

CONVOLUTION OPERATORS WITH HOMOGENEOUS SINGULAR MEASURES ON \mathbb{R}^3 OF POLYNOMIAL TYPE. THE REMAINDER CASE.

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Abstract

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Abstract

Let $\varphi(y_1, y_2) = y_2^l P(y_1, y_2)$ where P is a polynomial function of degree l such that $P(1, 0) \neq 0$. Let μ_δ be the Borel measure on \mathbb{R}^3 defined by $\mu_\delta(E) = \int_{V_\delta} \chi_E(x, \varphi(x)) dx$ where

$$V_\delta = \{x = (x_1, x_2) \in \mathbb{R}^2 : |x_1| \leq 1, \text{ and } |x_1| \leq \delta |x_2|\}$$

and let T_{μ_δ} be the convolution operator with the measure μ_δ . In this paper we explicitly describe the type set

$$E_{\mu_\delta} := \left\{ \left(\frac{1}{p}, \frac{1}{q} \right) \in [0, 1] \times [0, 1] : \|T_{\mu_\delta}\|_{p,q} < \infty \right\},$$

for δ small enough.

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1. Introduction

Let $\varphi : \mathbb{R}^2 \rightarrow \mathbb{R}$ be a homogeneous polynomial function of degree $m \geq 2$ and let $D = \{y \in \mathbb{R}^2 : |y| \leq 1\}$. Let μ be the Borel measure on \mathbb{R}^3 given by

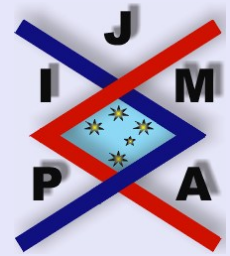
$$(1.1) \quad \mu(E) = \int_D \chi_E(y, \varphi(y)) dy$$

and let T_μ be the operator defined, for $f \in S(\mathbb{R}^3)$, by $T_\mu f = \mu * f$. Let E_μ be the set of the pairs $\left(\frac{1}{p}, \frac{1}{q}\right) \in [0, 1] \times [0, 1]$ such that there exists a positive constant c satisfying $\|Tf\|_q \leq c \|f\|_p$ for all $f \in S(\mathbb{R}^3)$, where the L^p spaces are taken with respect to the Lebesgue measure on \mathbb{R}^3 . For $\left(\frac{1}{p}, \frac{1}{q}\right) \in E_\mu$, T can be extended to a bounded operator, still denoted by T , from $L^p(\mathbb{R}^3)$ into $L^q(\mathbb{R}^3)$.

Let $\varphi = \varphi_1^{e_1} \dots \varphi_n^{e_n}$ be a decomposition of φ in irreducible factors with $\varphi_i \nmid \varphi_j$ for $i \neq j$. In [3] we could give a complete description of the set E_μ under the assumption that $e_i \neq \frac{m}{2}$ for each φ_i of degree 1. If $\det \varphi''(y)$ is not identically zero and if it vanishes somewhere on $\mathbb{R}^2 - \{0\}$, the set of the points y where $\det \varphi''(y)$ vanishes is a finite union of lines L_1, \dots, L_k through the origin. So, after a possibly linear change of variables, we localized the problem to the x axes and we studied the type set corresponding to measures μ_δ defined by

$$\mu_\delta(E) = \int_{V_\delta} \chi_E(y, \varphi(y)) dy,$$

where $V_\delta = D \cap \{(y_1, y_2) \in \mathbb{R}^2 : |y_2| \leq \delta |y_1|\}$ and δ is small enough such that $\det \varphi''(y)$ only vanishes, on V_δ , along the x axes. The only case left was the one



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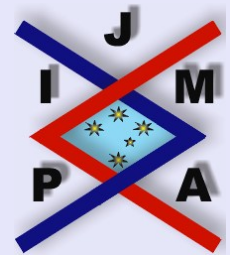
corresponding to functions φ of the form $\varphi(y_1, y_2) = y_2^l P(y_1, y_2)$ with $l = \frac{m}{2}$, P being a homogeneous polynomial function of degree l such that $P(1, 0) \neq 0$.

In this paper we characterize E_{μ_δ} in this remainder case.

L^p improving properties of convolution operators with singular measures supported on hypersurfaces in \mathbb{R}^n have been widely studied in [2], [5], [6]. In particular, in [5], the type set was studied under our actual hypothesis, but the endpoint problem was left open there. Our proof of the main result involves a biparametric family of dilations and will be based on a suitable adaptation of arguments due to M. Christ, developed in [1], where the author studied the type set associated to the two dimensional measure supported on the parabola.

