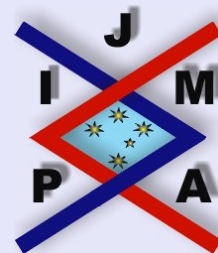
Let us write

$$\mathcal{R}_T[\mu, \alpha, \beta, \gamma, \lambda, A, B] = \mathcal{R}[\mu, \alpha, \beta, \gamma, \lambda, A, B] \cap T$$

and

$$\mathcal{C}_T[\mu, \alpha, \beta, \gamma, \lambda, A, B] = \mathcal{C}[\mu, \alpha, \beta, \gamma, \lambda, A, B] \cap T$$

where T is the class of functions of the form (1.10) that are analytic and univalent in U . The idea of unifying the study of classes $\mathcal{R}_T[\mu, \alpha, \beta, \gamma, \lambda, A, B]$



A Unified Treatment of Certain Subclasses of Prestarlike Functions

Maslina Darus

Title Page

Contents



Go Back

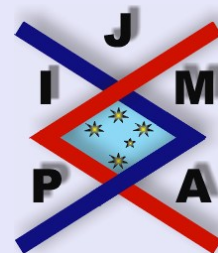
Close

Quit

Page 5 of 16

and $\mathcal{C}_{\mathcal{T}}[\mu, \alpha, \beta, \gamma, \lambda, A, B]$ thus, forming a new class $\mathcal{U}[\mu, \alpha, \beta, \gamma, \lambda, \eta, A, B]$ is somewhat or rather motivated from the work of [1] and [2].

In this paper, we will study the unified presentation of prestarlike functions belonging to $\mathcal{U}[\mu, \alpha, \beta, \gamma, \lambda, \eta, A, B]$ which include growth and distortion theorem, radii of convexity, radii of starlikeness and radii of close-to-convexity.



**A Unified Treatment of Certain
Subclasses of Prestarlike
Functions**

Maslina Darus

Title Page

Contents



Go Back

Close

Quit

Page 6 of 16

2. Coefficient Inequality

Our main tool in this paper is the following result, which can be easily proven, and the details are omitted.

Lemma 2.1. *Let the function f be defined by (1.10). Then $f \in \mathcal{R}_{\mathcal{T}}[\mu, \alpha, \beta, \gamma, \lambda, A, B]$ if and only if*

$$(2.1) \quad \sum_{n=2}^{\infty} \Lambda(n, \lambda) D[n, \beta, \gamma, A, B] |a_n| c_n(\alpha) \leq E[\beta, \gamma, \mu, A, B]$$

where

$$\begin{aligned} \Lambda(n, \lambda) &= (1 + (n - 1)\lambda), \\ D[n, \beta, \gamma, A, B] &= n - 1 + 2\beta\gamma(n - \mu)(B - A) - B\beta(n - 1), \\ E[\beta, \gamma, \mu, A, B] &= 2\beta\gamma(1 - \mu)(B - A). \end{aligned}$$

The result is sharp.

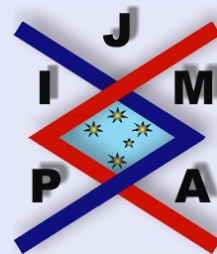
Next, by observing that

$$(2.2) \quad f \in \mathcal{C}_{\mathcal{T}}[\mu, \alpha, \beta, \gamma, \lambda, A, B] \Leftrightarrow zf'(z) \in \mathcal{R}_{\mathcal{T}}[\mu, \alpha, \beta, \gamma, \lambda, A, B],$$

we gain the following Lemma 2.2.

Lemma 2.2. *Let the function f be defined by (1.10). Then $f \in \mathcal{C}_{\mathcal{T}}[\mu, \alpha, \beta, \gamma, \lambda, A, B]$ if and only if*

$$(2.3) \quad \sum_{n=2}^{\infty} n\Lambda(n, \lambda) D[n, \beta, \gamma, A, B] |a_n| c_n(\alpha) \leq E[\beta, \gamma, \mu, A, B]$$



A Unified Treatment of Certain
Subclasses of Prestarlike
Functions

Maslina Darus

Title Page

Contents



Go Back

Close

Quit

Page 7 of 16

where

$$\begin{aligned}\Lambda(n, \lambda) &= (1 + (n - 1)\lambda), \\ D[n, \beta, \gamma, A, B] &= n - 1 + 2\beta\gamma(n - \mu)(B - A) - B\beta(n - 1), \\ E[\beta, \gamma, \mu, A, B] &= 2\beta\gamma(1 - \mu)(B - A)\end{aligned}$$

and $c_n(\alpha)$ given by (1.5).

