

TIME SCALE INTEGRAL INEQUALITIES SIMILAR TO QI'S INEQUALITY

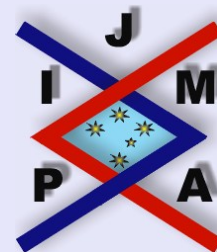
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Abstract

In this study, some integral inequalities and Qi's inequalities of which is proved by the Bougoffa [5] – [7] are extended to the general time scale.

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1. Introduction

The unification and extension of continuous calculus, discrete calculus, q -calculus, and indeed arbitrary real-number calculus to time scale calculus was first accomplished by Hilger in his PhD. thesis [8]. The purpose of this work is to extend some integral inequalities and Qi inequalities proved by Bougoffa [5] – [7]. The following definitions will serve as a short primer on time scale calculus; they can be found in [1] – [4]. A time scale \mathbb{T} is any nonempty closed subset of \mathbb{R} . Within that set, define the jump operators $\rho, \sigma : \mathbb{T} \rightarrow \mathbb{T}$ by

$$\rho(t) = \sup\{s \in \mathbb{T} : s < t\} \quad \text{and} \quad \sigma(t) = \inf\{s \in \mathbb{T} : s > t\},$$

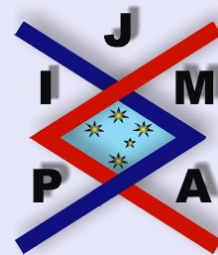
where $\inf \phi := \sup \mathbb{T}$ and $\sup \phi := \inf \mathbb{T}$. If $\rho(t) = t$ and $\rho(t) < t$, then the point $t \in \mathbb{T}$ is left-dense, left-scattered. If $\sigma(t) = t$ and $\sigma(t) > t$, then the point $t \in \mathbb{T}$ is right-dense, right-scattered. If \mathbb{T} has a right-scattered minimum m , define $\mathbb{T}_k := \mathbb{T} - \{m\}$; otherwise, set $\mathbb{T}_k = \mathbb{T}$. If \mathbb{T} has a left-scattered maximum M , define $\mathbb{T}^k := \mathbb{T} - \{M\}$; otherwise, set $\mathbb{T}^k = \mathbb{T}$. The so-called graininess functions are $\mu(t) := \sigma(t) - t$ and $\nu(t) := t - \rho(t)$.

For $f : \mathbb{T} \rightarrow \mathbb{R}$ and $t \in \mathbb{T}^k$, the delta derivative in [3, 4] of f at t , denoted $f^\Delta(t)$, is the number (provided it exists) with the property that given any $\varepsilon > 0$, there is a neighborhood U of t such that

$$|f(\sigma(t)) - f(s) - f^\Delta(t)[\sigma(t) - s]| \leq \varepsilon |\sigma(t) - s|$$

for all $s \in U$. For $\mathbb{T} = \mathbb{R}$, $f^\Delta = f'$, the usual derivative; for $\mathbb{T} = \mathbb{Z}$ the delta derivative is the forward difference operator, $f^\Delta(t) = f(t + 1) - f(t)$; in the case of q -difference equations with $q > 1$,

$$f^\Delta(t) = \frac{f(qt) - f(t)}{(q - 1)t}, \quad f^\Delta(0) = \lim_{s \rightarrow 0} \frac{f(s) - f(0)}{s}.$$



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A function $f : \mathbb{T} \rightarrow \mathbb{R}$ is right-dense continuous or rd-continuous provided it is continuous at right-dense points in \mathbb{T} and its left-sided limits exist (finite) at left-dense points in \mathbb{T} . If $\mathbb{T} = \mathbb{R}$, then f is rd-continuous if and only if f is continuous. It is known from Theorem 1.74 in [3] that if f is right-dense continuous, there is a function F such that $F^\Delta(t) = f(t)$ and

$$\int_a^b f(t)\Delta t = F(b) - F(a).$$

Note that we have

$$\sigma(t) = t, \quad \mu(t) \equiv 0, \quad f^\Delta = f', \quad \int_a^b f(t)\Delta t = \int_a^b f(t)dt, \quad \text{when } \mathbb{T} = \mathbb{R}$$

while

$$\sigma(t) = t+1, \quad \mu(t) \equiv 1, \quad f^\Delta = \Delta f, \quad \int_a^b f(t)\Delta t = \sum_{t=a}^{b-1} f(t), \quad \text{when } \mathbb{T} = \mathbb{Z}.$$

Much more information concerning time scales and dynamic equations on time scales can be found in the books [3, 4].

