



INTEGRAL MEAN ESTIMATES FOR POLYNOMIALS WHOSE ZEROS ARE WITHIN A CIRCLE

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Abstract:

Let $p(z)$ be a polynomial of degree n . Zygmund [11] has shown that for $s \geq 1$

$$\left(\int_0^{2\pi} |p'(e^{i\theta})|^s d\theta \right)^{1/s} \leq n \left(\int_0^{2\pi} |p(e^{i\theta})|^s d\theta \right)^{1/s}.$$

In this paper, we have obtained inequalities in the reverse direction for the polynomials having a zero of order m at the origin. We also consider a problem for the class of polynomials $p(z) = a_n z^n + \sum_{\nu=\mu}^n a_{n-\nu} z^{n-\nu}$ not vanishing outside the disk $|z| < k$, $k \leq 1$ and obtain a result which, besides yielding some interesting results as corollaries, includes a result due to Aziz and Shah [*Indian J. Pure Appl. Math.*, 28 (1997), 1413–1419] as a special case.

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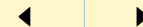
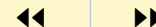
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1. Introduction and Statement of Results

Let $p(z)$ be a polynomial of degree n and $p'(z)$ its derivative. It was shown by Turán [10] that if $p(z)$ has all its zeros in $|z| \leq 1$, then

$$(1.1) \quad \max_{|z|=1} |p'(z)| \geq \frac{n}{2} \max_{|z|=1} |p(z)|.$$

More generally, if the polynomial $p(z)$ has all its zeros in $|z| \leq k$, $k \leq 1$, it was proved by Malik [5] that the inequality (1.1) can be replaced by

$$(1.2) \quad \max_{|z|=1} |p'(z)| \geq \frac{n}{1+k} \max_{|z|=1} |p(z)|.$$

Malik [6] obtained a L^p analogue of (1.1) by proving that if $p(z)$ has all its zeros in $|z| \leq 1$, then for each $r > 0$

$$(1.3) \quad n \left\{ \int_0^{2\pi} |p(e^{i\theta})|^r d\theta \right\}^{\frac{1}{r}} \leq \left\{ \int_0^{2\pi} |1 + e^{i\theta}|^r d\theta \right\}^{\frac{1}{r}} \max_{|z|=1} |p'(z)|.$$

As an extension of (1.3) and a generalization of (1.2), Aziz [1] proved that if $p(z)$ has all its zeros in $|z| \leq k$, $k \leq 1$, then for each $r > 0$

$$(1.4) \quad n \left\{ \int_0^{2\pi} |p(e^{i\theta})|^r d\theta \right\}^{\frac{1}{r}} \leq \left\{ \int_0^{2\pi} |1 + ke^{i\theta}|^r d\theta \right\}^{\frac{1}{r}} \max_{|z|=1} |p'(z)|.$$

If we let $r \rightarrow \infty$ in (1.3) and (1.4) and make use of the well known fact from analysis (see for example [8, p. 73] or [9, p. 91]) that

$$(1.5) \quad \left\{ \int_0^{2\pi} |p(e^{i\theta})|^r d\theta \right\}^{\frac{1}{r}} \rightarrow \max_{0 \leq \theta < 2\pi} |p(e^{i\theta})| \quad \text{as } r \rightarrow \infty,$$

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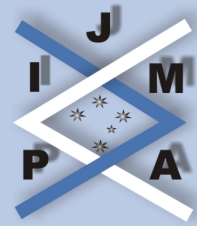
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we get inequalities (1.1) and (1.2) respectively.

In this paper, we will first obtain a Zygmund [11] type integral inequality, but in the reverse direction, for polynomials having a zero of order m at the origin. More precisely, we prove

Theorem 1.1. *Let $p(z) = z^m \sum_{j=0}^{n-m} a_j z^j$ be a polynomial of degree n , having all its zeros in $|z| \leq k$, $k \leq 1$, with a zero of order m at $z = 0$. Then for β with $|\beta| < k^{n-m}$ and $s \geq 1$*

$$(1.6) \quad \left(\int_0^{2\pi} \left| p'(e^{i\theta}) + \frac{mm'}{k^n} \bar{\beta} e^{i(m-1)\theta} \right|^s d\theta \right)^{\frac{1}{s}} \\ \geq \{n - (n - m)C_s^{(k)}\} \left(\int_0^{2\pi} \left| p(e^{i\theta}) + \frac{m'}{k^n} \bar{\beta} e^{im\theta} \right|^s d\theta \right)^{\frac{1}{s}},$$

