

Journal of Inequalities in Pure and Applied Mathematics

ISOMETRIES ON LINEAR n -NORMED SPACES

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©2000 Victoria University
ISSN (electronic): 1443-5756
385-05



volume 7, issue 5, article 168,
2006.

*Received 05 December, 2005;
accepted 25 April, 2006.*

Communicated by: K. Nikodem

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Abstract

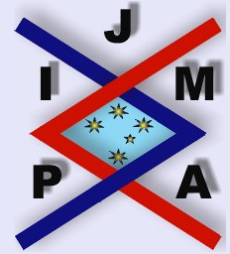
The aim of this article is to generalize the Aleksandrov problem to the case of linear n -normed spaces.

2000 Mathematics Subject Classification: Primary 46B04, 46B20, 51K05.

Key words: Linear n -normed space, n -isometry, n -Lipschitz mapping.

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1. Introduction

Let X and Y be metric spaces. A mapping $f : X \rightarrow Y$ is called an isometry if f satisfies

$$d_Y(f(x), f(y)) = d_X(x, y)$$

for all $x, y \in X$, where $d_X(\cdot, \cdot)$ and $d_Y(\cdot, \cdot)$ denote the metrics in the spaces X and Y , respectively. For some fixed number $r > 0$, suppose that f preserves distance r ; i.e., for all x, y in X with $d_X(x, y) = r$, we have $d_Y(f(x), f(y)) = r$. Then r is called a conservative (or preserved) distance for the mapping f . Aleksandrov [1] posed the following problem:

Remark 1. *Examine whether the existence of a single conservative distance for some mapping T implies that T is an isometry.*

The Aleksandrov problem has been investigated in several papers (see [3] – [10]). Th.M. Rassias and P. Šemrl [9] proved the following theorem for mappings satisfying the strong distance one preserving property (SDOPP), i.e., for every $x, y \in X$ with $\|x - y\| = 1$ it follows that $\|f(x) - f(y)\| = 1$ and conversely.

Theorem 1.1 ([9]). *Let X and Y be real normed linear spaces with dimension greater than one. Suppose that $f : X \rightarrow Y$ is a Lipschitz mapping with Lipschitz constant $\kappa = 1$. Assume that f is a surjective mapping satisfying (SDOPP). Then f is an isometry.*

Definition 1.1 ([2]). *Let X be a real linear space with $\dim X \geq n$ and $\|\cdot, \dots, \cdot\| : X^n \rightarrow \mathbb{R}$ a function. Then $(X, \|\cdot, \dots, \cdot\|)$ is called a linear n -normed space if*

$$(nN_1) \quad \|x_1, \dots, x_n\| = 0 \iff x_1, \dots, x_n \text{ are linearly dependent}$$



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$$(nN_2) \quad \|x_1, \dots, x_n\| = \|x_{j_1}, \dots, x_{j_n}\|$$

for every permutation (j_1, \dots, j_n) of $(1, \dots, n)$

$$(nN_3) \quad \|\alpha x_1, \dots, x_n\| = |\alpha| \|x_1, \dots, x_n\|$$

$$(nN_4) \quad \|x + y, x_2, \dots, x_n\| \leq \|x, x_2, \dots, x_n\| + \|y, x_2, \dots, x_n\|$$

for all $\alpha \in \mathbb{R}$ and all $x, y, x_1, \dots, x_n \in X$. The function $\|\cdot, \dots, \cdot\|$ is called the n -norm on X .

In [3], Chu *et al.* defined the notion of weak n -isometry and proved the Rassias and Šemrl's theorem in linear n -normed spaces.

Definition 1.2 ([3]). We call $f : X \rightarrow Y$ a weak n -Lipschitz mapping if there is a $\kappa \geq 0$ such that

$$\|f(x_1) - f(x_0), \dots, f(x_n) - f(x_0)\| \leq \kappa \|x_1 - x_0, \dots, x_n - x_0\|$$

for all $x_0, x_1, \dots, x_n \in X$. The smallest such κ is called the weak n -Lipschitz constant.

Definition 1.3 ([3]). Let X and Y be linear n -normed spaces and $f : X \rightarrow Y$ a mapping. We call f a weak n -isometry if

$$\|x_1 - x_0, \dots, x_n - x_0\| = \|f(x_1) - f(x_0), \dots, f(x_n) - f(x_0)\|$$

for all $x_0, x_1, \dots, x_n \in X$.