Also, oscillatory integral estimates are involved. A very careful study of this kind of estimate can be found in [4] where the authors study the boundedness of maximal operators associated to mixed homogeneous hypersurfaces.

Throughout this paper c will denote a positive constant, not the same at each occurrence.



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2. The Main Result

We assume $\varphi(y_1, y_2) = y_2^l P(y_1, y_2)$, where $l = \frac{m}{2}$ and P is a homogeneous polynomial function of degree l such that $P(1, 0) \neq 0$. We take $\delta_1 > 0$ such that, for $y \in V_{\delta_1}$ such that $y_2 \neq 0$, $\det \varphi''(y) \neq 0$. Moreover, since $P(1, 0) \neq 0$ we can assume that $P(y) \neq 0$ and $P_1(y) \neq 0$ for all $y \in V_{\delta_1}$. Now, if $\max_{V_{\delta_1}} |P_2(y_1, y_2)| \neq 0$, we choose $\delta < \min\left(\frac{l \min_{V_{\delta_1}} |P(y_1, y_2)|}{2 \max_{V_{\delta_1}} |P_2(y_1, y_2)|}, \delta_1\right)$. In the other case we take $\delta = \delta_1$.

The main result we prove is the following.

Theorem 2.1. *Let $\varphi(y_1, y_2) = y_2^l P(y_1, y_2)$ where $l = \frac{m}{2}$ and P is a homogeneous polynomial function of degree l such that $P(1, 0) \neq 0$ and $y_2 \nmid P(y_1, y_2)$. Let V_δ be defined as above and let E_{V_δ} be the corresponding type set. Then E_{V_δ} is the closed polygonal region with vertices $(0, 0)$, $(1, 1)$, $(\frac{2l+1}{2l+2}, \frac{2l-1}{2l+2})$ and $(\frac{3}{2l+2}, \frac{1}{2l+2})$.*

Standard arguments (see, for example Lemma 2 and Lemma 3 in [3]) imply the following result.

Lemma 2.2. *If $(\frac{1}{p}, \frac{1}{q}) \in E_{\mu_\delta}$ then $\frac{1}{q} \leq \frac{1}{p}$, $\frac{1}{q} \geq \frac{3}{p} - 2$ and $\frac{1}{q} \geq \frac{1}{p} - \frac{1}{l+1}$.*

So, since $\|T_{\mu_\delta}\|_{1,1} < \infty$, by duality arguments it only remains to prove that

$$(2.1) \quad \|T_{\mu_\delta}\|_{\frac{2l+2}{2l+1}, \frac{2l+2}{2l-1}} < \infty.$$

We set $Q_0 = [\frac{1}{4}, 2] \times [\frac{\delta}{64}, \frac{\delta}{8}]$. We take a truncation function $\theta \in C^\infty(\mathbb{R}^2)$, $\theta(y_1, y_2) \geq 0$, $\text{supp } \theta \subset Q_0$ and $\theta(y_1, y_2) = 1$ on $[\frac{1}{2}, 1] \times [\frac{\delta}{32}, \frac{\delta}{16}]$. We define,



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for $\varepsilon, \gamma > 0$, the biparametric family of dilations on \mathbb{R}^2 and \mathbb{R}^3 given by $(\varepsilon, \gamma) \circ (y_1, y_2) = (\varepsilon y_1, \gamma y_2)$ and $(\varepsilon, \gamma) \circ (y_1, y_2, y_3) = (\varepsilon y_1, \gamma y_2, \varepsilon^l \gamma^l y_3)$ respectively. Also, for $j, k \geq 0$, we set $Q_{j,k} = (2^{-j}, 2^{-k}) \circ Q_0$.

For $f \in S(\mathbb{R}^3)$, we define

$$(2.2) \quad T_{j,k}f(x_1, x_2, x_3) = \int f(x_1 - y_1, x_2 - y_2, x_3 - \varphi(y_1, y_2)) \theta(2^j y_1, 2^k y_2) dy_1 dy_2$$

so for $f \geq 0$,

$$(2.3) \quad T_{\mu_{\frac{\delta}{8}}} f \leq c \sum_{0 \leq j \leq k} T_{j,k} f.$$