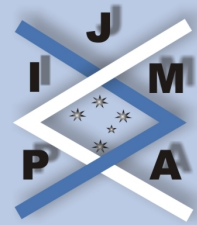
where $m' = \min_{|z|=k} |p(z)|$,

$$C_s^{(k)} = \left(\frac{1}{2\pi} \int_0^{2\pi} |S_c + e^{i\theta}|^s d\theta \right)^{-\frac{1}{s}} \quad \text{and} \quad S_c = \frac{\left(\frac{1}{n-m}\right) \left| \frac{a_{n-m-1}}{a_{n-m}} \right| + 1}{k^2 + \left(\frac{1}{n-m}\right) \left| \frac{a_{n-m-1}}{a_{n-m}} \right|}.$$

By taking $k = 1$ and $\beta = 0$ in Theorem 1.1, we obtain:

Corollary 1.2. *If $p(z)$ is a polynomial of degree n , having all its zeros in $|z| \leq 1$, with a zero of order m at $z = 0$, then for $s \geq 1$*

$$(1.7) \quad \left(\int_0^{2\pi} |p(e^{i\theta})|^s d\theta \right)^{\frac{1}{s}} \geq \{n - (n - m)C_s^{(1)}\} \left(\int_0^{2\pi} |p'(e^{i\theta})|^s d\theta \right)^{\frac{1}{s}},$$



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where

$$C_s^{(1)} = \frac{1}{\left(\frac{1}{2\pi} \int_0^{2\pi} |1 + e^{i\theta}|^s d\theta\right)^{\frac{1}{s}}}.$$

By letting $s \rightarrow \infty$ in Theorem 1.1, we obtain

Corollary 1.3. Let $p(z) = z^m \sum_{j=0}^{n-m} a_j z^j$ be a polynomial of degree n , having all its zeros in $|z| \leq k$, $k \leq 1$, with a zero of order m at $z = 0$. Then for β with $|\beta| < k^{n-m}$

$$(1.8) \quad \max_{|z|=1} \left| p'(z) + \frac{mm'}{k^n} \bar{\beta} z^{m-1} \right| \geq \left(\frac{m + nS_c}{1 + S_c} \right) \max_{|z|=1} \left| p(z) + \frac{m'}{k^n} \bar{\beta} z^m \right|,$$

where m' and S_c are as defined in Theorem 1.1.

By choosing the argument of β suitably and letting $|\beta| \rightarrow k^{n-m}$ in Corollary 1.3, we obtain the following result.

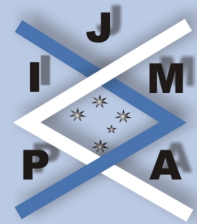
Corollary 1.4. Let $p(z) = z^m \sum_{j=0}^{n-m} a_j z^j$ be a polynomial of degree n , having all its zeros in $|z| \leq k$, $k \leq 1$, with a zero of order m at $z = 0$. Then

$$(1.9) \quad \max_{|z|=1} |p'(z)| \geq \left(\frac{m + nS_c}{1 + S_c} \right) \max_{|z|=1} |p(z)| + \frac{(n - m)S_c}{1 + S_c} \frac{m'}{k^m},$$

where m' and S_c are as defined in Theorem 1.1.

Let $\mathcal{D}_\alpha p(z)$ denote the polar derivative of the polynomial $p(z)$ of degree n with respect to the point α . Then

$$\mathcal{D}_\alpha p(z) = np(z) + (\alpha - z)p'(z).$$



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The polynomial $\mathcal{D}_\alpha p(z)$ is of degree at most $(n - 1)$ and it generalizes the ordinary derivative in the sense that

$$(1.10) \quad \lim_{\alpha \rightarrow \infty} \frac{\mathcal{D}_\alpha p(z)}{\alpha} = p'(z).$$

Our next result generalizes as well as improving upon the inequality (1.4), which in turns, gives a generalization as well as improvements of inequalities (1.3), (1.2) and (1.1) in terms of the polar derivatives of L^p inequalities.

Theorem 1.5. *If $p(z) = a_n z^n + \sum_{j=\mu}^n a_{n-j} z^{n-j}$, $1 \leq \mu \leq n$, is a polynomial of degree n , having all its zeros in $|z| \leq k$, $k \leq 1$, then for every real or complex numbers α and β with $|\alpha| \geq k^\mu$ and $|\beta| \leq 1$ and for each $r > 0$*

$$(1.11) \quad \max_{|z|=1} |\mathcal{D}_\alpha p(z)| \geq \frac{n(|\alpha| - k^\mu)}{\left(\int_0^{2\pi} |1 + k^\mu e^{i\theta}|^r d\theta\right)^{\frac{1}{r}}} \left(\int_0^{2\pi} \left|p(e^{i\theta}) + \frac{\beta m'}{k^{n-\mu}} e^{i(n-1)\theta}\right|^r d\theta\right)^{\frac{1}{r}} + \frac{n}{k^{n-\mu}} m',$$

where $m' = \min_{|z|=k} |p(z)|$.