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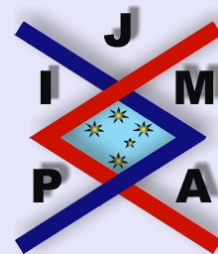
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For a mapping $f : X \rightarrow Y$, consider the following condition which is called the *weak n -distance one preserving property*: For $x_0, x_1, \dots, x_n \in X$ with $\|x_1 - y_1, \dots, x_n - y_n\| = 1$, $\|f(x_1) - f(x_0), \dots, f(x_n) - f(x_0)\| = 1$.

Theorem 1.2 ([3]). *Let $f : X \rightarrow Y$ be a weak n -Lipschitz mapping with weak n -Lipschitz constant $\kappa \leq 1$. Assume that if x_0, x_1, \dots, x_m are m -colinear then $f(x_0), f(x_1), \dots, f(x_m)$ are m -colinear, $m = 2, n$, and that f satisfies the weak n -distance one preserving property. Then f is a weak n -isometry.*

In this paper, we introduce the concept of n -isometry which is suitable for representing the notion of n -distance preserving mappings in linear n -normed spaces. We prove also that the Rassias and Šemrl theorem holds under some conditions when X and Y are linear n -normed spaces.



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2. The Aleksandrov Problem in Linear n -normed Spaces

In this section, let X and Y be linear n -normed spaces with dimension greater than $n - 1$.

Definition 2.1. Let X and Y be linear n -normed spaces and $f : X \rightarrow Y$ a mapping. We call f an n -isometry if

$$\|x_1 - y_1, \dots, x_n - y_n\| = \|f(x_1) - f(y_1), \dots, f(x_n) - f(y_n)\|$$

for all $x_1, \dots, x_n, y_1, \dots, y_n \in X$.

For a mapping $f : X \rightarrow Y$, consider the following condition which is called the n -distance one preserving property : For $x_1, \dots, x_n, y_1, \dots, y_n \in X$ with $\|x_1 - y_1, \dots, x_n - y_n\| = 1, \|f(x_1) - f(y_1), \dots, f(x_n) - f(y_n)\| = 1$.

Lemma 2.1 ([3, Lemma 2.3]). Let x_1, x_2, \dots, x_n be elements of a linear n -normed space X and γ a real number. Then

$$\|x_1, \dots, x_i, \dots, x_j, \dots, x_n\| = \|x_1, \dots, x_i, \dots, x_j + \gamma x_i, \dots, x_n\|.$$

for all $1 \leq i \neq j \leq n$.

Definition 2.2 ([3]). The points x_0, x_1, \dots, x_n of X are said to be n -colinear if for every $i, \{x_j - x_i \mid 0 \leq j \neq i \leq n\}$ is linearly dependent.

Remark 2. The points x_0, x_1 and x_2 are 2-colinear if and only if $x_2 - x_0 = t(x_1 - x_0)$ for some real number t .



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Theorem 2.2. Let $f : X \rightarrow Y$ be a weak n -Lipschitz mapping with weak n -Lipschitz constant $\kappa \leq 1$. Assume that if x_0, x_1, \dots, x_m are m -colinear then $f(x_0), f(x_1), \dots, f(x_m)$ are m -colinear, $m = 2, n$, and that f satisfies the weak n -distance one preserving property. Then f satisfies

$$\|x_1 - y_1, \dots, x_n - y_n\| = \|f(x_1) - f(y_1), \dots, f(x_n) - f(y_n)\|$$

for all $x_1, \dots, x_n, y_1, \dots, y_n \in X$ with x_1, y_1, y_j 2-colinear for $j = 2, 3, \dots, n$.

Proof. By Theorem 1.2, f is a weak n -isometry. Hence

$$(2.1) \quad \|x_1 - y, \dots, x_n - y\| = \|f(x_1) - f(y), \dots, f(x_n) - f(y)\|$$

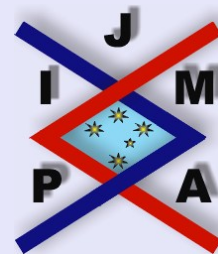
for all $x_1, \dots, x_n, y \in X$.

