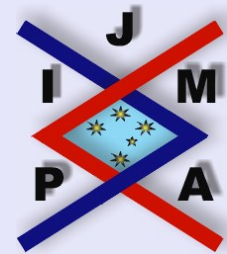


CORRIGENDUM TO “ON SHORT SUMS OF CERTAIN MULTIPLICATIVE FUNCTIONS”

OLIVIER BORDELLÈS

2 Allée de la Combe
43000 AIGUILHE
FRANCE.

EMail: borde43@wanadoo.fr



volume 5, issue 3, article 81,
2004.

*Received 27 August, 2003;
accepted 28 April, 2004.*

Communicated by: S.S. Dragomir

Abstract

Contents



Home Page

Go Back

Close

Quit

Abstract

This note is a corrigendum of the main result of the paper "On short sums of certain multiplicative functions" (J. Ineq. Pure & Appl. Math., 3(5), Art. 70 (2002)).

2000 Mathematics Subject Classification: 11N37, 11P21

Key words: Corrigendum, Short sums, Integer points

The author is indebted to the referee for insightful comments.

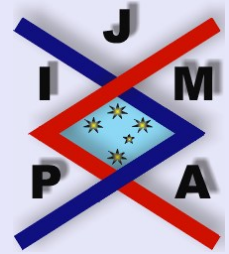
The purpose of this note is both to give a corrected statement of the main result in [1] and to provide the necessary changes to the arguments in that paper to justify this corrected statement. The result we now assert is the following :

Theorem 1. *Let $\varepsilon, c_0 > 0$ and $2 \leq y \leq c_0 x^{1/2}$ be real numbers. Let f be a multiplicative function satisfying $0 \leq f(n) \leq 1$ for any positive integer n and $f(p) = 1$ for any prime number p . We have as $x \rightarrow +\infty$:*

$$\sum_{x < n \leq x+y} f(n) = y\mathcal{P}(f) + O_\varepsilon(x^{1/15+\varepsilon}y^{2/3}),$$

where

$$\mathcal{P}(f) := \prod_p \left(1 - \frac{1}{p}\right) \left(1 + \sum_{l=1}^{\infty} \frac{f(p^l)}{p^l}\right).$$



Corrigendum to "On Short Sums of Certain Multiplicative Functions"

Olivier Bordellès

Title Page

Contents



Go Back

Close

Quit

Page 2 of 7

Before giving the proof, we note that if $y < x^{1/5}$, then $x^{1/15} > y^{1/3}$ so that the expression $x^{1/15+\varepsilon}y^{2/3}$ in the error term exceeds the main term (as well as the trivial bound of $y + 1$ on the sum).

Proof. On page 5 of [1], the sum

$$S_1 := \sum_{\substack{y < d \leq x+y \\ d \text{ squarefull}}} |g(d)| \left(\left[\frac{x+y}{d} \right] - \left[\frac{x}{d} \right] \right)$$

has been bounded by the sum

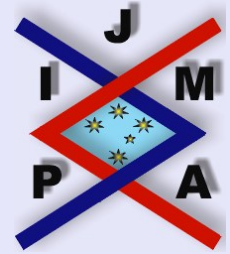
$$S_2 := \sum_{b \leq (x+y)^{1/3}} \sum_{\left(\frac{y}{b^3}\right)^{1/2} < a \leq \left(\frac{x+y}{b^3}\right)^{1/2}} \left(\left[\frac{(x+y)b^{-3}}{a^2} \right] - \left[\frac{xb^{-3}}{a^2} \right] \right)$$

by using $d = a^2b^3$ with $\mu^2(b) = 1$, and S_2 has been bounded by

$$\ll_{\varepsilon} x^{\varepsilon} \max_{1 \leq B \leq (x+y)^{1/3}} \mathcal{R} \left(\frac{x}{b^3}, B, \frac{y}{B^3} \right)$$

for any (small) positive real number ε , where we defined $\mathcal{R}(f, N, \delta)$ to be the number of integer points (n, m) verifying $n \in]N; 2N]$ and $|f(n) - m| \leq \delta$. Unfortunately, the part $\sum_{b \leq y^{1/3}}$ of S_2 cannot be estimated by

$$\max_{1 \leq B \leq (x+y)^{1/3}} \mathcal{R} \left(\frac{x}{b^3}, B, \frac{y}{B^3} \right),$$



Corrigendum to “On Short Sums of Certain Multiplicative Functions”

Olivier Bordellès

Title Page

Contents



Go Back

Close

Quit

Page 3 of 7

hence we have to proceed differently: for any positive integer r , we set

$$\tau_{(r)}(n) := \sum_{d^r | n} 1$$

and recall that

$$\tau_{(r)}(n) \ll_{\varepsilon} n^{\varepsilon/r}$$

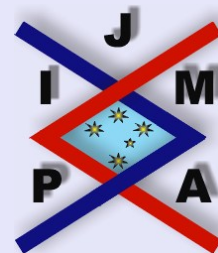
for any positive integers n, r .