Dividing both sides of (1.11) by $|\alpha|$, letting $|\alpha| \rightarrow \infty$ and noting that (1.10), we obtain

Corollary 1.6. *If $p(z) = a_n z^n + \sum_{j=\mu}^n a_{n-j} z^{n-j}$, $1 \leq \mu \leq n$, is a polynomial of degree n , having all its zeros in $|z| \leq k$, $k \leq 1$, then for every real or complex*



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number β with $|\beta| \leq 1$, for each $r > 0$

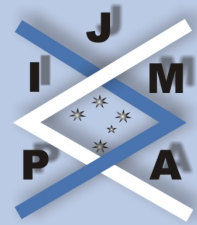
$$(1.12) \quad \max_{|z|=1} |p'(z)| \geq \frac{n}{\left(\int_0^{2\pi} |1 + k^\mu e^{i\theta}|^r d\theta\right)^{\frac{1}{r}}} \left(\int_0^{2\pi} \left|p(e^{i\theta}) + \frac{\beta m'}{k^{n-\mu}} e^{i(n-1)\theta}\right|^r d\theta\right)^{\frac{1}{r}},$$

where $m' = \min_{|z|=k} |p(z)|$.

Remark 1. Letting $r \rightarrow \infty$ in (1.12) and choosing the argument of β suitably with $|\beta| = 1$, it follows that, if $p(z) = a_n z^n + \sum_{j=\mu}^n a_{n-j} z^{n-j}$, $1 \leq \mu \leq n$, is a polynomial of degree n , having all its zeros in $|z| \leq k$, $k \leq 1$, then

$$(1.13) \quad \max_{|z|=1} |p'(z)| \geq \frac{n}{(1 + k^\mu)} \left[\max_{|z|=1} |p(z)| + \frac{1}{k^{n-\mu}} \min_{|z|=k} |p(z)| \right].$$

Inequality (1.13) was already proved by Aziz and Shah [2].



2. Lemmas

For the proofs of these theorems we need the following lemmas.

Lemma 2.1. *Let $p(z) = \sum_{j=0}^n a_j z^j$ be a polynomial of degree n having no zeros in $|z| < k$, $k \geq 1$. Then for $s \geq 1$*

$$(2.1) \quad \left\{ \int_0^{2\pi} |p'(e^{i\theta})|^s d\theta \right\}^{\frac{1}{s}} \leq n S_s \left\{ \int_0^{2\pi} |p(e^{i\theta})|^s d\theta \right\}^{\frac{1}{s}},$$

where

$$S_s = \left\{ \frac{1}{2\pi} \int_0^{2\pi} |S'_c + e^{i\theta}|^s d\theta \right\}^{\frac{1}{s}} \quad \text{and} \quad S'_c = \frac{k^2 \left[\frac{1}{n} \left| \frac{a_1}{a_0} \right| + 1 \right]}{1 + \frac{1}{n} \left| \frac{a_1}{a_0} \right| k^2}.$$

The above lemma is due to Dewan, Bhat and Pukhta [3].

The following lemma is due to Rather [7].

Lemma 2.2. *Let $p(z) = a_n z^n + \sum_{j=\mu}^n a_{n-j} z^{n-j}$, $1 \leq \mu \leq n$, be a polynomial of degree n having all its zero in $|z| \leq k$, $k \leq 1$. Then*

$$(2.2) \quad k^\mu |p'(z)| \geq |q'(z)| + \frac{n}{k^{n-\mu}} \min_{|z|=k} |p(z)| \quad \text{for } |z| = 1,$$

where $q(z) = z^n \overline{p\left(\frac{1}{z}\right)}$.

