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A GENERALIZED CLASS OF k-STARLIKE FUNCTIONS WITH VARYING ARGUMENTS OF COEFFICIENTS

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ABSTRACT. In terms of Wright generalized hypergeometric function we define a class of analytic functions. The class generalize well known classes of k-starlike functions and k-uniformly convex functions. Necessary and sufficient coefficient bounds are given for functions in this class. Further distortion bounds, extreme points and results on partial sums are investigated.

Key words and phrases: Univalent functions, starlike functions, varying arguments, coefficient estimates.

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1. INTRODUCTION

Let A denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1.1}$$

which are analytic in the open unit disc $U = \{z : |z| < 1\}$. We denote by S the subclass of A consisting of functions f which are univalent in U.

Also we denote by V, the class of analytic functions with varying arguments (introduced by Silverman [16]) consisting of functions f of the form (1.1) for which there exists a real number η such that

$$\theta_n + (n-1)\eta = \pi (\mathrm{mod}\ 2\pi), \quad \text{where} \quad \arg(a_n) = \theta_n \quad \text{for all} \quad n \geq 2. \tag{1.2}$$

Let k, γ be real parameters with $k \geq 0,\ -1 \leq \gamma < 1$.

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Definition 1.1. A function $f \in A$ is said to be in the class $UCV(k,\gamma)$ of k-uniformly convex functions of order γ if it satisfies the condition

$$\mathrm{Re}\left\{1+\frac{zf''(z)}{f'(z)}-\gamma\right\} > k\left|\frac{zf''(z)}{f'(z)}\right|, \quad z \in U.$$

In particular, the classes $UCV := UCV(1,0)$, $k-UCV := UCV(k,0)$ were introduced by Goodman [6] (see also [10, 13]), and Kanas and Wisniowska [8] (see also [7]), respectively, where their geometric definition and connections with the conic domains were considered.

Related to the class $UCV(k,\gamma)$ by means of the well-known Alexander equivalence between the usual classes of convex and starlike functions, we define the class $SP(k,\gamma)$ of k-starlike functions of order γ.

Definition 1.2. A function $f \in A$ is said to be in the class $SP(k,\gamma)$ of k-starlike functions of order γ if it satisfies the condition

$$\mathrm{Re}\left\{\frac{zf'(z)}{f(z)}-\gamma\right\} > k\left|\frac{zf'(z)}{f(z)}-1\right|, \quad z \in U.$$

The classes $S_p := SP(1,0)$, $k-ST := SP(k,0)$ were investigated by Rønning [13, 14], Kanas and Wisniowska [9], Kanas and Srivastava [7].

Note that the classes

$$ST := SP(0,0), \quad CV := UCV(0,0)$$

are the well known classes of starlike and convex functions, respectively.

For functions $f \in A$ given by (1.1) and $g \in A$ given by

$$g(z) = z + \sum_{n=2}^{\infty} b_n z^n, \quad z \in U,$$

we define the Hadamard product (or convolution) of f and g by

$$(f * g)(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n, \quad z \in U.$$

For positive real parameters $\alpha_1, A_1, \ldots, \alpha_p, A_p$ and $\beta_1, B_1, \ldots, \beta_q, B_q$ $(p,q \in N = 1,2,3,\ldots)$ such that

$$1 + \sum_{n=1}^{q} B_n - \sum_{n=1}^{p} A_n \geq 0, \tag{1.3}$$

the Wright generalized hypergeometric function [24]

$${}_p\Psi_q[(\alpha_1,A_1),\ldots,(\alpha_p,A_p);(\beta_1,B_1),\ldots,(\beta_q,B_q);z] = {}_p\Psi_q[(\alpha_n,A_n)_{1,p};(\beta_n,B_n)_{1,q};z]$$

is defined by

$${}_p\Psi_q[(\alpha_t,A_t)_{1,p};(\beta_t,B_t)_{1,q};z] = \sum_{n=0}^{\infty}\left\{\prod_{t=0}^{p}\Gamma(\alpha_t+nA_t\right\}\left\{\prod_{t=0}^{q}\Gamma(\beta_t+nB_t\right\}^{-1}\frac{z^n}{n!}, \quad z \in U.$$

