

A NOTE ON THE MODULUS OF U -CONVEXITY AND MODULUS OF W^* -CONVEXITY

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Abstract: We present some sufficient conditions for which a Banach space X has normal structure in term of the modulus of U -convexity, modulus of W^* -convexity and the coefficient of weak orthogonality. Some known results are improved.

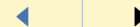
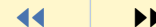
Modulus of U -Convexity and
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Zhanfei Zuo and Yunan Cui

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1. Introduction

We assume that X and X^* stand for a Banach space and its dual space, respectively. By S_X and B_X we denote the unit sphere and the unit ball of a Banach space X , respectively. Let C be a nonempty bounded closed convex subset of a Banach space X . A mapping $T : C \rightarrow C$ is said to be nonexpansive provided the inequality

$$\|Tx - Ty\| \leq \|x - y\|$$

holds for every $x, y \in C$. A Banach space X is said to have the fixed point property if every nonexpansive mapping $T : C \rightarrow C$ has a fixed point, where C is a nonempty bounded closed convex subset of a Banach space X .

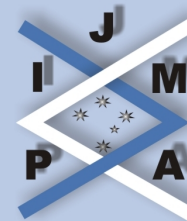
Recall that a Banach space X is said to be uniformly non-square if there exists $\delta > 0$ such that $\|x + y\|/2 \leq 1 - \delta$ or $\|x - y\|/2 \leq 1 - \delta$ whenever $x, y \in S_X$. A bounded convex subset K of a Banach space X is said to have normal structure if for every convex subset H of K that contains more than one point, there exists a point $x_0 \in H$ such that

$$\sup\{\|x_0 - y\| : y \in H\} < \sup\{\|x - y\| : x, y \in H\}.$$

A Banach space X is said to have weak normal structure if every weakly compact convex subset of X that contains more than one point has normal structure. In reflexive spaces, both notions coincide. A Banach space X is said to have uniform normal structure if there exists $0 < c < 1$ such that for any closed bounded convex subset K of X that contains more than one point, there exists $x_0 \in K$ such that

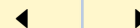
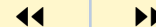
$$\sup\{\|x_0 - y\| : y \in K\} < c \sup\{\|x - y\| : x, y \in K\}.$$

It was proved by W.A. Kirk that every reflexive Banach space with normal structure has the fixed point property (see [9]).



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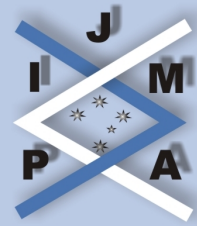


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The WORTH property was introduced by B. Sims in [15] as follows: a Banach space X has the WORTH property if

$$\lim_{n \rightarrow \infty} \left| \|x_n + x\| - \|x_n - x\| \right| = 0$$

for all $x \in X$ and all weakly null sequences $\{x_n\}$. In [16], Sims introduced the following geometric constant

$$\omega(X) = \sup \left\{ \lambda > 0 : \lambda \cdot \liminf_{n \rightarrow \infty} \|x_n + x\| \leq \liminf_{n \rightarrow \infty} \|x_n - x\| \right\},$$

where the supremum is taken over all the weakly null sequences $\{x_n\}$ in X and all elements x of X . It was proved that $\frac{1}{3} \leq \omega(X) \leq 1$. It is known that X has the WORTH property if and only if $\omega(X) = 1$. We also note here that $\omega(X) = \omega(X^*)$ in a reflexive Banach space (see [7]).

In [1] and [2], Gao introduced the modulus of U -convexity and modulus of W^* -convexity of a Banach space X , respectively, as follows:

$$U_X(\epsilon) := \inf \left\{ 1 - \frac{1}{2} \|x + y\| : x, y \in S_X, f(x - y) \geq \epsilon \text{ for some } f \in \nabla_x \right\},$$

$$W_X^*(\epsilon) := \inf \left\{ \frac{1}{2} f(x - y) : x, y \in S_X, \|x - y\| \geq \epsilon \text{ for some } f \in \nabla_x \right\}.$$

Here $\nabla_x := \{f \in S_{X^*} : f(x) = \|x\|\}$. S. Saejung (see [11], [12]) studied the above modulus extensively, and obtained some useful results as follows :

- (1) If $U_X(\epsilon) > 0$ or $W^*(\epsilon) > 0$ for some $\epsilon \in (0, 2)$, then X is uniformly non-square.
- (2) If $U_X(\epsilon) > \frac{1}{2} \max\{0, \epsilon - 1\}$ for some $\epsilon \in (0, 2)$, then X has uniform normal structure. Further, if $U_X(\epsilon) > \max\{0, \epsilon - 1\}$ for some $\epsilon \in (0, 2)$, then X and X^* has uniform normal structure.