Theorem 1.1 (Hölder's inequality on time scales [3]). *Let $a, b \in \mathbb{T}$. For rd-continuous functions $f, g : [a, b] \rightarrow \mathbb{R}$ we have*

$$\int_a^b |f(x)g(x)| \Delta x \leq \left(\int_a^b |f(x)|^p \Delta x \right)^{\frac{1}{p}} \left(\int_a^b |g(x)|^q \Delta x \right)^{\frac{1}{q}},$$

where $p > 1$ and $q = \frac{p}{p-1}$.



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2. Main Results

In this section, we will state our main results and give their proofs.

Lemma 2.1. *Let $a, b \in \mathbb{T}$, and $p > 1$ and $q > 1$ with $\frac{1}{p} + \frac{1}{q} = 1$. If two positive functions $f, g : [a, b] \rightarrow \mathbb{R}$ are rd-continuous and satisfying $0 < m \leq \frac{f^p}{g^q} \leq M < \infty$ on the set $[a, b]$, then we have the following inequality*

$$(2.1) \quad \left(\int_a^b f^p \Delta x \right)^{\frac{1}{p}} \left(\int_a^b g^q \Delta x \right)^{\frac{1}{q}} \leq \left(\frac{M}{m} \right)^{\frac{1}{pq}} \int_a^b fg \Delta x.$$

Inequality (2.1) is called the reverse Hölder inequality.

Proof. Since $\frac{f^p}{g^q} \leq M$, $g \geq M^{-\frac{1}{q}} f^{\frac{p}{q}}$, therefore

$$fg \geq M^{-\frac{1}{q}} f^{1+\frac{p}{q}} = M^{-\frac{1}{q}} f^p$$

and so,

$$(2.2) \quad \left(\int_a^b f^p \Delta x \right)^{\frac{1}{p}} \leq M^{\frac{1}{pq}} \left(\int_a^b fg \Delta x \right)^{\frac{1}{p}}.$$

On the other hand, since $m \leq \frac{f^p}{g^q}$, $f \geq m^{\frac{1}{p}} g^{\frac{q}{p}}$, hence

$$\int_a^b fg \Delta x \geq \int_a^b m^{\frac{1}{p}} g^{1+\frac{q}{p}} \Delta x \geq m^{\frac{1}{p}} \int_a^b g^q \Delta x$$



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and so,

$$\left(\int_a^b fg \Delta x \right)^{\frac{1}{q}} \geq m^{\frac{1}{pq}} \left(\int_a^b g^q \Delta x \right)^{\frac{1}{q}}.$$

Combining with (2.2), we have the desired inequality

$$\begin{aligned} \left(\int_a^b f^p \Delta x \right)^{\frac{1}{p}} \left(\int_a^b g^q \Delta x \right)^{\frac{1}{q}} &\leq M^{\frac{1}{pq}} \left(\int_a^b fg \Delta x \right)^{\frac{1}{p}} m^{-\frac{1}{pq}} \left(\int_a^b g^q \Delta x \right)^{\frac{1}{q}} \\ &= \left(\frac{M}{m} \right)^{\frac{1}{pq}} \int_a^b fg \Delta x. \end{aligned}$$

□

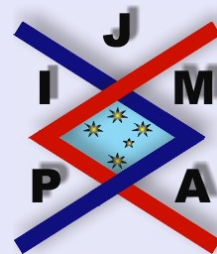
Corollary 2.2. In Lemma 2.1, replacing f^p and g^q by f and g , respectively, we obtain the reverse Hölder type inequality,

$$(2.3) \quad \left(\int_a^b f \Delta x \right)^{\frac{1}{p}} \left(\int_a^b g \Delta x \right)^{\frac{1}{q}} \leq \left(\frac{m}{M} \right)^{-\frac{1}{pq}} \int_a^b f^{\frac{1}{p}} g^{\frac{1}{q}} \Delta x.$$

The proof of this corollary can be obtained from (2.1).