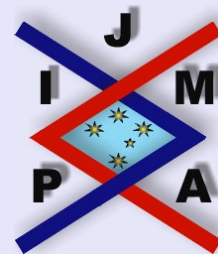
To study $\sum_{0 \leq j \leq k} T_{j,k} f$, we will adapt the argument developed by M. Christ (see [1]) to the setting of biparametric dilations. First of all, we prove the following

Proposition 2.3. *There exists a positive constant $c > 0$ such that for $0 \leq j \leq k$,*

$$\|T_{j,k}\|_{\frac{2l+2}{2l+1}, \frac{2l+2}{2l-1}} \leq c.$$

Proof.

$$\begin{aligned} T_{j,k}f(x_1, x_2, x_3) &= \int f(x_1 - y_1, x_2 - y_2, x_3 - \varphi(y_1, y_2)) \theta(2^j y_1, 2^k y_2) dy_1 dy_2 \\ &= 2^{-(j+k)} \int f(x_1 - 2^{-j} y_1, x_2 - 2^{-k} y_2, x_3 - \varphi(2^{-j} y_1, 2^{-k} y_2)) \theta(y_1, y_2) dy_1 dy_2 \\ &= 2^{-(j+k)} T^{(j-k)} f_{j,k}(2^j x_1, 2^k x_2, 2^{(j+k)l} x_3), \end{aligned}$$



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where we denote

$$T^{(j)} f(x_1, x_2, x_3) = \int f(x_1 - y_1, x_2 - y_2, x_3 - y_2^l P(y_1, 2^j y_2)) \theta(y_1, y_2) dy_1 dy_2$$

and

$$f_{j,k}(x_1, x_2, x_3) = f((2^{-j}, 2^{-k}) \circ (x_1, x_2, x_3)).$$

So

$$(2.4) \quad \|T_{j,k} f(x_1, x_2, x_3)\|_q = 2^{(j+k)(\frac{1+l}{p} - \frac{1+l}{q} - 1)} \|T^{(j-k)}\|_{p,q} \|f\|_p.$$

Now,

$$\det(y_2^l P(y_1, 2^{j-k} y_2))'' = 2^{(2-2l)(j-k)} \det(\varphi)''(y_1, 2^{j-k} y_2).$$

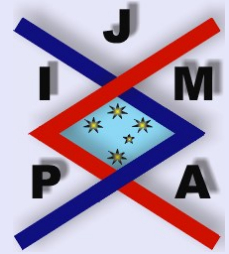
so as in the proof of Lemma 4 in [3] we obtain that there exists $c > 0$ such that $\|T^{(j-k)}\|_{\frac{2l+2}{2l+1}, \frac{2l+2}{2l-1}} \leq c$ for $0 \leq j \leq k$, and the proposition follows. \square

We take $0 \leq j \leq k$, and denote by $\mu_{j,k}$ and $\mu^{(j)}$ the measures associated to $T_{j,k}$ and $T^{(j)}$ respectively. For $\xi = (\xi_1, \xi_2, \xi_3)$,

$$\widehat{\mu^{(j-k)}}(\xi) = \int e^{-i(\xi_1 y_1 + \xi_2 y_2 + \xi_3 y_2^l P(y_1, 2^{j-k} y_2))} \theta(y_1, y_2) dy_1 dy_2.$$

If for some ξ on the unit sphere, $\Omega_{\xi}^{(j-k)}(y_1, y_2) = \xi_1 y_1 + \xi_2 y_2 + \xi_3 y_2^l P(y_1, 2^{j-k} y_2)$ has a critical point belonging to the $\text{supp } \theta$, then

$$\xi_1 + \xi_3 y_2^l P_1(y_1, 2^{j-k} y_2) = 0$$



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and

$$\xi_2 + \xi_3 (2^{j-k} y_2^l P_2(y_1, 2^{j-k} y_2) + l y_2^{l-1} P(y_1, 2^{j-k} y_2)) = 0,$$

but then, since $P_1(y) \neq 0$ for $y \in V_{\delta_1}$, from the first equation we obtain that there exist constants $a, b \in \mathbb{Z}$ with $a < b$ such that $2^a |\xi_3| \leq |\xi_1| \leq 2^b |\xi_3|$, and, from the second one and the choice of δ we obtain constants $c, d \in \mathbb{Z}^2$ with $c < d$ such that $2^c |\xi_3| \leq |\xi_2| \leq 2^d |\xi_3|$. So ξ belongs to the cone

$$C_0 = \{ \xi \in \mathbb{R}^3 : 2^a |\xi_3| < |\xi_1| < 2^b |\xi_3|, 2^c |\xi_3| < |\xi_2| < 2^d |\xi_3| \}.$$