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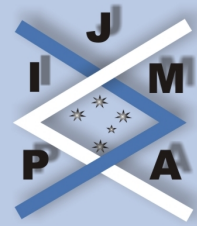
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3. Proofs of The Theorems

Proof of Theorem 1.1. Let

$$p(z) = z^m \sum_{j=0}^{n-m} a_j z^j = z^m \phi(z), \quad (\text{say})$$

where $\phi(z)$ is a polynomial of degree $n - m$, with the property that

$$\phi(0) \neq 0.$$

Then

$$q(z) = z^n \overline{p\left(\frac{1}{\bar{z}}\right)} = z^{n-m} \overline{\phi\left(\frac{1}{\bar{z}}\right)}$$

is also a polynomial of degree $n - m$ and has no zeros in $|z| < \frac{1}{k}, \frac{1}{k} \geq 1$. Now if

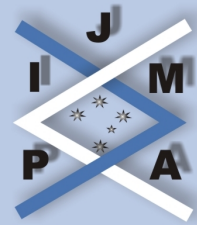
$$m_0 = \min_{|z|=\frac{1}{k}} |q(z)| = \min_{|z|=\frac{1}{k}} \left| z^n \overline{p\left(\frac{1}{\bar{z}}\right)} \right| = \frac{1}{k^n} \min_{|z|=k} |p(z)| = \frac{m'}{k^n},$$

then, by Rouché's theorem, the polynomial

$$q(z) + m_0 \beta z^{n-m}, \quad |\beta| < k^{n-m},$$

of degree $n - m$, will also have no zeros in $|z| < \frac{1}{k}, \frac{1}{k} \geq 1$. Hence, by Lemma 2.1, we have for $s \geq 1$ and $|\beta| < k^{n-m}$

$$\begin{aligned} & \left(\int_0^{2\pi} \left| q'(e^{i\theta}) + \frac{m'}{k^n} \beta e^{i(n-m-1)\theta} (n-m) \right|^s d\theta \right)^{\frac{1}{s}} \\ & \leq (n-m) C_s^{(k)} \left(\int_0^{2\pi} \left| q(e^{i\theta}) + \frac{m'}{k^n} \beta e^{i(n-m)\theta} \right|^s d\theta \right)^{\frac{1}{s}}, \end{aligned}$$



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which implies

$$(3.1) \quad \left(\int_0^{2\pi} \left| np(e^{i\theta}) - e^{i\theta} p'(e^{i\theta}) + \bar{\beta} \frac{m'}{k^n} (n-m) e^{im\theta} \right|^s d\theta \right)^{\frac{1}{s}} \leq (n-m) C_s^{(k)} \left(\int_0^{2\pi} \left| p(e^{i\theta}) + \frac{m'}{k^n} \bar{\beta} e^{im\theta} \right|^s d\theta \right)^{\frac{1}{s}}.$$

Now by Minkowski's inequality, we have for $s \geq 1$ and $|\beta| < k^{n-m}$

$$n \left(\int_0^{2\pi} \left| p(e^{i\theta}) + \frac{m'}{k^n} \bar{\beta} e^{im\theta} \right|^s d\theta \right)^{\frac{1}{s}} \leq \left(\int_0^{2\pi} \left| np(e^{i\theta}) + \frac{m'}{k^n} \bar{\beta} (n-m) e^{im\theta} - e^{i\theta} p'(e^{i\theta}) \right|^s d\theta \right)^{\frac{1}{s}} + \left(\int_0^{2\pi} \left| e^{i\theta} p'(e^{i\theta}) + \frac{mm'}{k^n} \bar{\beta} e^{im\theta} \right|^s d\theta \right)^{\frac{1}{s}},$$

which implies, by using inequality (3.1)

$$n \left(\int_0^{2\pi} \left| p(e^{i\theta}) + \frac{m'}{k^n} \bar{\beta} e^{im\theta} \right|^s d\theta \right)^{\frac{1}{s}} \leq (n-m) C_s^{(k)} \left(\int_0^{2\pi} \left| p(e^{i\theta}) + \frac{m'}{k^n} \bar{\beta} e^{im\theta} \right|^s d\theta \right)^{\frac{1}{s}} + \left(\int_0^{2\pi} \left| p'(e^{i\theta}) + m \frac{m'}{k^n} \bar{\beta} e^{i(m-1)\theta} \right|^s d\theta \right)^{\frac{1}{s}},$$

and the Theorem 1.1 follows. □



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Proof of Theorem 1.5. Since $q(z) = z^n \overline{p\left(\frac{1}{\bar{z}}\right)}$ so that $p(z) = z^n \overline{q\left(\frac{1}{\bar{z}}\right)}$, therefore, we have

$$(3.2) \quad p'(z) = nz^{n-1} \overline{q\left(\frac{1}{\bar{z}}\right)} - z^{n-2} \overline{q'\left(\frac{1}{\bar{z}}\right)},$$

which implies

$$(3.3) \quad |p'(z)| = |nq(z) - zq'(z)| \quad \text{for } |z| = 1.$$

Using (3.2) in (2.2), we get for $1 \leq \mu \leq n$

$$|q'(z)| + \frac{m'n}{k^{n-\mu}} \leq k^\mu |nq(z) - zq'(z)| \quad \text{for } |z| = 1.$$