If $x_1, y_1, y_2 \in X$ are 2-colinear then there exists a $t \in \mathbb{R}$ such that $y_1 - y_2 = t(y_1 - x_1)$. By Lemma 2.1,

$$\begin{aligned} \|x_1 - y_1, x_2 - y_2, \dots, x_n - y_n\| &= \|x_1 - y_1, (x_2 - y_1) + (y_1 - y_2), \dots, x_n - y_n\| \\ &= \|x_1 - y_1, (x_2 - y_1) + (-t)(x_1 - y_1), \dots, x_n - y_n\| \\ &= \|x_1 - y_1, x_2 - y_1, \dots, x_n - y_n\| \end{aligned}$$

for all $x_1, \dots, x_n, y_1, \dots, y_n \in X$ with x_1, y_1, y_2 2-colinear. By the same method as above, one can obtain that if x_1, y_1, y_j are 2-colinear for $j = 3, \dots, n$ then

$$(2.2) \quad \|x_1 - y_1, x_2 - y_2, x_3 - y_3, \dots, x_n - y_n\| = \|x_1 - y_1, x_2 - y_1, x_3 - y_3, \dots, x_n - y_n\|$$



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$$\begin{aligned}
&= \|x_1 - y_1, x_2 - y_1, x_3 - y_1, \dots, x_n - y_n\| \\
&= \dots \\
&= \|x_1 - y_1, x_2 - y_1, x_3 - y_1, \dots, x_n - y_1\|
\end{aligned}$$

for all $x_1, \dots, x_n, y_1, \dots, y_n \in X$ with x_1, y_1, y_j 2-colinear for $j = 2, 3, \dots, n$.

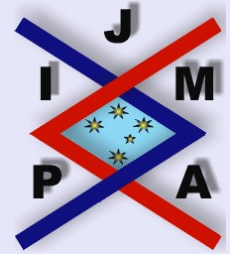
By the assumption, if $x_1, y_1, y_2 \in X$ are 2-colinear then $f(x_1), f(y_1), f(y_2) \in Y$ are 2-colinear. So there exists a $t \in \mathbb{R}$ such that $f(y_1) - f(y_2) = t(f(y_1) - f(x_1))$. By Lemma 2.1,

$$\begin{aligned}
&\|f(x_1) - f(y_1), f(x_2) - f(y_2), \dots, f(x_n) - f(y_n)\| \\
&= \|f(x_1) - f(y_1), (f(x_2) - f(y_1)) + (f(y_1) - f(y_2)), \dots, f(x_n) - f(y_n)\| \\
&= \|f(x_1) - f(y_1), (f(x_2) - f(y_1)) + (-t)(f(x_1) - f(y_1)), \dots, f(x_n) - f(y_n)\| \\
&= \|f(x_1) - f(y_1), f(x_2) - f(y_1), \dots, f(x_n) - f(y_n)\|
\end{aligned}$$

for all $x_1, \dots, x_n, y_1, \dots, y_n \in X$ with x_1, y_1, y_2 2-colinear. If x_1, y_1, y_j are 2-colinear for $j = 3, \dots, n$ then $f(x_1), f(y_1), f(y_j)$ are 2-colinear for $j = 3, \dots, n$. By the same method as above, one can obtain that if $f(x_1), f(y_1), f(y_j)$ are 2-colinear for $j = 3, \dots, n$, then

$$\begin{aligned}
(2.3) \quad &\|f(x_1) - f(y_1), f(x_2) - f(y_2), f(x_3) - f(y_3), \dots, f(x_n) - f(y_n)\| \\
&= \|f(x_1) - f(y_1), f(x_2) - f(y_1), f(x_3) - f(y_3), \dots, f(x_n) - f(y_n)\| \\
&= \|f(x_1) - f(y_1), f(x_2) - f(y_1), f(x_3) - f(y_1), \dots, f(x_n) - f(y_n)\| \\
&= \dots \\
&= \|f(x_1) - f(y_1), f(x_2) - f(y_1), f(x_3) - f(y_1), \dots, f(x_n) - f(y_1)\|
\end{aligned}$$

for all $x_1, \dots, x_n, y_1, \dots, y_n \in X$ with x_1, y_1, y_j 2-colinear for $j = 2, 3, \dots, n$.



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By (2.1), (2.2) and (2.3),

$$\|x_1 - y_1, \dots, x_n - y_n\| = \|f(x_1) - f(y_1), \dots, f(x_n) - f(y_n)\|$$

for all $x_1, \dots, x_n, y_1, \dots, y_n \in X$ with x_1, y_1, y_j 2-colinear for $j = 2, 3, \dots, n$. \square

Now we introduce the concept of n -Lipschitz mapping and prove that the n -Lipschitz mapping satisfying the n -distance one preserving property is an n -isometry under some conditions.