Lemma 2.4. *Suppose C_0 is as above. Then the family of cones $\{ (2^j, 2^k) \circ C_0 \}_{j,k \in \mathbb{Z}}$ has finite overlapping (i.e., $\# \{ (j, k) \in \mathbb{Z}^2 : C_0 \cap ((2^j, 2^k) \circ C_0) \neq \emptyset \} < \infty$).*

Proof. We suppose $\xi \in C_0$ and $(2^j, 2^k) \circ \xi \in C_0$, then

$$2^a |\xi_3| < |\xi_1| < 2^b |\xi_3|, \quad 2^c |\xi_3| < |\xi_2| < 2^d |\xi_3|$$

and

$$2^{(j+k)l+a} |\xi_3| < 2^j |\xi_1| < 2^{(j+k)l+b} |\xi_3|,$$

$$2^{(j+k)l+c} |\xi_3| < 2^k |\xi_2| < 2^{(j+k)l+d} |\xi_3|$$

so

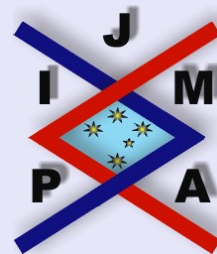
$$2^j |\xi_1| < 2^{(j+k)l+b} |\xi_3| < 2^{(j+k)l+b-a} |\xi_1|$$

and

$$2^b |\xi_3| > |\xi_1| > 2^{-j} 2^{(j+k)l+a} |\xi_3|,$$

so

$$a - b - kl < j(l - 1) < b - a - kl,$$



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analogously we obtain

$$c - d - jl < k(l - 1) < d - c - jl,$$

thus

$$\frac{(c - d)(l - 1) + (a - b)l}{l^2 - (l - 1)^2} < k < \frac{(d - c)(l - 1) + (b - a)l}{l^2 - (l - 1)^2}$$

and so

$$\frac{a - b}{l - 1} - l \frac{(d - c)(l - 1) + (b - a)l}{(l^2 - (l - 1)^2)(l - 1)} < j < \frac{(b - a)}{l - 1} + l \frac{(d - c)(l - 1) + (b - a)l}{(l^2 - (l - 1)^2)(l - 1)}.$$

□

We define $m_0(\xi) = n(\xi_1, \xi_3) r(\xi_2, \xi_3)$ where n and r belong to $C^\infty(R^2 - \{0\})$, are homogeneous of degree zero with respect to the isotropic dilations,

$$\text{supp } n \subset \{(\xi_1, \xi_3) : 2^{a-1} |\xi_3| < |\xi_1| < 2^{b+1} |\xi_3|\}$$

$$n \geq 0 \text{ and } n \equiv 1 \text{ on } \{(\xi_1, \xi_3) : 2^a |\xi_3| < |\xi_1| < 2^b |\xi_3|\},$$

$$\text{supp } r \subset \{(\xi_2, \xi_3) : 2^{c-1} |\xi_3| < |\xi_2| < 2^{d+1} |\xi_3|\},$$



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$r \geq 0$ and $r \equiv 1$ on $\{(\xi_2, \xi_3) : 2^c |\xi_3| < |\xi_2| < 2^d |\xi_3|\}$, so m_0 is homogeneous of degree zero with respect to the isotropic dilations, it belongs to C^∞ on each octant of \mathbb{R}^3 , $m_0 \geq 0$, $m_0 \equiv 1$ on C_0 and

$$\begin{aligned} \text{supp } m_0 &\subset \widetilde{C}_0 \\ &= \{\xi \in \mathbb{R}^3 : 2^{a-1} |\xi_3| < |\xi_1| < 2^{b+1} |\xi_3|, 2^{c-1} |\xi_3| < |\xi_2| < 2^{d+1} |\xi_3|\}. \end{aligned}$$