If $p \leq q+1$, $A_n = 1$ $(n = 1,\ldots,p)$ and $B_n = 1$ $(n = 1,\ldots,q)$, we have the relationship:

$$\Omega\,{}_p\Psi_q[(\alpha_n,1)_{1,p};(\beta_n,1)_{1,q};z] = {}_pF_q(\alpha_1,\ldots,\alpha_p;\beta_1,\ldots,\beta_q;z), \quad z \in U, \tag{1.4}$$

where ${}_pF_q(\alpha_1,\ldots,\alpha_p;\beta_1,\ldots,\beta_q;z)$ is the generalized hypergeometric function and

$$\Omega = \left(\prod_{t=0}^{p}\Gamma(\alpha_t)\right)^{-1}\left(\prod_{t=0}^{q}\Gamma(\beta_t)\right). \tag{1.5}$$

In [3] Dziok and Raina defined the linear operator by using Wright generalized hypergeometric function. Let

$$ {}_p\phi_q[(\alpha_t, A_t)_{1,p}; (\beta_t, B_t)_{1,q}; z] = \Omega z\ {}_p\Psi_q[(\alpha_t, A_t)_{1,p}(\beta_t, B_t)_{1,q}; z], \quad z \in U, $$

and

$$ \mathcal{W} = \mathcal{W}[(\alpha_n, A_n)_{1,p}; (\beta_n, B_n)_{1,q}] : A \to A $$

be a linear operator defined by

$$ \mathcal{W}f(z) := z\ {}_p\phi_q[(\alpha_t, A_t)_{1,p}; (\beta_t, B_t)_{1,q}; z] * f(z), \quad z \in U. $$

We observe that, for f of the form (1.1), we have

$$ \mathcal{W}f(z) = z + \sum_{n=2}^{\infty} \sigma_n\, a_n z^n, \quad z \in U, \tag{1.6} $$

where

$$ \sigma_n = \frac{\Omega\, \Gamma(\alpha_1 + A_1(n-1)) \cdots \Gamma(\alpha_p + A_p(n-1))}{(n-1)!\Gamma(\beta_1 + B_1(n-1)) \cdots \Gamma(\beta_q + B_q(n-1))}\ , $$

and Ω is given by (1.5).

In view of the relationship (1.4), the linear operator (1.6) includes the Dziok-Srivastava operator (see [5]) and other operators. For more details on these operators, see [1], [2], [4], [11], [12], [15] and [19].

Motivated by the earlier works of Kanas and Srivastava [7], Srivastava and Mishra [20] and Vijaya and Murugusundaramoorthy [23], we define a new class of functions based on generalized hypergeometric functions.

Corresponding to the family $SP(\gamma, k)$, we define the class $W_q^p(k, \gamma)$ for a function f of the form (1.1) such that

$$ \operatorname{Re}\left\{ \frac{z(\mathcal{W}f(z))'}{\mathcal{W}f(z)} - \gamma \right\} \geq k \left| \frac{z(\mathcal{W}f(z))'}{\mathcal{W}f(z)} - 1 \right|, \quad z \in U. \tag{1.7} $$

We also let

$$ VW_q^p(k, \gamma) = V \cap W_q^p(k, \gamma). $$

The class $W_q^p(k, \gamma)$ generalizes the classes of k-uniformly convex functions and k-starlike functions. If $p = 2,\ q = 1,\ A_1 = A_2 = B_1 = \alpha_1 = \beta_1 = 1$, then for $\alpha_2 = 2$ we have

$$ W_1^2(k, 0) = k - UCV, $$

and for $\alpha_2 = 1$ we have

$$ W_1^2(k, 0) = k - ST. $$

In this paper we obtain a sufficient coefficient condition for functions f given by (1.1) to be in the class $W_q^p(k, \gamma)$ and we show that it is also a necessary condition for functions to belong to this class. Distortion results and extreme points for functions in $VW_q^p(k, \gamma)$ are obtained. Finally, we investigate partial sums for the class $VW_q^p(k, \gamma)$.

2. MAIN RESULTS

First we obtain a sufficient condition for functions from the class A to belong to the class $W_q^p(k,\gamma)$.

Theorem 2.1. *Let f be given by (1.1). If*

$$\sum_{n=2}^{\infty}(kn+n-k-\gamma)\sigma_n|a_n| \leq 1-\gamma, \tag{2.1}$$

then $f \in W_q^p(k,\gamma)$.