Definition 2.3. We call $f : X \rightarrow Y$ an n -Lipschitz mapping if there is a $\kappa \geq 0$ such that

$$\|f(x_1) - f(y_1), \dots, f(x_n) - f(y_n)\| \leq \kappa \|x_1 - y_1, \dots, x_n - y_n\|$$

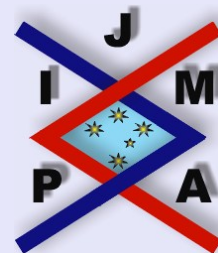
for all $x_1, \dots, x_n, y_1, \dots, y_n \in X$. The smallest such κ is called the n -Lipschitz constant.

Lemma 2.3 ([3, Lemma 2.4]). For $x_1, x'_1 \in X$, if x_1 and x'_1 are linearly dependent with the same direction, that is, $x'_1 = \alpha x_1$ for some $\alpha > 0$, then

$$\|x_1 + x'_1, x_2, \dots, x_n\| = \|x_1, x_2, \dots, x_n\| + \|x'_1, x_2, \dots, x_n\|$$

for all $x_2, \dots, x_n \in X$.

Lemma 2.4. Assume that if x_0, x_1 and x_2 are 2-colinear then $f(x_0), f(x_1)$ and $f(x_2)$ are 2-colinear, and that f satisfies the n -distance one preserving property. Then f preserves the n -distance k for each $k \in \mathbb{N}$.



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Proof. Suppose that there exist $x_0, x_1 \in X$ with $x_0 \neq x_1$ such that $f(x_0) = f(x_1)$. Since $\dim X \geq n$, there are $x_2, \dots, x_n \in X$ such that $x_1 - x_0, x_2 - x_0, \dots, x_n - x_0$ are linearly independent. Since $\|x_1 - x_0, x_2 - x_0, \dots, x_n - x_0\| \neq 0$, we can set

$$z_2 := x_0 + \frac{x_2 - x_0}{\|x_1 - x_0, x_2 - x_0, \dots, x_n - x_0\|}.$$

Then we have

$$\begin{aligned} & \|x_1 - x_0, z_2 - x_0, x_3 - x_0, \dots, x_n - x_0\| \\ &= \left\| x_1 - x_0, \frac{x_2 - x_0}{\|x_1 - x_0, x_2 - x_0, \dots, x_n - x_0\|}, x_3 - x_0, \dots, x_n - x_0 \right\| \\ &= 1. \end{aligned}$$

Since f preserves the n -distance 1,

$$\|f(x_1) - f(x_0), f(z_2) - f(x_0), \dots, f(x_n) - f(x_0)\| = 1.$$

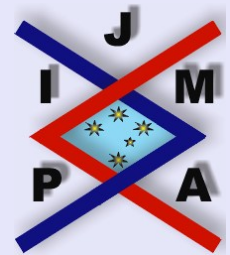
But it follows from $f(x_0) = f(x_1)$ that

$$\|f(x_1) - f(x_0), f(z_2) - f(x_0), \dots, f(x_n) - f(x_0)\| = 0,$$

which is a contradiction. Hence f is injective.

Let $x_1, \dots, x_n, y_1, \dots, y_n \in X, k \in \mathbb{N}$ and

$$\|x_1 - y_1, x_2 - y_2, \dots, x_n - y_n\| = k.$$



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We put

$$z_i = y_1 + \frac{i}{k}(x_1 - y_1), \quad i = 0, 1, \dots, k.$$

Then

$$\begin{aligned} & \|z_{i+1} - z_i, x_2 - y_2, \dots, x_n - y_n\| \\ &= \left\| y_1 + \frac{i+1}{k}(x_1 - y_1) - \left(y_1 + \frac{i}{k}(x_1 - y_1) \right), x_2 - y_2, \dots, x_n - y_n \right\| \\ &= \left\| \frac{1}{k}(x_1 - y_1), x_2 - y_2, \dots, x_n - y_n \right\| \\ &= \frac{1}{k} \|x_1 - y_1, x_2 - y_2, \dots, x_n - y_n\| = \frac{k}{k} = 1 \end{aligned}$$

for all $i = 0, 1, \dots, k - 1$. Since f satisfies the n -distance one preserving property,