(3) If $W_X^*(\epsilon) > \frac{1}{2} \max\{0, \epsilon - 1\}$ for some $\epsilon \in (0, 2)$, then X and X^* has uniform normal structure.

In a recent paper [4], Gao introduced the following quadratic parameter, which is defined as

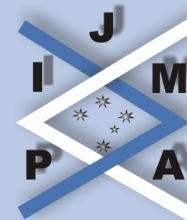
$$E(X) = \sup \{ \|x + y\|^2 + \|x - y\|^2 : x, y \in S_X \}.$$

The constant is also a significant tool in the geometric theory of Banach spaces. Furthermore, Gao obtained the values of $E(X)$ for some classical Banach spaces. In terms of the constant, he obtained some sufficient conditions for a Banach space X to have uniform normal structure, which plays an important role in fixed point theory.

In this paper, we will show that a Banach space X has uniform normal structure whenever

$$U_X(1 + \omega(X)) > \frac{1 - \omega(X)}{2} \quad \text{or} \quad W_X^*(1 + \omega(X)) > \frac{1 - \omega(X)}{2}.$$

These results improve S. Saejung's and Gao's results. Furthermore, sufficient conditions for uniform normal structure in terms of $E(X)$ and $\omega(X)$ have been obtained which improve the results in [3].



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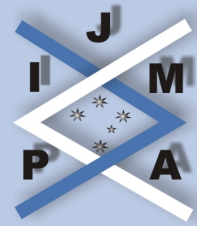
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2. Uniform Normal Structure

As our proof uses the ultraproduct technique, we start by making some basic definitions. Let \mathcal{U} be a filter on I . Then, $\{x_i\}$ is said to be convergent to x with respect to \mathcal{U} , denoted by $\lim_{\mathcal{U}} x_i = x$, if for each neighborhood V of x , $\{i \in I : x_i \in V\} \in \mathcal{U}$. A filter \mathcal{U} on I is called an ultrafilter if it is maximal with respect to the ordering of set inclusion. An ultrafilter is called trivial if it is of the form $\{A : A \subseteq I, i_0 \in A\}$ for some $i_0 \in I$. We will use the fact that if \mathcal{U} is an ultrafilter, then

- (1) for any $A \subseteq I$, either $A \in \mathcal{U}$ or $I \setminus A \in \mathcal{U}$;
- (2) if $\{x_i\}$ has a cluster point x , then $\lim_{\mathcal{U}} x_i$ exists and equals x .

Let $\{X_i\}$ be a family of Banach spaces and $l_{\infty}(I, X_i)$ denote the subspace of the product space equipped with the norm $\|(x_i)\| = \sup_{i \in I} \|x_i\| < \infty$. Let \mathcal{U} be an ultrafilter on I and $N_{\mathcal{U}} = \{(x_i) \in l_{\infty}(I, X_i) : \lim_{\mathcal{U}} \|x_i\| = 0\}$. The ultraproduct of $\{X_i\}_{i \in I}$ is the quotient space $l_{\infty}(I, X_i)/N_{\mathcal{U}}$ equipped with the quotient norm. We will use $\widetilde{(x_i)}_{\mathcal{U}}$ to denote the element of the ultraproduct. In the following, we will restrict our set I to be \mathbb{N} (the set of \mathcal{U} natural numbers), and let $X_i = X$, $i \in \mathbb{N}$, for some Banach space X . For an ultrafilter \mathcal{U} on \mathbb{N} , we use $\widetilde{X}_{\mathcal{U}}$ to denote the ultraproduct. Note that if \mathcal{U} is nontrivial, then X can be embedded into $\widetilde{X}_{\mathcal{U}}$ isometrically.