In view of Lemma 2.1 and Lemma 2.2, we unified the classes $\mathcal{R}_{\mathcal{T}}[\mu, \alpha, \beta, \gamma, \lambda, A, B]$ and $\mathcal{C}_{\mathcal{T}}[\mu, \alpha, \beta, \gamma, \lambda, A, B]$ and so a new class $\mathcal{U}[\mu, \alpha, \beta, \gamma, \lambda, \eta, A, B]$ is formed. Thus we say that a function f defined by (1.10) belongs to $\mathcal{U}[\mu, \alpha, \beta, \gamma, \lambda, \eta, A, B]$ if and only if,

$$(2.4) \quad \sum_{n=2}^{\infty} (1 - \eta + n\eta)\Lambda(n, \lambda)D[n, \beta, \gamma, A, B]|a_n|c_n(\alpha) \leq E[\beta, \gamma, \mu, A, B],$$

$$(0 \leq \alpha < 1; 0 < \beta \leq 1; \eta \geq 0; \lambda \geq 0; -1 \leq A \leq B \leq 1 \text{ and } 0 < B \leq 1),$$

where $\Lambda(n, \lambda)$, $D[n, \beta, \gamma, A, B]$, $E[\beta, \gamma, \mu, A, B]$ and $c_n(\alpha)$ are given in (Lemma 2.1 and Lemma 2.2) and given by (1.5), respectively.

Clearly, we obtain

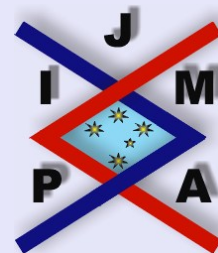
$$\mathcal{U}[\mu, \alpha, \beta, \gamma, \lambda, \eta, A, B] = (1 - \eta)\mathcal{R}_{\mathcal{T}}[\mu, \alpha, \beta, \gamma, A, B] + \eta\mathcal{C}_{\mathcal{T}}[\mu, \alpha, \beta, \gamma, A, B],$$

so that

$$\mathcal{U}[\mu, \alpha, \beta, \gamma, \lambda, 0, A, B] = \mathcal{R}_{\mathcal{T}}[\mu, \alpha, \beta, \gamma, A, B],$$

and

$$\mathcal{U}[\mu, \alpha, \beta, \gamma, \lambda, 1, A, B] = \mathcal{C}_{\mathcal{T}}[\mu, \alpha, \beta, \gamma, A, B].$$



A Unified Treatment of Certain Subclasses of Prestarlike Functions

Maslina Darus

Title Page

Contents



Go Back

Close

Quit

Page 8 of 16

3. Growth and Distortion Theorem

A distortion property for function f in the class $\mathcal{U}[\mu, \alpha, \beta, \gamma, \lambda, \eta, A, B]$ is given as follows:

Theorem 3.1. *Let the function f defined by (1.10) be in the class $\mathcal{U}[\mu, \alpha, \beta, \gamma, \lambda, \eta, A, B]$, then*

$$(3.1) \quad r - \frac{E[\beta, \gamma, \mu, A, B]}{2(1 + \eta)\Lambda(2, \lambda)D[2, \beta, \gamma, A, B](1 - \alpha)} r^2 \leq |f(z)| \leq r + \frac{E[\beta, \gamma, \mu, A, B]}{2(1 + \eta)\Lambda(2, \lambda)D[2, \beta, \gamma, A, B](1 - \alpha)} r^2,$$

$$(\eta \geq 0; \quad 0 \leq \alpha < 1; \quad 0 < \beta \leq 1; \quad z \in U)$$

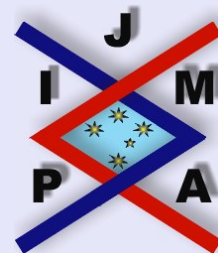
and

$$(3.2) \quad 1 - \frac{E[\beta, \gamma, \mu, A, B]}{(1 + \eta)\Lambda(2, \lambda)D[2, \beta, \gamma, A, B](1 - \alpha)} r \leq |f'(z)| \leq 1 + \frac{E[\beta, \gamma, \mu, A, B]}{(1 + \eta)\Lambda(2, \lambda)D[2, \beta, \gamma, A, B](1 - \alpha)} r,$$

$$(\eta \geq 0; \quad 0 \leq \alpha < 1; \quad 0 < \beta \leq 1; \quad z \in U).$$