Theorem 2.3. Let $a, b \in \mathbb{T}$, $p > 1$ and $q > 1$ with $\frac{1}{p} + \frac{1}{q} = 1$. If $f : [a, b] \rightarrow \mathbb{R}$ is rd-continuous and $0 < m^{\frac{1}{p}} \leq f \leq M^{\frac{1}{p}} < \infty$ on $[a, b]$, then we have the following inequality

$$(2.4) \quad \left(\int_a^b f^{\frac{1}{p}} \Delta x \right)^p \geq (b - a)^{\frac{p+1}{q}} \left(\frac{m}{M} \right)^{\frac{p+1}{pq}} \left(\int_a^b f^p \Delta x \right)^{\frac{1}{p}}.$$



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Proof. Putting $g \equiv 1$ in Lemma 2.1, we obtain

$$\left(\int_a^b f^p \Delta x \right)^{\frac{1}{p}} [b - a]^{\frac{1}{q}} \leq \left(\frac{m}{M} \right)^{-\frac{1}{pq}} \int_a^b f \Delta x.$$

Therefore, we get

$$(2.5) \quad \left(\int_a^b f^p \Delta x \right)^{\frac{1}{p}} \leq \left(\frac{m}{M} \right)^{-\frac{1}{pq}} [b - a]^{-\frac{1}{q}} \int_a^b f \Delta x.$$

Again, substituting $g \equiv 1$ in Corollary 2.2 leads to

$$\left(\int_a^b f \Delta x \right)^{\frac{1}{p}} \leq \left(\frac{m}{M} \right)^{-\frac{1}{pq}} [b - a]^{-\frac{1}{q}} \int_a^b f^{\frac{1}{p}} \Delta x,$$

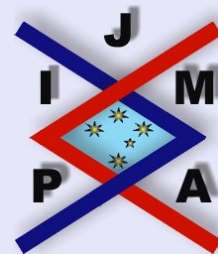
and so,

$$(2.6) \quad \int_a^b f \Delta x \leq \left(\frac{m}{M} \right)^{-\frac{1}{q}} [b - a]^{-\frac{p}{q}} \left(\int_a^b f^{\frac{1}{p}} \Delta x \right)^p.$$

Combining (2.5) with (2.6), we obtain

$$\left(\int_a^b f^{\frac{1}{p}} \Delta x \right)^p \geq (b - a)^{\frac{p+1}{q}} \left(\frac{m}{M} \right)^{\frac{p+1}{pq}} \left(\int_a^b f^p \Delta x \right)^{\frac{1}{p}}.$$

□



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Corollary 2.4. If $0 < m^{\frac{1}{p}} \leq f \leq M^{\frac{1}{p}} < \infty$ on $[a, b]$ and $\frac{m}{M} = [b - a]^{-p}$ for $p > 1$, then

$$(2.7) \quad \left(\int_a^b f^{\frac{1}{p}} \Delta x \right)^p \geq \left(\int_a^b f^p \Delta x \right)^{\frac{1}{p}}.$$

Remark 1. For $\mathbb{T} = \mathbb{R}$, (2.7) is Qi's inequality [9].

Theorem 2.5. If $f : [a, b] \rightarrow \mathbb{R}$ is rd-continuous and $0 < m \leq f(x) \leq M$ on $[a, b]$, then we have the following inequality

$$(2.8) \quad \int_a^b f^{\frac{1}{p}} \Delta x \geq B \left(\int_a^b f \Delta x \right)^{\frac{1}{p}-1},$$

where $B = m(b - a)^{1+\frac{1}{q}} \left(\frac{m}{M}\right)^{\frac{1}{pq}}$ and $p > 1, q > 1$ with $\frac{1}{p} + \frac{1}{q} = 1$.