Proof. By definition of the class $W_q^p([\alpha_1],\gamma)$, it suffices to show that

$$k\left|\frac{z(\mathcal{W}f(z))'}{\mathcal{W}f(z)}-1\right| - \operatorname{Re}\left\{\frac{z(\mathcal{W}f(z))'}{\mathcal{W}f(z)}-1\right\} \leq 1-\gamma, \quad z \in U.$$

Simple calculations give

$$\begin{aligned} k\left|\frac{z(\mathcal{W}f(z))'}{\mathcal{W}f(z)}-1\right| &- \operatorname{Re}\left\{\frac{z(\mathcal{W}f(z))'}{\mathcal{W}f(z)}-\gamma\right\} \\ &\leq (k+1)\left|\frac{z(\mathcal{W}f(z))'}{\mathcal{W}f(z)}-1\right| \\ &\leq (k+1)\,\frac{\sum_{n=2}^{\infty}(n-1)\sigma_n|a_n||z|^{n-1}}{1-\sum_{n=2}^{\infty}\sigma_n|a_n||z|^{n-1}}. \end{aligned}$$

Now the last expression is bounded above by $(1-\gamma)$ if (2.1) holds. □

In the next theorem, we show that the condition (2.1) is also necessary for functions from the class $VW_q^p(k,\gamma)$.

Theorem 2.2. *Let f be given by (1.1) and satisfy (1.2).Then the function f belongs to the class $VW_q^p(k,\gamma)$ if and only if (2.1) holds.*

Proof. In view of Theorem 2.1 we need only to show that $f \in VW_q^p(k,\gamma)$ satisfies the coefficient inequality (2.1). If $f \in VW_q^p(k,\gamma)$ then by definition, we have

$$k\left|\frac{z+\sum_{n=2}^{\infty}n\sigma_n a_n z^n}{z+\sum_{n=2}^{\infty}\sigma_n a_n z^n}-1\right| \leq \operatorname{Re}\left\{\frac{z+\sum_{n=2}^{\infty}n\sigma_n a_n z^n}{z+\sum_{n=2}^{\infty}\sigma_n a_n z^n}-\gamma\right\},$$

or

$$k\left|\frac{\sum_{n=2}^{\infty}(n-1)\sigma_n a_n z^{n-1}}{1+\sum_{n=2}^{\infty}\sigma_n a_n z^{n-1}}\right| \leq \operatorname{Re}\left\{\frac{(1-\gamma)+\sum_{n=2}^{\infty}(n-\gamma)\sigma_n a_n z^{n-1}}{1+\sum_{n=2}^{\infty}\sigma_n a_n z^{n-1}}\right\}.$$

In view of (1.2), we set $z = r^{i\eta}$ in the above inequality to obtain

$$\frac{\sum_{n=2}^{\infty}k(n-1)\sigma_n|a_n|r^{n-1}}{1-\sum_{n=2}^{\infty}\sigma_n|a_n|r^{n-1}} \leq \frac{(1-\gamma)-\sum_{n=2}^{\infty}(n-\gamma)\sigma_n|a_n|r^{n-1}}{1-\sum_{n=2}^{\infty}\sigma_n|a_n|r^{n-1}}.$$

Thus

$$\sum_{n=2}^{\infty}(kn+n-k-\gamma)\sigma_n|a_n|r^{n-1} \leq 1-\gamma, \tag{2.2}$$

and letting $r \to 1^-$ in (2.2), we obtain the desired inequality (2.1). □

Corollary 2.3. *If a function f of the form (1.1) belongs to the class $VW_q^p(k,\gamma)$, then*

$$|a_n| \le \frac{1-\gamma}{(kn+n-k-\gamma)\sigma_n}, \quad n = 2,3,\dots.$$

The equality holds for the functions

$$h_{n,\eta}(z) = z - \frac{(1-\gamma)\, e^{i(1-n)\eta}}{(kn+n-k-\gamma)\sigma_n} z^n, \quad z \in U;\ 0 \le \eta < 2\pi,\ n = 2,3,\dots. \tag{2.3}$$

Next we obtain the distortion bounds for functions belonging to the class $VW_q^p(k,\gamma)$.

Theorem 2.4. *Let f be in the class $VW_q^p(k,\gamma),\ |z| = r < 1$. If the sequence*

$$\{(kn+n-k-\gamma)\sigma_n\}_{n=2}^{\infty}$$

is nondecreasing, then

$$r - \frac{1-\gamma}{(k-\gamma+2)\sigma_2} r^2 \le |f(z)| \le r + \frac{1-\gamma}{(k-\gamma+2)\sigma_2} r^2. \tag{2.4}$$

If the sequence $\left\{\frac{kn+n-k-\gamma}{n}\sigma_n\right\}_{n=2}^{\infty}$ is nondecreasing, then

$$1 - \frac{2(1-\gamma)}{(k-\gamma+2)\sigma_2} r \le |f'(z)| \le 1 + \frac{2(1-\gamma)}{(k-\gamma+2)\sigma_2} r. \tag{2.5}$$

The result is sharp. The extremal functions are the functions $h_{2,\eta}$ of the form (2.3).