The bounds in (3.1) and (3.2) are attained for the function f given by

$$f(z) = z - \frac{E[\beta, \gamma, \mu, A, B]}{2(1 + \eta)\Lambda(2, \lambda)D[2, \beta, \gamma, A, B](1 - \alpha)} z^2.$$



A Unified Treatment of Certain Subclasses of Prestarlike Functions

Maslina Darus

Title Page

Contents



Go Back

Close

Quit

Page 9 of 16

Proof. Observing that $c_n(\alpha)$ defined by (1.5) is nondecreasing for $(0 \leq \alpha < 1)$, we find from (2.4) that

$$(3.3) \quad \sum_{n=2}^{\infty} |a_n| \leq \frac{E[\beta, \gamma, \mu, A, B]}{2(1 + \eta)\Lambda(2, \lambda)D[2, \beta, \gamma, A, B](1 - \alpha)}.$$

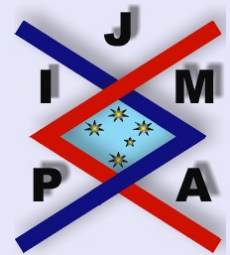
Using (1.10) and (3.3), we readily have ($z \in U$)

$$\begin{aligned} |f(z)| &\geq |z| - \sum_{n=2}^{\infty} |a_n|c_n(\alpha)|z^n| \\ &\geq |z| - |z^2| \sum_{n=2}^{\infty} |a_n|c_n(\alpha), \\ &\geq r - \frac{E[\beta, \gamma, \mu, A, B]}{2(1 + \eta)\Lambda(2, \lambda)D[2, \beta, \gamma, A, B](1 - \alpha)}r^2, \quad |z| = r < 1 \end{aligned}$$

and

$$\begin{aligned} |f(z)| &\leq |z| + \sum_{n=2}^{\infty} |a_n|c_n(\alpha)|z^n| \\ &\leq |z| + |z^2| \sum_{n=2}^{\infty} |a_n|c_n(\alpha), \\ &\leq r + \frac{E[\beta, \gamma, \mu, A, B]}{2(1 + \eta)\Lambda(2, \lambda)D[2, \beta, \gamma, A, B](1 - \alpha)}r^2, \quad |z| = r < 1, \end{aligned}$$

which proves the assertion (3.1) of Theorem 3.1.



A Unified Treatment of Certain Subclasses of Prestarlike Functions

Maslina Darus

Title Page

Contents



Go Back

Close

Quit

Page 10 of 16

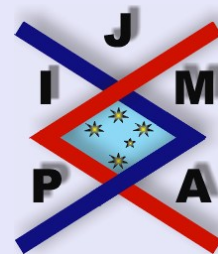
Also, from (1.10), we find for $z \in U$ that

$$\begin{aligned}
 |f'(z)| &\geq 1 - \sum_{n=2}^{\infty} n|a_n|c_n(\alpha)|z^{n-1}| \\
 &\geq 1 - |z| \sum_{n=2}^{\infty} n|a_n|c_n(\alpha), \\
 &\geq 1 - \frac{E[\beta, \gamma, \mu, A, B]}{(1 + \eta)\Lambda(2, \lambda)D[2, \beta, \gamma, A, B](1 - \alpha)^r}, \quad |z| = r < 1
 \end{aligned}$$

and

$$\begin{aligned}
 |f'(z)| &\leq 1 + \sum_{n=2}^{\infty} n|a_n|c_n(\alpha)|z^{n-1}| \\
 &\leq 1 + |z| \sum_{n=2}^{\infty} n|a_n|c_n(\alpha), \\
 &\leq 1 + \frac{E[\beta, \gamma, \mu, A, B]}{2(1 + \eta)\Lambda(2, \lambda)D[2, \beta, \gamma, A, B](1 - \alpha)^r}, \quad |z| = r < 1,
 \end{aligned}$$

which proves the assertion (3.2) of Theorem 3.1. □



A Unified Treatment of Certain Subclasses of Prestarlike Functions

Maslina Darus

Title Page

Contents



Go Back

Close

Quit

Page 11 of 16

4. Radii Convexity and Starlikeness

The radii of convexity for class $\mathcal{U}[\mu, \alpha, \beta, \gamma, \lambda, \eta, A, B]$ is given by the following theorem.