Lemma 2.1 (see [5]). *Let X be a Banach space without weak normal structure, then there exists a weakly null sequence $\{x_n\}_{n=1}^{\infty} \subseteq S_X$ such that*

$$\lim_n \|x_n - x\| = 1 \text{ for all } x \in \text{co}\{x_n\}_{n=1}^{\infty}$$

Theorem 2.2. *If $U_X(1 + \omega(X)) > \frac{1 - \omega(X)}{2}$, then X has uniform normal structure.*

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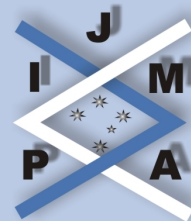
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Proof. It suffices to prove that X has weak normal structure whenever

$$U_X(1 + \omega(X)) > \frac{1 - \omega(X)}{2}.$$

In fact, since $\frac{1}{3} \leq \omega(X) \leq 1$, we have

$$U_X(\epsilon) > \frac{1 - \omega(X)}{2} \geq 0$$

for some $\epsilon \in (0, 2)$. This implies that X is super-reflexive, and then $U_X(\epsilon) = U_{\bar{X}}(\epsilon)$ (see [11]). Now suppose that X fails to have weak normal structure. Then, by the Lemma 2.1, there exists a weakly null sequence $\{x_n\}_{n=1}^{\infty}$ in S_X such that

$$\lim_n \|x_n - x\| = 1 \text{ for all } x \in \text{co}\{x_n\}_{n=1}^{\infty}.$$

Take $\{f_n\} \subset S_{X^*}$ such that $f_n \in \nabla_{x_n}$ for all $n \in \mathbb{N}$. By the reflexivity of X^* , without loss of generality we may assume that $f_n \rightharpoonup f$ for some $f \in B_{X^*}$ (where \rightharpoonup denotes weak star convergence). We now choose a subsequence of $\{x_n\}_{n=1}^{\infty}$, denoted again by $\{x_n\}_{n=1}^{\infty}$, such that

$$\lim_n \|x_{n+1} - x_n\| = 1, \quad |(f_{n+1} - f)(x_n)| < \frac{1}{n}, \quad f_n(x_{n+1}) < \frac{1}{n}$$

for all $n \in \mathbb{N}$. It follows that

$$\lim_n f_{n+1}(x_n) = \lim_n (f_{n+1} - f)(x_n) + f(x_n) = 0.$$

Put $\tilde{x} = (x_{n+1} - x_n)_U$, $\tilde{y} = [\omega(X)(x_{n+1} + x_n)]_U$, and $\tilde{f} = (-f_n)_U$. By the definition of $\omega(X)$ and Lemma 2.1, then

$$\|\tilde{f}\| = \tilde{f}(\tilde{x}) = \|\tilde{x}\| = 1$$

and

$$\|\tilde{y}\| = \|\omega(X)(x_{n+1} + x_n)\| \leq \|x_{n+1} - x_n\| = 1.$$

Furthermore, we have

$$\begin{aligned} \tilde{f}(\tilde{x} - \tilde{y}) &= \lim_U (-f_n) \left((1 - \omega(X))x_{n+1} - (1 + \omega(X))x_n \right) \\ &= 1 + \omega(X), \end{aligned}$$

$$\begin{aligned} \|\tilde{x} + \tilde{y}\| &= \lim_U \|(1 + \omega(X))x_{n+1} - (1 - \omega(X))x_n\| \\ &\geq \lim_U (f_{n+1}) \left((1 + \omega(X))x_{n+1} - (1 - \omega(X))x_n \right) \\ &= 1 + \omega(X). \end{aligned}$$

From the definition of $U_X(\epsilon)$, we have

$$U_X(1 + \omega(X)) = U_{\tilde{X}}(1 + \omega(X)) \leq \frac{1 - \omega(X)}{2},$$

which is a contradiction. Therefore

$$U_X(1 + \omega(X)) > \frac{1 - \omega(X)}{2}$$

implies that X has uniform normal structure. \square

Remark 1. Compare to the result of S. Saejung (2). Let $\epsilon = 1 + \omega(X)$. Then $U_X(\epsilon) > \frac{2-\epsilon}{2}$ implies that X has uniform normal structure from Theorem 2.2. It is well known that $\frac{1}{3} \leq \omega(X) \leq 1$, therefore $\frac{\epsilon-1}{2} > \frac{2-\epsilon}{2}$ whenever $\omega(X) > \frac{1}{2}$, therefore Theorem 2.2 strengthens the result of S. Saejung (2).