Proof. In Corollary 2.2, putting $g \equiv 1$ yields

$$\left(\int_a^b f \Delta x \right)^{\frac{1}{p}} [b - a]^{\frac{1}{q}} \leq \left(\frac{m}{M} \right)^{-\frac{1}{pq}} \int_a^b f^{\frac{1}{p}} \Delta x,$$

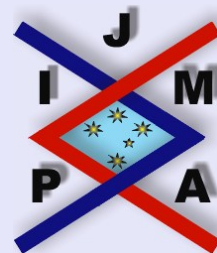
and so,

$$\int_a^b f^{\frac{1}{p}} \Delta x \geq \left(\frac{m}{M} \right)^{-\frac{1}{pq}} [b - a]^{\frac{1}{q}} \left(\int_a^b f \Delta x \right)^{\frac{1}{p}-1} \left(\int_a^b f \Delta x \right)^{\frac{1}{p}}.$$

Since $0 < m \leq f(x)$, we have

$$\int_a^b f^{\frac{1}{p}} \Delta x \geq \left(\frac{m}{M} \right)^{\frac{1}{pq}} m [b - a]^{1+\frac{1}{q}} \left(\int_a^b f \Delta x \right)^{\frac{1}{p}-1}.$$

This proves inequality (2.8). □



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Corollary 2.6. Let $p > 1$ and $q > 1$ with $\frac{1}{p} + \frac{1}{q} = 1$. If

$$m \left(\frac{m}{M} \right)^{\frac{1}{pq}} = \frac{1}{[b-a]^{1+\frac{1}{q}}}$$

and $0 < m \leq f(x) \leq M$ on $[a, b]$, then

$$(2.9) \quad \int_a^b f^{\frac{1}{p}} \Delta x \geq \left(\int_a^b f \Delta x \right)^{\frac{1}{p}-1}.$$

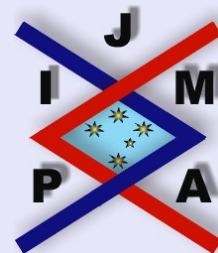
Remark 2. For $\mathbb{T} = \mathbb{R}$, (2.9) is Qi's inequality [9].

Lemma 2.7. Let $a, b \in \mathbb{T}$, and $f, g : [a, b] \rightarrow \mathbb{R}$ be rd-continuous and nonnegative functions with $0 < m \leq \frac{f}{g} \leq M < \infty$ on $[a, b]$. Then for $p > 1$ and $q > 1$ with $\frac{1}{p} + \frac{1}{q} = 1$ we have the following inequality

$$(2.10) \quad \int_a^b [f(x)]^{\frac{1}{p}} [g(x)]^{\frac{1}{q}} \Delta x \leq M^{\frac{1}{p^2}} m^{-\frac{1}{q^2}} \int_a^b [f(x)]^{\frac{1}{q}} [g(x)]^{\frac{1}{p}} \Delta x$$

and

$$(2.11) \quad \int_a^b [f(x)]^{\frac{1}{p}} [g(x)]^{\frac{1}{q}} \Delta x \leq M^{\frac{1}{p^2}} m^{-\frac{1}{q^2}} \left(\int_a^b f(x) \Delta x \right)^{\frac{1}{q}} \left(\int_a^b g(x) \Delta x \right)^{\frac{1}{p}}.$$



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Proof. From Hölder's inequality, we obtain

$$\int_a^b [f(x)]^{\frac{1}{p}} [g(x)]^{\frac{1}{q}} \Delta x \leq \left(\int_a^b f(x) \Delta x \right)^{\frac{1}{q}} \left(\int_a^b g(x) \Delta x \right)^{\frac{1}{p}},$$

that is,

$$\int_a^b [f(x)]^{\frac{1}{p}} [g(x)]^{\frac{1}{q}} \Delta x \leq \left(\int_a^b [f(x)]^{\frac{1}{p}} [f(x)]^{\frac{1}{q}} \Delta x \right)^{\frac{1}{q}} \left(\int_a^b [g(x)]^{\frac{1}{p}} [g(x)]^{\frac{1}{q}} \Delta x \right)^{\frac{1}{p}}.$$