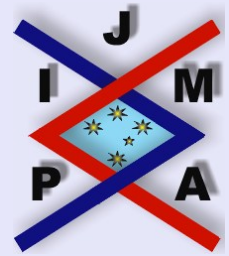
$$(2.4) \quad \|f(z_{i+1}) - f(z_i), f(x_2) - f(y_2), \dots, f(x_n) - f(y_n)\| = 1$$

for all $i = 0, 1, \dots, k - 1$. Since z_0, z_1 and z_2 are 2-colinear, $f(z_0), f(z_1)$ and $f(z_2)$ are also 2-colinear. Thus there is a real number t_0 such that

$$f(z_2) - f(z_1) = t_0(f(z_1) - f(z_0)).$$

By (2.4),

$$\begin{aligned} & \|f(z_1) - f(z_0), f(x_2) - f(y_2), \dots, f(x_n) - f(y_n)\| \\ &= \|f(z_2) - f(z_1), f(x_2) - f(y_2), \dots, f(x_n) - f(y_n)\| \\ &= \|t_0(f(z_1) - f(z_0)), f(x_2) - f(y_2), \dots, f(x_n) - f(y_n)\| \\ &= |t_0| \|f(z_1) - f(z_0), f(x_2) - f(y_2), \dots, f(x_n) - f(y_n)\|. \end{aligned}$$



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So we have $t_0 = \pm 1$. If $t_0 = -1$, $f(z_2) - f(z_1) = -f(z_1) + f(z_0)$, that is,

$$f(z_2) = f(z_0).$$

Since f is injective, $z_2 = z_0$, which is a contradiction. Thus $t_0 = 1$. Hence

$$f(z_2) - f(z_1) = f(z_1) - f(z_0).$$

Similarly, one can obtain that

$$f(z_{i+1}) - f(z_i) = f(z_i) - f(z_{i-1})$$

for all $i = 2, 3, \dots, k - 1$. Thus

$$f(z_{i+1}) - f(z_i) = f(z_1) - f(z_0)$$

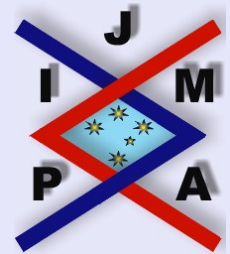
for all $i = 1, 2, \dots, k - 1$. Hence

$$\begin{aligned} f(x_1) - f(y_1) &= f(z_k) - f(z_0) \\ &= f(z_k) - f(z_{k-1}) + f(z_{k-1}) - f(z_{k-2}) + \dots + f(z_1) - f(z_0) \\ &= k(f(z_1) - f(z_0)). \end{aligned}$$

Therefore,

$$\begin{aligned} &\|f(x_1) - f(y_1), f(x_2) - f(y_2), \dots, f(x_n) - f(y_n)\| \\ &= \|k(f(z_1) - f(z_0)), f(x_2) - f(y_2), \dots, f(x_n) - f(y_n)\| \\ &= k\|(f(z_1) - f(z_0)), f(x_2) - f(y_2), \dots, f(x_n) - f(y_n)\| \\ &= k, \end{aligned}$$

which completes the proof. □



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Theorem 2.5. Let $f : X \rightarrow Y$ be an n -Lipschitz mapping with n -Lipschitz constant $\kappa = 1$. Assume that if x_0, x_1, x_2 are 2-colinear then $f(x_0), f(x_1), f(x_2)$ are 2-colinear, and that if $x_1 - y_1, \dots, x_n - y_n$ are linearly dependent then $f(x_1) - f(y_1), \dots, f(x_n) - f(y_n)$ are linearly dependent. If f satisfies the n -distance one preserving property, then f is an n -isometry.