Theorem 4.1. *Let the function f be in the class $\mathcal{U}[\mu, \alpha, \beta, \gamma, \lambda, \eta, A, B]$. Then the function f is convex of order ρ ($0 \leq \rho < 1$) in the disk $|z| < r_1(\mu, \alpha, \beta, \gamma, \lambda, \eta, A, B) = r_1$, where*

$$(4.1) \quad r_1 = \inf_n \left\{ \frac{2(1-\alpha)(1-\rho)\Lambda(n, \lambda)D[n, \beta, \gamma, A, B](1-\eta+n\eta)}{n(n-\rho)E[\beta, \gamma, \mu, A, B]} \right\}^{\frac{1}{n-1}}.$$

Proof. It sufficient to show that

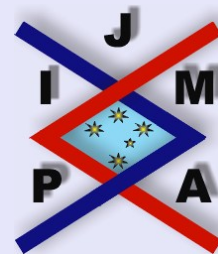
$$(4.2) \quad \left| \frac{zf''(z)}{f'(z)} \right| = \left| \frac{-\sum_{n=2}^{\infty} n(n-1)a_n z^{n-1}}{1 - \sum_{n=2}^{\infty} na_n z^{n-1}} \right| \leq \frac{\sum_{n=2}^{\infty} n(n-1)a_n |z|^{n-1}}{1 - \sum_{n=2}^{\infty} na_n |z|^{n-1}}$$

which implies that

$$(4.3) \quad (1-\rho) - \left| \frac{zf''(z)}{f'(z)} \right| \geq (1-\rho) - \frac{\sum_{n=2}^{\infty} n(n-1)|a_n||z|^{n-1}}{1 - \sum_{n=2}^{\infty} na_n z^{n-1}} = \frac{(1-\rho) - \sum_{n=2}^{\infty} n(n-\rho)a_n |z|^{n-1}}{1 - \sum_{n=2}^{\infty} na_n |z|^{n-1}}.$$

Hence from (4.1), if

$$(4.4) \quad |z|^{n-1} \leq \frac{(1-\rho)}{n(n-\rho)} \cdot \frac{2(1-\alpha)\Lambda(n, \lambda)D[n, \beta, \gamma, A, B](1-\eta+n\eta)}{E[\beta, \gamma, \mu, A, B]},$$



A Unified Treatment of Certain Subclasses of Prestarlike Functions

Maslina Darus

Title Page

Contents



Go Back

Close

Quit

Page 12 of 16

and according to (2.4)

$$(4.5) \quad 1 - \rho - \sum_{n=2}^{\infty} n(n - \rho)a_n|z|^{n-1} > 1 - \rho - (1 - \rho) = \rho.$$

Hence from (4.3), we obtain

$$\left| \frac{zf''(z)}{f'(z)} \right| < 1 - \rho$$

Therefore

$$\operatorname{Re} \left\{ 1 + \frac{zf''(z)}{f'(z)} \right\} > 0,$$

which shows that f is convex in the disk $|z| < r_1(\mu, \alpha, \beta, \gamma, \lambda, \eta, \rho, A, B)$. \square

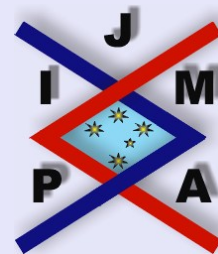
By setting $\eta = 0$ and $\eta = 1$, we have the Corollary 4.2 and the Corollary 4.3, respectively.

Corollary 4.2. *Let the function f be in the class $\mathcal{R}_{\mathcal{T}}(\mu, \alpha, \beta, \gamma, \lambda, \rho, A, B)$. Then the function f is convex of order ρ ($0 \leq \rho < 1$) in the disk $|z| < r_2(\mu, \alpha, \beta, \gamma, \lambda, \rho, A, B) = r_2$, where*

$$(4.6) \quad r_2 = \inf_n \left\{ \frac{2(1 - \alpha)(1 - \rho)\Lambda(n, \lambda)D[n, \beta, \gamma, A, B]}{n(n - \rho)E[\beta, \gamma, \mu, A, B]} \right\}^{\frac{1}{n-1}}.$$

Corollary 4.3. *Let the function f be in the class $\mathcal{C}_{\mathcal{T}}(\mu, \alpha, \beta, \gamma, \lambda, \rho, A, B)$. Then the function f is convex of order ρ ($0 \leq \rho < 1$) in the disk $|z| < r_3(\mu, \alpha, \beta, \gamma, \lambda, \rho, A, B) = r_3$, where*

$$(4.7) \quad r_3 = \inf_n \left\{ \frac{2(1 - \alpha)(1 - \rho)\Lambda(n, \lambda)D[n, \beta, \gamma, A, B]}{(n - \rho)E[\beta, \gamma, \mu, A, B]} \right\}^{\frac{1}{n-1}}.$$