Proof. Since $f \in VW_q^p(k,\gamma)$, we apply Theorem 2.2 to obtain

$$(k-\gamma+2)\sigma_2 \sum_{n=2}^{\infty} |a_n| \le \sum_{n=2}^{\infty} (kn+n-k-\gamma)\sigma_n |a_n| \le 1-\gamma.$$

Thus

$$|f(z)| \le |z| + |z|^2 \sum_{n=2}^{\infty} |a_n| \le r + \frac{1-\gamma}{(k-\gamma+2)\sigma_2} r^2.$$

Also we have

$$|f(z)| \ge |z| - |z|^2 \sum_{n=2}^{\infty} |a_n| \ge r - \frac{1-\gamma}{(k-\gamma+2)\sigma_2} r^2$$

and (2.4) follows. In similar manner for f', the inequalities

$$|f'(z)| \le 1 + \sum_{n=2}^{\infty} n|a_n||z|^{n-1} \le 1 + |z| \sum_{n=2}^{\infty} n a_n$$

and

$$\sum_{n=2}^{\infty} n|a_n| \le \frac{2(1-\gamma)}{(k-\gamma+2)\sigma_2}$$

lead to (2.5). This completes the proof. □

Corollary 2.5. *Let f be in the class $VW_q^p(k,\gamma),\ |z| = r < 1$. If*

$$p > q,\ \alpha_{q+1} \ge 1,\ \alpha_j \ge \beta_j \quad \text{and} \quad A_j \ge B_j\ \ (j = 2,\dots,q), \tag{2.6}$$

then the assertions (2.4), (2.5) hold true.

Proof. From (2.6) we have that the sequences $\{(kn+n-k-\gamma)\sigma_n\}_{n=2}^{\infty}$ and $\left\{\frac{kn+n-k-\gamma}{n}\sigma_n\right\}_{n=2}^{\infty}$ are nondecreasing. Thus, by Theorem 2.4, we have Corollary 2.5. □

Theorem 2.6. *Let f be given by (1.1) and satisfy (1.2). Then the function f belongs to the class $VW_q^p(k,\gamma)$ if and only if f can be expressed in the form*

$$f(z) = \sum_{n=1}^{\infty} \mu_n h_{n,\eta}(z), \qquad \mu_n \geq 0 \quad \text{and} \quad \sum_{n=1}^{\infty} \mu_n = 1, \tag{2.7}$$

where $h_1(z) = z$ and $h_{n,\eta}$ are defined by (2.3).

Proof. If a function f is of the form (2.7), then by (1.2) we have

$$f(z) = z + \sum_{n=2}^{\infty} \frac{(1-\gamma)\, e^{i\theta_n}}{(kn+n-k-\gamma)\sigma_n} \mu_n z^n, \quad z \in U.$$

Since

$$\sum_{n=2}^{\infty} (kn+n-k-\gamma)\sigma_n \frac{1-\gamma}{(kn+n-k-\gamma)\sigma_n} \mu_n = \sum_{n=2}^{\infty} \mu_n (1-\gamma) = (1-\mu_1)(1-\gamma) \leq 1-\gamma,$$

by Theorem 2.2 we have $f \in VW_q^p(k,\gamma)$.

Conversely, if f is in the class $VW_q^p(k,\gamma)$, then we may set $\mu_n = \frac{(kn+n-k-\gamma)\sigma_n}{1-\gamma}$, $n \geq 2$ and $\mu_1 = 1 - \sum_{n=2}^{\infty} \mu_n$. Then the function f is of the form (2.7) and this completes the proof. $\square$

3. PARTIAL SUMS

For a function $f \in A$ given by (1.1), Silverman [17] and Silvia [18] investigated the partial sums f_1 and f_m defined by

$$f_1(z) = z; \quad \text{and} \quad f_m(z) = z + \sum_{n=2}^{m} a_n z^n, \quad (m = 2, 3 \ldots). \tag{3.1}$$

We consider in this section partial sums of functions in the class $VW_q^p(k,\gamma)$ and obtain sharp lower bounds for the ratios of the real part of f to $f_m(z)$ and f' to f'_m.