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The modulus of convexity of X is the function $\delta_X(\epsilon) : [0, 2] \rightarrow [0, 1]$ defined by

$$\begin{aligned}\delta_X(\epsilon) &= \inf \left\{ 1 - \frac{\|x + y\|}{2} : x, y \in S_X, \|x - y\| = \epsilon \right\} \\ &= \inf \left\{ 1 - \frac{\|x + y\|}{2} : \|x\| \leq 1, \|y\| \leq 1, \|x - y\| \geq \epsilon \right\}.\end{aligned}$$

The function $\delta_X(\epsilon)$ is strictly increasing on $[\epsilon_0(X), 2]$. Here $\epsilon_0(X) = \sup\{\epsilon : \delta_X(\epsilon) = 0\}$ is the characteristic of convexity of X . Also, X is uniformly non-square provided $\epsilon_0(X) < 2$. Some sufficient conditions for which a Banach space X has uniform normal structure in terms of the modulus of convexity have been widely studied in [3], [5], [13], [18]. It is easy to prove that $U_X(\epsilon) \geq \delta_X(\epsilon)$, therefore we have the following corollary which strengthens Theorem 6 of Gao [3].

Corollary 2.3. *If $\delta_X((1 + \omega(X))) > \frac{1 - \omega(X)}{2}$, then X has uniform normal structure.*

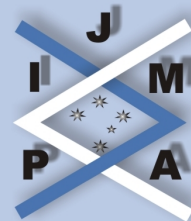
Remark 2. In fact, it is well known that $J(X) < \epsilon$ if and only if $\delta_X(\epsilon) > 1 - \frac{\epsilon}{2}$ (see [6]). Therefore Corollary 2.3 is equivalent to $J(X) < 1 + \omega(X)$ implies that X has uniform normal structure (see [7, Theorem 2]). Moreover, if X is the Bynum space $b_{2, \infty}$, then X does not have normal structure and $\delta_X((1 + \omega(X))) = \frac{1 - \omega(X)}{2}$. Hence Theorem 2.2 and Corollary 2.3 are sharp.

It is well known that $\epsilon_0(X) = 2\rho'_{X^*}(0)$. Here, $\rho'_X(0) = \lim_{t \rightarrow 0} \frac{\rho_X(t)}{t}$, where $\rho_X(t)$ is the modulus of smoothness defined as

$$\rho_X(t) = \sup \left\{ \frac{\|x + ty\| + \|x - ty\|}{2} - 1 : x, y \in S_X \right\}.$$

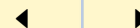
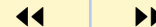
Therefore we have the following corollary.

Corollary 2.4. *If $\delta_X(2\omega(X)) > \frac{1 - \omega(X)}{2}$, then X and X^* have uniform normal structure.*



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Proof. From $2\omega(X) \leq 1 + \omega(X)$ and the monotonicity of $\delta_X(\epsilon)$, we have that X has uniform normal structure from Corollary 2.3. It is well known that $\omega(X) = \omega(X^*)$ in a reflexive Banach space. So the inequality $\rho'_{X^*}(0) < \omega(X)$, or, equivalently, $\epsilon_0(X) < 2\omega(X)$ imply X^* has uniform normal structure (see [10], [13]). From the definition of $\epsilon_0(X)$, obviously the condition $\delta_X(2\omega(X)) > \frac{1-\omega(X)}{2}$ implies that $\epsilon_0(X) < 2\omega(X)$. So X^* have uniform normal structure. \square

Theorem 2.5. *If $W_X^*(1 + \omega(X)) > \frac{1-\omega(X)}{2}$, then X has uniform normal structure.*

Proof. It suffices to prove that X has weak normal structure whenever $W_X^*(1 + \omega(X)) > \frac{1-\omega(X)}{2}$. In fact, since $\frac{1}{3} \leq \omega(X) \leq 1$, we have $W_X^*(2\epsilon) > \frac{1-\omega(X)}{2} \geq 0$ for some $\epsilon \in (0, 2)$. This implies that X is super-reflexive, and $W_X^*(\epsilon) = W_{\tilde{X}}^*(\epsilon)$ (see [12]). Repeating the arguments in the proof of Theorem 2.2, and $\tilde{x} = (x_n - x_{n+1})_{\mathcal{U}}$, $\tilde{y} = [\omega(X)(x_{n+1} + x_n)]_{\mathcal{U}}$, and $\tilde{f} = (f_n)_{\mathcal{U}}$. Then

$$f(\tilde{x}) = \|\tilde{x}\| = 1 \quad \text{and} \quad \|\tilde{y}\| \leq 1.$$