A Unified Treatment of Certain Subclasses of Prestarlike Functions

Maslina Darus

Title Page

Contents



Go Back

Close

Quit

Page 13 of 16

Theorem 4.4. Let the function f be in the class $\mathcal{U}[\mu, \alpha, \beta, \gamma, \lambda, \eta, A, B]$. Then the function f is starlike of order ρ ($0 \leq \rho < 1$) in the disk $|z| < r_4(\mu, \alpha, \beta, \gamma, \lambda, \eta, A, B) = r_4$, where

$$(4.8) \quad r_4 = \inf_n \left\{ \frac{2(1-\alpha)(1-\rho)\Lambda(n, \lambda)D[n, \beta, \gamma, A, B](1-\eta+n\eta)}{(n-\rho)E[\beta, \gamma, \mu, A, B]} \right\}^{\frac{1}{n-1}}.$$

Proof. It sufficient to show that

$$\left| \frac{zf'(z)}{f(z)} - 1 \right| < 1 - \rho$$

Using a similar method to Theorem 4.1 and making use of (2.4), we get (4.8). \square

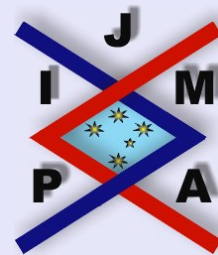
Letting $\eta = 0$ and $\eta = 1$, we have the Corollary 4.5 and the Corollary 4.6, respectively.

Corollary 4.5. Let the function f be in the class $\mathcal{R}_{\mathcal{T}}(\mu, \alpha, \beta, \gamma, \lambda, \rho, A, B)$. Then the function f is starlike of order ρ ($0 \leq \rho < 1$) in the disk $|z| < r_5(\mu, \alpha, \beta, \gamma, \lambda, \rho, A, B) = r_5$, where

$$(4.9) \quad r_5 = \inf_n \left\{ \frac{2(1-\alpha)(1-\rho)\Lambda(n, \lambda)D[n, \beta, \gamma, A, B]}{(n-\rho)E[\beta, \gamma, \mu, A, B]} \right\}^{\frac{1}{n-1}}.$$

Corollary 4.6. Let the function f be in the class $\mathcal{C}_{\mathcal{T}}(\mu, \alpha, \beta, \gamma, \lambda, \rho, A, B)$. Then the function f is starlike of order ρ ($0 \leq \rho < 1$) in the disk $|z| < r_6(\mu, \alpha, \beta, \gamma, \lambda, \rho, A, B) = r_6$, where

$$(4.10) \quad r_6 = \inf_n \left\{ \frac{2n(1-\alpha)(1-\rho)\Lambda(n, \lambda)D[n, \beta, \gamma, A, B]}{(n-\rho)E[\beta, \gamma, \mu, A, B]} \right\}^{\frac{1}{n-1}}.$$



A Unified Treatment of Certain
Subclasses of Prestarlike
Functions

Maslina Darus

Title Page

Contents



Go Back

Close

Quit

Page 14 of 16

Last, but not least we give the following result.

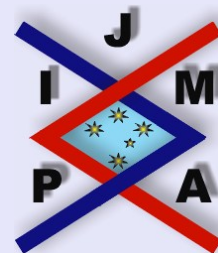
Theorem 4.7. *Let the function f be in the class $\mathcal{U}[\mu, \alpha, \beta, \gamma, \lambda, \eta, A, B]$. Then the function f is close-to-convex of order ρ ($0 \leq \rho < 1$) in the disk $|z| < r_7(\mu, \alpha, \beta, \gamma, \lambda, \eta, A, B) = r_7$, where*

$$(4.11) \quad r_7 = \inf_n \left\{ \frac{2(1-\alpha)(1-\rho)\Lambda(n, \lambda)D[n, \beta, \gamma, A, B](1-\eta+n\eta)}{nE[\beta, \gamma, \mu, A, B]} \right\}^{\frac{1}{n-1}}.$$

Proof. It sufficient to show that

$$|f'(z) - 1| < 1 - \rho.$$

Using a similar technique to Theorem 4.1 and making use of (2.4), we get (4.11). \square



A Unified Treatment of Certain
Subclasses of Prestarlike
Functions

Maslina Darus

Title Page

Contents



Go Back

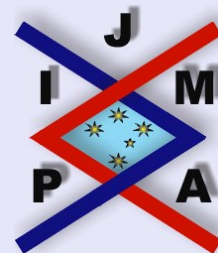
Close

Quit

Page 15 of 16

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A Unified Treatment of Certain Subclasses of Prestarlike Functions

Maslina Darus

Title Page

Contents



Go Back

Close

Quit

Page 16 of 16