Theorem 3.1. *Let a function f of the form (1.1) belong to the class $VW_q^p(k,\gamma)$ and assume (2.6). Then*

$$\operatorname{Re}\left\{\frac{f(z)}{f_m(z)}\right\} \geq 1 - \frac{1}{d_{m+1}}, \quad z \in U,\ m \in N \tag{3.2}$$

and

$$\operatorname{Re}\left\{\frac{f_m(z)}{f(z)}\right\} \geq \frac{d_{m+1}}{1+d_{m+1}}, \quad z \in U,\ m \in N, \tag{3.3}$$

where

$$d_n := \frac{kn+n-k-\gamma}{1-\gamma}\sigma_n. \tag{3.4}$$

Proof. By (2.6) it is not difficult to verify that

$$d_{n+1} > d_n > 1, \quad n = 2, 3, \ldots. \tag{3.5}$$

Thus by Theorem 2.1 we have

$$\sum_{n=2}^{m} |a_n| + d_{m+1} \sum_{n=m+1}^{\infty} |a_n| \leq \sum_{n=2}^{\infty} d_n |a_n| \leq 1. \tag{3.6}$$

Setting

$$g(z)=d_{m+1}\left\{\frac{f(z)}{f_m(z)}-\left(1-\frac{1}{d_{m+1}}\right)\right\}=1+\frac{d_{m+1}\sum_{n=m+1}^{\infty}a_nz^{n-1}}{1+\sum_{n=2}^{m}a_nz^{n-1}}, \tag{3.7}$$

it suffices to show that

$$\operatorname{Re} g(z)\geq 0,\quad z\in U.$$

Applying (3.6), we find that

$$\left|\frac{g(z)-1}{g(z)+1}\right|\leq\frac{d_{m+1}\sum_{n=m+1}^{\infty}|a_n|}{2-2\sum_{n=2}^{n}|a_n|-d_{m+1}\sum_{n=m+1}^{\infty}|a_n|}\leq 1,\quad z\in U,$$

which readily yields the assertion (3.2) of Theorem 3.1. In order to see that

$$f(z)=z+\frac{z^{m+1}}{d_{m+1}},\quad z\in U, \tag{3.8}$$

gives sharp the result, we observe that for $z=re^{i\pi/m}$ we have

$$\frac{f(z)}{f_m(z)}=1+\frac{z^m}{d_{m+1}}\xrightarrow{z\to 1^-}1-\frac{1}{d_{m+1}}.$$

Similarly, if we take

$$\begin{aligned}h(z)&=(1+d_{m+1})\left\{\frac{f_m(z)}{f(z)}-\frac{d_{m+1}}{1+d_{m+1}}\right\}\\&=1-\frac{(1+d_{n+1})\sum_{n=m+1}^{\infty}a_nz^{n-1}}{1+\sum_{n=2}^{\infty}a_nz^{n-1}},\quad z\in U,\end{aligned}$$

and making use of (3.6), we can deduce that

$$\left|\frac{h(z)-1}{h(z)+1}\right|\leq\frac{(1+d_{m+1})\sum_{n=m+1}^{\infty}|a_n|}{2-2\sum_{n=2}^{m}|a_n|-(1-d_{m+1})\sum_{n=m+1}^{\infty}|a_n|}\leq 1,\quad z\in U,$$

which leads us immediately to the assertion (3.3) of Theorem 3.1. The bound in (3.3) is sharp for each $m\in N$ with the extremal function f given by (3.8), and the proof is complete. □

Theorem 3.2. *Let a function f of the form (1.1) belong to the class $VW_q^p(k,\gamma)$ and assume (2.6). Then*

$$\operatorname{Re}\left\{\frac{f'(z)}{f'_m(z)}\right\}\geq 1-\frac{m+1}{d_{m+1}} \tag{3.9}$$

and

$$\operatorname{Re}\left\{\frac{f'_m(z)}{f'(z)}\right\}\geq\frac{d_{m+1}}{m+1+d_{m+1}}, \tag{3.10}$$

where d_m is defined by (3.4)

Proof. By setting

$$g(z)=d_{m+1}\left\{\frac{f'(z)}{f'_m(z)}-\left(1-\frac{m+1}{d_{m+1}}\right)\right\},\quad z\in U,$$

and

$$h(z)=[(m+1)+d_{m+1}]\left\{\frac{f'_m(z)}{f'(z)}-\frac{d_{m+1}}{m+1+d_{m+1}}\right\},\quad z\in U,$$

the proof is analogous to that of Theorem 3.1, and we omit the details. □

Concluding Remarks: Observe that, if we specialize the parameters of the class $VW_q^p(k,\gamma)$, we obtain various classes introduced and studied by Goodman [6], Kanas and Srivastava [7], Ma and Minda [10], Rønning [13, 14], Murugusundaramoorthy *et al.* [22, 23], and others.

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