Furthermore, we have

$$\begin{aligned} \|\tilde{x} - \tilde{y}\| &= \lim_{\mathcal{U}} \|(1 + \omega(X))x_{n+1} - (1 - \omega(X))x_n\| \\ &\geq \lim_{\mathcal{U}} (f_{n+1}) \left((1 + \omega(X))x_{n+1} - (1 - \omega(X))x_n \right) = 1 + \omega(X), \end{aligned}$$

$$\begin{aligned} \frac{1}{2} \tilde{f}(\tilde{x} - \tilde{y}) &= \frac{1}{2} \lim_{\mathcal{U}} (f_n) \left((1 - \omega(X))x_n - (1 + \omega(X))x_{n+1} \right) \\ &= \frac{1 - \omega(X)}{2}. \end{aligned}$$

However, this implies

$$W_X^*(1 + \omega(X)) = W_{\tilde{X}}^*(1 + \omega(X)) \leq \frac{1 - \omega(X)}{2}$$

which is a contradiction. Therefore

$$W_X^*(1 + \omega(X)) > \frac{1 - \omega(X)}{2}$$

implies that X has uniform normal structure. □

Remark 3. Similarly, the above theorem strengthens the result of S. Saejung (3), whenever $\omega(X) > \frac{1}{2}$. Since $W_X^*(\epsilon) \geq \delta_X(\epsilon)$, therefore we also obtain Corollary 2.3 from Theorem 2.5.

The following theorem can be found in [14].

Theorem 2.6. *Let X be a Banach space, we have*

$$E(X) = \sup\{\epsilon^2 + 4(1 - \delta_X(\epsilon))^2 : \epsilon \in (0, 2]\}$$

Remark 4. Letting $\epsilon \rightarrow 2^-$ in Theorem 2.6, we obtain the following inequality

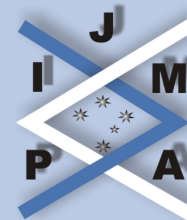
$$E(X) \geq 4 + [\epsilon_0(X)]^2.$$

Corollary 2.7. *If $E(X) < 2(1 + \omega(X))^2$, then X and X^* have uniform normal structure.*

Proof. From Theorem 2.6, $E(X) < 2(1 + \omega(X))^2$ implies that $\delta_X((1 + \omega(X)) > \frac{1 - \omega(X)}{2})$, so X has uniform normal structure from Corollary 2.3. It is well known that $\epsilon_0(X) < 2\omega(X)$ implies that X^* have uniform normal structure. Therefore, from Remark 4, $E(X) < 4(1 + \omega(X)^2)$ implies that X^* have uniform normal structure. Obviously

$$E(X) < 2(1 + \omega(X))^2 \leq 4(1 + \omega(X)^2)$$

implies X^* have uniform normal structure. □



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Remark 5. In [3], Gao obtained that if $E(X) < 1 + 2\omega(X) + 5(\omega(X))^2$, then X has uniform normal structure. Comparing the result of Gao and Corollary 2.7, we have the following equality

$$2(1 + \omega(X))^2 - 1 - 2\omega(X) - 5(\omega(X))^2 = (1 - \omega(X))(3\omega(X) + 1).$$

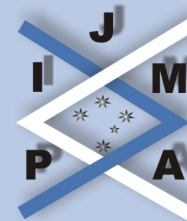
It is well known that $\frac{1}{3} \leq \omega(X) \leq 1$, so when $\omega(X) < 1$, we have

$$(1 - \omega(X))(3\omega(X) + 1) > 0.$$

Therefore Corollary 2.7 is strict generalization of Gao's result. Moreover this is extended to conclude uniform normal structure for X^* . In fact repeating the arguments in [7], we have that $E(b_{2, \infty}) = 3 + 2\sqrt{2}$, where $b_{2, \infty}$ is the Bynum space which does not have normal structure and $E(X) = 2(1 + \omega(X))^2$ (note that $\omega(b_{2, \infty}) = \frac{\sqrt{2}}{2}$). Therefore Corollary 2.7 is sharp.

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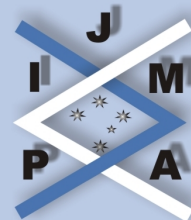
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