In S_1 , $d = a^2 b^3 > y$ implies $a > y^{1/5}$ or $b > y^{1/5}$. Since

$$\begin{aligned} \sum_{y^{1/5} < a \leq (x+y)^{1/2}} \sum_{\left(\frac{y}{a^2}\right)^{1/3} < b \leq \left(\frac{x+y}{a^2}\right)^{1/3}} \left(\left[\frac{(x+y)a^{-2}}{b^3} \right] - \left[\frac{xa^{-2}}{b^3} \right] \right) \\ \leq \sum_{y^{1/5} < a \leq (x+y)^{1/2}} \sum_{\frac{x}{a^2} < n \leq \frac{x+y}{a^2}} \tau_{(3)}(n) \end{aligned}$$

and we have the same if $b > y^{1/5}$, then

$$\begin{aligned} S_1 &\leq \sum_{y^{1/5} < b \leq (x+y)^{1/3}} \sum_{\frac{x}{b^3} < n \leq \frac{x+y}{b^3}} \tau_{(2)}(n) + \sum_{y^{1/5} < a \leq (x+y)^{1/2}} \sum_{\frac{x}{a^2} < n \leq \frac{x+y}{a^2}} \tau_{(3)}(n) \\ &= \sum_{y^{1/5} < b \leq (2y)^{1/3}} \sum_{\frac{x}{b^3} < n \leq \frac{x+y}{b^3}} \tau_{(2)}(n) + \sum_{(2y)^{1/3} < b \leq (x+y)^{1/3}} \sum_{\frac{x}{b^3} < n \leq \frac{x+y}{b^3}} \tau_{(2)}(n) \\ &\quad + \sum_{y^{1/5} < a \leq (2y)^{1/2}} \sum_{\frac{x}{a^2} < n \leq \frac{x+y}{a^2}} \tau_{(3)}(n) + \sum_{(2y)^{1/2} < a \leq (x+y)^{1/2}} \sum_{\frac{x}{a^2} < n \leq \frac{x+y}{a^2}} \tau_{(3)}(n) \\ &:= \Sigma_1 + \Sigma_2 + \Sigma_3 + \Sigma_4. \end{aligned}$$



Corrigendum to “On Short Sums of Certain Multiplicative Functions”

Olivier Bordellès

Title Page

Contents



Go Back

Close

Quit

Page 4 of 7

- For Σ_1 and Σ_3 we use the trivial bound:

$$\begin{aligned} \Sigma_1 + \Sigma_3 &\ll_{\varepsilon} x^{\varepsilon/2} \sum_{y^{1/5} < b \leq (2y)^{1/3}} \left(\left[\frac{x+y}{b^3} \right] - \left[\frac{x}{b^3} \right] \right) \\ &\quad + x^{\varepsilon/3} \sum_{y^{1/5} < a \leq (2y)^{1/2}} \left(\left[\frac{x+y}{a^2} \right] - \left[\frac{x}{a^2} \right] \right) \\ &\ll_{\varepsilon} x^{\varepsilon/2} y \left(\sum_{b > y^{1/5}} \frac{1}{b^3} + \sum_{a > y^{1/5}} \frac{1}{a^2} \right) \ll_{\varepsilon} y^{4/5} x^{\varepsilon/2}. \end{aligned}$$

- For Σ_2 , we use the method of [1] to get

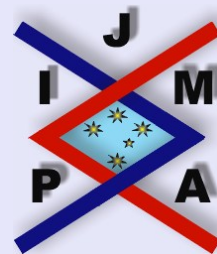
$$\Sigma_2 \ll_{\varepsilon} x^{\varepsilon} (x^{1/6} + y^{1/3}).$$

- For Σ_4 , if we suppose $y \leq c_0 x^{1/2}$ (where $c_0 > 0$ is sufficiently small), we have using Lemmas 2.1 and 2.2 of [1]:

$$\begin{aligned} \Sigma_4 &\ll_{\varepsilon} x^{\varepsilon} \left\{ \max_{(2y)^{1/2} < A \leq c_0^{-1} y} \mathcal{R} \left(\frac{x}{a^2}, A, \frac{y}{A^2} \right) + \max_{c_0^{-1} y < A \leq x^{1/2}} \mathcal{R} \left(\frac{x}{a^2}, A, \frac{y}{A^2} \right) \right\} \\ &\ll_{\varepsilon} x^{\varepsilon} \left((xy)^{1/6} + x^{1/5} + x^{1/15} y^{2/3} \right). \end{aligned}$$

Hence we finally have

$$\begin{aligned} S_1 &\ll_{\varepsilon} x^{\varepsilon} \left(x^{1/15} y^{2/3} + y^{4/5} + (xy)^{1/6} + x^{1/5} + x^{1/6} + y^{1/3} \right) \\ &\ll_{\varepsilon} x^{\varepsilon} \left(x^{1/15} y^{2/3} + y^{4/5} \right) \end{aligned}$$



Corrigendum to “On Short Sums of Certain Multiplicative Functions”

Olivier Bordellès

Title Page

Contents



Go Back

Close

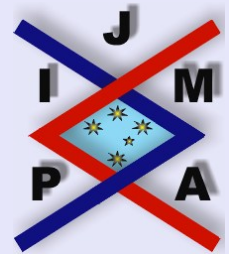
Quit

Page 5 of 7

if $y \geq x^{1/5}$. Note that $y^{4/5} \ll x^{1/15}y^{2/3}$ if $y \leq c_0x^{1/2}$ and that

$$\left| \sum_{x < n \leq x+y} f(n) - y\mathcal{P}(f) \right| \ll y \ll x^{1/15}y^{2/3}$$

if $y < x^{1/5}$. This concludes the proof of the theorem. □



Corrigendum to “On Short Sums of Certain Multiplicative Functions”

Olivier Bordellès

Title Page

Contents



Go Back

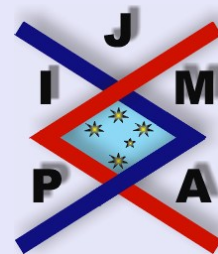
Close

Quit

Page 6 of 7

References

- [1] O. BORDELLÈS, On short sums of certain multiplicative functions, *J. Inequal. Pure and Appl. Math.*, **3**(5) (2002), Art. 70. [ONLINE: <http://jipam.vu.edu.au/article.php?sid=222>]



Corrigendum to “On Short Sums of Certain Multiplicative Functions”

Olivier Bordellès

Title Page

Contents



Go Back

Close

Quit

Page 7 of 7