Since $[f(x)]^{\frac{1}{p}} \leq M^{\frac{1}{p}} [g(x)]^{\frac{1}{p}}$ and $[g(x)]^{\frac{1}{q}} \leq m^{-\frac{1}{q}} [f(x)]^{\frac{1}{q}}$, from the above inequality it follows that

$$\begin{aligned} & \int_a^b [f(x)]^{\frac{1}{p}} [g(x)]^{\frac{1}{q}} \Delta x \\ & \leq M^{\frac{1}{p^2}} m^{-\frac{1}{q^2}} \left(\int_a^b [g(x)]^{\frac{1}{p}} [f(x)]^{\frac{1}{q}} \Delta x \right)^{\frac{1}{q}} \left(\int_a^b [g(x)]^{\frac{1}{p}} [f(x)]^{\frac{1}{q}} \Delta x \right)^{\frac{1}{p}}, \end{aligned}$$

and so,

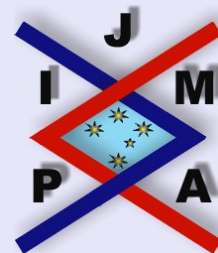
$$(2.12) \quad \int_a^b [f(x)]^{\frac{1}{p}} [g(x)]^{\frac{1}{q}} \Delta x \leq M^{\frac{1}{p^2}} m^{-\frac{1}{q^2}} \int_a^b [f(x)]^{\frac{1}{q}} [g(x)]^{\frac{1}{p}} \Delta x.$$

Hence, the inequality (2.10) is proved.

The inequality (2.11) follows from substituting the following

$$\int_a^b [f(x)]^{\frac{1}{p}} [g(x)]^{\frac{1}{q}} \Delta x \leq \left(\int_a^b f(x) \Delta x \right)^{\frac{1}{q}} \left(\int_a^b g(x) \Delta x \right)^{\frac{1}{p}}$$

into (2.12), which can be obtained by Hölder's inequality on time scales. \square



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Lemma 2.8. Let $a, b \in \mathbb{T}$. For a given positive integer $p \geq 2$, if $f : [a, b] \rightarrow \mathbb{R}$ is rd-continuous and $0 < m \leq \frac{f}{g} \leq M < \infty$ on $[a, b]$, then

$$(2.13) \quad \int_a^b [f(x)]^{\frac{1}{p}} \Delta x \leq \left(\int_a^b f(x) \Delta x \right)^{1 - \frac{1}{p}}.$$

Proof. Putting $g(x) \equiv 1$ in (2.11) yields

$$\int_a^b [f(x)]^{\frac{1}{p}} \Delta x \leq K \left(\int_a^b f(x) \Delta x \right)^{1 - \frac{1}{p}},$$

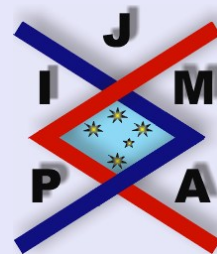
where $K = \frac{M^{\frac{1}{p^2}}(b-a)^{\frac{1}{p}}}{m^{(1-\frac{1}{p})^2}}$. From $M \leq \frac{m^{(p-1)^2}}{(b-a)^p}$, we conclude that $K \leq 1$. Thus the inequality (2.13) is proved. \square

In the following we generalize to arbitrary time scales a result in [6].

Theorem 2.9. Let $a, b \in \mathbb{T}$. If $f, g : [a, b] \rightarrow \mathbb{R}$ is rd-continuous and satisfying $0 < m \leq \frac{f}{g} \leq M < \infty$ on $[a, b]$, then we have the following inequality

$$(2.14) \quad \left(\int_a^b f^p(x) \Delta x \right)^{\frac{1}{p}} + \left(\int_a^b g^p(x) \Delta x \right)^{\frac{1}{p}} \leq c \left(\int_a^b (f(x) + g(x))^p \Delta x \right)^{1 - \frac{1}{p}},$$

where $c = \left(\frac{m}{M} \right)^{\frac{1}{pq}}$.