Now, from the above inequality, for every complex β with $|\beta| \leq 1$, we get, for $|z| = 1$

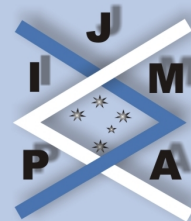
$$(3.4) \quad \left| q'(z) + \bar{\beta} \frac{m'n}{k^{n-\mu}} \right| \leq |q'(z)| + \frac{m'n}{k^{n-\mu}} \leq k^\mu |nq(z) - zq'(z)|.$$

For every real or complex number α with $|\alpha| \geq k^\mu$, we have

$$\begin{aligned} |\mathcal{D}_\alpha p(z)| &= |np(z) + (\alpha - z)p'(z)| \\ &\geq |\alpha| |p'(z)| - |np(z) - zp'(z)|, \end{aligned}$$

which gives by interchanging the roles of $p(z)$ and $q(z)$ in (3.3) for $|z| = 1$

$$(3.5) \quad \begin{aligned} |\mathcal{D}_\alpha p(z)| &\geq |\alpha| |p'(z)| - |q'(z)| \\ &\geq |\alpha| |p'(z)| - k^\mu |p'(z)| + \frac{m'n}{k^{n-\mu}} \quad (\text{using (2.2)}). \end{aligned}$$



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Since $p(z)$ has all its zeros in $|z| \leq k \leq 1$, by the Gauss-Lucas theorem, all the zeros of $p'(z)$ also lie in $|z| \leq 1$. This implies that the polynomial

$$z^{n-1} \overline{p'\left(\frac{1}{\bar{z}}\right)} = nq(z) - zq'(z)$$

has all its zeros in $|z| \geq \frac{1}{k} \geq 1$. Therefore, it follows from (3.4) that the function

$$(3.6) \quad w(z) = \frac{zq'(z) + \bar{\beta} \frac{m'n}{k^{n-\mu}} z}{k^\mu (nq(z) - zq'(z))}$$

is analytic for $|z| \leq 1$ and $|w(z)| \leq 1$ for $|z| \leq 1$. Furthermore $w(0) = 0$. Thus the function $1 + k^\mu w(z)$ is a subordinate to the function $1 + k^\mu z$ in $|z| \leq 1$. Hence by a well-known property of subordination [4], we have for $r > 0$ and for $0 \leq \theta < 2\pi$,

$$(3.7) \quad \int_0^{2\pi} |1 + k^\mu w(e^{i\theta})|^r d\theta \leq \int_0^{2\pi} |1 + k^\mu e^{i\theta}|^r d\theta.$$

Also from (3.6), we have

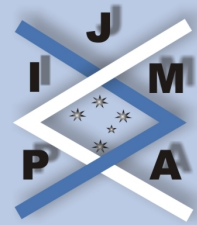
$$1 + k^\mu w(z) = \frac{nq(z) + \bar{\beta} \frac{m'n}{k^{n-\mu}} z}{nq(z) - zq'(z)},$$

or

$$\left| nq(z) + \bar{\beta} \frac{m'n}{k^{n-\mu}} z \right| = |1 + k^\mu w(z)| |p'(z)| \quad \text{for } |z| = 1,$$

which implies

$$(3.8) \quad n \left| p(z) + \beta \frac{m'}{k^{n-\mu}} z^{n-1} \right| = |1 + k^\mu w(z)| |p'(z)| \quad \text{for } |z| = 1.$$



Now combining (3.7) and (3.8), we get

$$n^r \int_0^{2\pi} \left| p(e^{i\theta}) + \beta \frac{m'}{k^{n-\mu}} e^{i(n-1)\theta} \right|^r d\theta \leq \int_0^{2\pi} |1 + k^\mu e^{i\theta}|^r |p'(e^{i\theta})|^r d\theta.$$

Using (3.5) in the above inequality, we obtain

$$\begin{aligned} n^r (|\alpha| - k^\mu)^r \int_0^{2\pi} \left| p(e^{i\theta}) + \beta \frac{m'}{k^{n-\mu}} e^{i(n-1)\theta} \right|^r d\theta \\ \leq \int_0^{2\pi} |1 + k^\mu e^{i\theta}|^r d\theta \left\{ \max_{|z|=1} |\mathcal{D}_\alpha p(z)| - \frac{nm'}{k^{n-\mu}} \right\}^r, \end{aligned}$$

from which we obtain the required result. □

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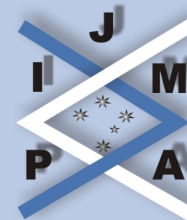
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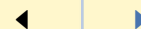
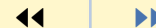


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