Proof. By Lemma 2.4, f preserves the n -distance k for each $k \in \mathbb{N}$. For $x_1, \dots, x_n, y_1, \dots, y_n \in X$, there are two cases depending upon whether $\|x_1 - y_1, \dots, x_n - y_n\| = 0$ or not. In the case $\|x_1 - y_1, \dots, x_n - y_n\| = 0$, $x_1 - y_1, \dots, x_n - y_n$ are linearly dependent. By the assumption, $f(x_1) - f(y_1), \dots, f(x_n) - f(y_n)$ are linearly dependent. Hence

$$\|f(x_1) - f(y_1), \dots, f(x_n) - f(y_n)\| = 0.$$

In the case $\|x_1 - y_1, \dots, x_n - y_n\| > 0$, there exists an $n_0 \in \mathbb{N}$ such that

$$\|x_1 - y_1, \dots, x_n - y_n\| < n_0.$$

Assume that

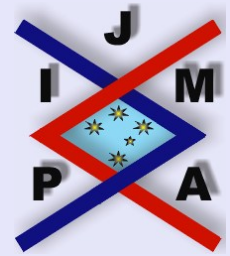
$$\|f(x_1) - f(y_1), \dots, f(x_n) - f(y_n)\| < \|x_1 - y_1, \dots, x_n - y_n\|.$$

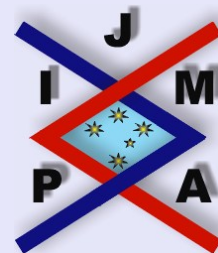
Set

$$w = y_1 + \frac{n_0}{\|x_1 - y_1, \dots, x_n - y_n\|} (x_1 - y_1).$$

Then we obtain that

$$\|w - y_1, x_2 - y_2, \dots, x_n - y_n\|$$





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$$\begin{aligned}
 &= \left\| y_1 + \frac{n_0}{\|x_1 - y_1, \dots, x_n - y_n\|} (x_1 - y_1) - y_1, x_2 - y_2, \dots, x_n - y_n \right\| \\
 &= \frac{n_0}{\|x_1 - y_1, \dots, x_n - y_n\|} \|x_1 - y_1, \dots, x_n - y_n\| = n_0.
 \end{aligned}$$

By Lemma 2.4,

$$\|f(w) - f(y_1), f(x_2) - f(y_2), \dots, f(x_n) - f(y_n)\| = n_0.$$

By the definition of w ,

$$w - x_1 = \left(\frac{n_0}{\|x_1 - y_1, \dots, x_n - y_n\|} - 1 \right) (x_1 - y_1).$$

Since

$$\frac{n_0}{\|x_1 - y_1, \dots, x_n - y_n\|} > 1,$$

$w - x_1$ and $x_1 - y_1$ have the same direction. By Lemma 2.3,

$$\begin{aligned}
 &\|w - y_1, x_2 - y_2, \dots, x_n - y_n\| \\
 &= \|w - x_1, x_2 - y_2, \dots, x_n - y_n\| + \|x_1 - y_1, x_2 - y_2, \dots, x_n - y_n\|.
 \end{aligned}$$

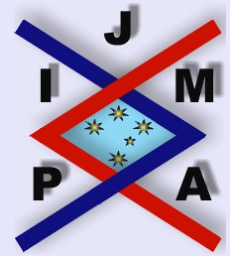
So we have

$$\begin{aligned}
 &\|f(w) - f(x_1), f(x_2) - f(y_2), \dots, f(x_n) - f(y_n)\| \\
 &\leq \|w - x_1, x_2 - y_2, \dots, x_n - y_n\| \\
 &= n_0 - \|x_1 - y_1, x_2 - y_2, \dots, x_n - y_n\|.
 \end{aligned}$$

By the assumption,

$$\begin{aligned}n_0 &= \|f(w) - f(y_1), f(x_2) - f(y_2), \dots, f(x_n) - f(y_n)\| \\ &\leq \|f(w) - f(x_1), f(x_2) - f(y_2), \dots, f(x_n) - f(y_n)\| \\ &\quad + \|f(x_1) - f(y_1), f(x_2) - f(y_2), \dots, f(x_n) - f(y_n)\| \\ &< n_0 - \|x_1 - y_1, x_2 - y_2, \dots, x_n - y_n\| + \|x_1 - y_1, x_2 - y_2, \dots, x_n - y_n\| \\ &= n_0,\end{aligned}$$

which is a contradiction. Hence f is an n -isometry. \square



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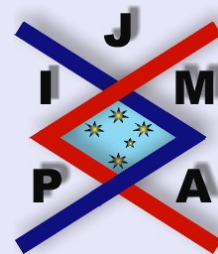
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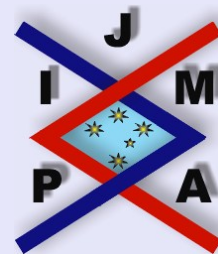
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