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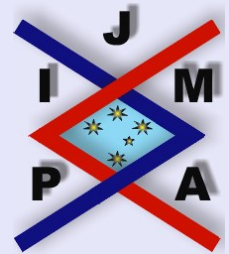
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Proof. It follows from Lemma 2.1 that

$$\begin{aligned}
 & \int_a^b (f(x) + g(x))^p \Delta x \\
 &= \int_a^b (f(x) + g(x))^{p-1} f(x) \Delta x + \int_a^b (f(x) + g(x))^{p-1} g(x) \Delta x \\
 &\geq \left(\frac{M}{m}\right)^{\frac{1}{pq}} \left(\int_a^b f^p(x) \Delta x\right)^{\frac{1}{p}} \left(\int_a^b (f(x) + g(x))^{q(p-1)} \Delta x\right)^{\frac{1}{q}} \\
 &\quad + \left(\frac{M}{m}\right)^{\frac{1}{pq}} \left(\int_a^b g^p(x) \Delta x\right)^{\frac{1}{p}} \left(\int_a^b (f(x) + g(x))^{q(p-1)} \Delta x\right)^{\frac{1}{q}} \\
 &= \left(\frac{M}{m}\right)^{\frac{1}{pq}} \left(\int_a^b (f(x) + g(x))^p \Delta x\right)^{\frac{1}{q}} \\
 &\quad \times \left[\left(\int_a^b f^p(x) \Delta x\right)^{\frac{1}{p}} + \left(\int_a^b g^p(x) \Delta x\right)^{\frac{1}{p}} \right].
 \end{aligned}$$

Therefore, we obtain

$$\begin{aligned}
 & \left[\left(\int_a^b f^p(x) \Delta x\right)^{\frac{1}{p}} + \left(\int_a^b g^p(x) \Delta x\right)^{\frac{1}{p}} \right] \\
 &\leq \left(\frac{m}{M}\right)^{\frac{1}{pq}} \left(\int_a^b (f(x) + g(x))^p \Delta x\right)^{1-\frac{1}{q}} \\
 &= \left(\frac{m}{M}\right)^{\frac{1}{pq}} \left(\int_a^b (f(x) + g(x))^p \Delta x\right)^p,
 \end{aligned}$$



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where $q(p - 1) = p$. □

Example 2.1. Let $\mathbb{T} = \mathbb{Z}$. Let $f(x) = 3^x$ and $g(x) = x^2$ on $[3, 4]$ with $M \approx 5.06$ and $m = 3$. Taking $p = 2$, we see that the conditions of Lemma 2.1 are fulfilled. Therefore, for

$$\left(\int_3^4 3^{2x} \Delta x \right)^{\frac{1}{2}} = \left(\frac{1}{8}(3^8 - 3^6) \right)^{\frac{1}{2}} = 3^3,$$

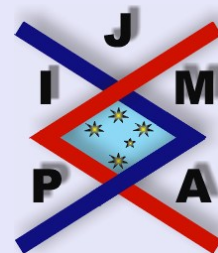
$$\left(\int_3^4 x^4 \Delta x \right)^{\frac{1}{2}} = \left(\sum_{x=3}^{4-1} x^4 \right)^{\frac{1}{2}} = 3^2$$

and

$$\int_3^4 3^x x^2 \Delta x = \sum_{x=3}^{4-1} 3^x x^2 = 3^5$$

we get

$$\left(\int_3^4 3^{2x} \Delta x \right)^{\frac{1}{2}} \left(\int_3^4 x^4 \Delta x \right)^{\frac{1}{2}} = 243 \leq \left(\frac{5.06}{3} \right)^{\frac{1}{4}} \int_3^4 3^x x^2 \Delta x \approx 274.6.$$



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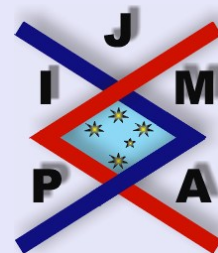
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