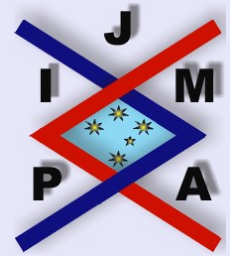
For $(j, k) \in \mathbb{Z}^2$, we define $m_{j,k}(\xi) = m_0((2^{-j}, 2^{-k}) \circ \xi)$ and $\mathfrak{Q}_{j,k}$ the operator with multiplier $m_{j,k}$. If ξ belongs to an open octant of \mathbb{R}^3 then ξ belongs to $(2^j, 2^k) \circ C_0$ for some $(j, k) \in \mathbb{Z}^2$ (indeed $2^{-k} \sim \frac{|\xi_1|}{|\xi_3|}$ and $2^{-j} \sim \frac{|\xi_2|}{|\xi_3|}$) and from the previous lemma, it belongs to a finite number of them (independent of ξ). So $\sum_{(j,k) \in \mathbb{Z}^2} m_{j,k}(\xi) \leq c$. Now it is easy to check that, for $1 < p < \infty$, there exists $A_p > 0$ such that for $f \in L^2 \cap L^p$ and any choice of $\varepsilon_{j,k} = \pm 1$,

$$(2.5) \quad \left\| \sum_{(j,k) \in \mathbb{Z}^2} \varepsilon_{j,k} \mathfrak{Q}_{j,k} f \right\|_p \leq A_p \|f\|_p.$$

Indeed, we now show that

$$m(\xi) = \sum_{(j,k) \in \mathbb{Z}^2} \varepsilon_{j,k} m_{j,k}(\xi)$$

satisfies the hypothesis of the Marcinkiewicz Theorem, as stated in Theorem 6' in [7].



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We have just observed that

$$|m(\xi)| \leq \sum_{(j,k) \in \mathbb{Z}^2} m_{j,k}(\xi) \leq c.$$

Now we want to estimate $\left| \frac{\partial}{\partial \xi_1} m(\xi) \right|$. We recall that $\frac{\partial}{\partial \xi_1} m_0$ is homogeneous of degree -1 . We pick ξ in an open octant. In a small neighborhood of ξ only finitely many $(j, k) \in \mathbb{Z}^2$ (independent of ξ) are involved. For each one of them,

$$\begin{aligned} \frac{\partial}{\partial \xi_1} m_{j,k}(\xi) &= 2^{-j} \frac{\partial}{\partial \xi_1} m_0(2^{-j} \xi_1, 2^{-k} \xi_2, 2^{-(j+k)l} \xi_3) \\ &\leq c 2^{-j} |2^{-j} \xi_1, 2^{-k} \xi_2, 2^{-(j+k)l} \xi_3|^{-1} \leq c 2^{-j} |2^{-j} \xi_1|^{-1}, \end{aligned}$$

so

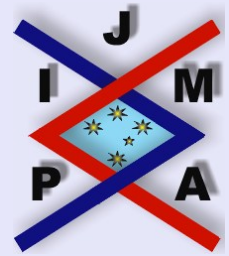
$$\sup_{\xi_2, \xi_3} \int_{2^s}^{2^{s+1}} \left| \frac{\partial}{\partial \xi_1} m(\xi) \right| d\xi_1 \leq c,$$

and in a similar way (using the homogeneity of the derivatives of $m_{j,k}$) we obtain that for each $0 < k \leq 3$,

$$\sup_{\xi_{k+1}, \dots, \xi_3} \int_{\rho} \left| \frac{\partial^k}{\partial \xi_1 \dots \partial \xi_k} m(\xi) \right| d\xi_1 \leq c,$$

as ρ ranges over dyadic rectangles of \mathbb{R}^k and that this inequality holds for every one of the six permutations of the variables ξ_1, ξ_2, ξ_3 .

We now define $h(\xi) \in C^\infty(\mathbb{R}^3)$, $h \geq 0$, $h \equiv 1$ on the unit ball of \mathbb{R}^3 , $h_{j,k}(\xi) = h((2^{-j}, 2^{-k}) \circ \xi)$ and $R_{j,k}$ the operators with multipliers $h_{j,k}$.



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Lemma 2.5. *There exists a constant $C > 0$, independent of K , such that*

$$\left\| \sum_{0 \leq j \leq k \leq K} T_{j,k} R_{j,k} \right\|_{\frac{2l+2}{2l+1}, \frac{2l+2}{2l-1}} \leq C.$$

Proof. Let $K_{j,k}$ be the kernel of $T_{j,k} R_{j,k}$. A computation shows that,

$$K_{j,k}(x) = 2^{(j+k)l} (\mu^{(j-k)} * h^{\wedge \vee}) ((2^j, 2^k) \circ x).$$

Thus

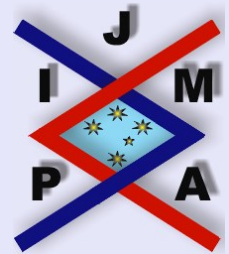
$$\sum_{0 \leq j \leq k \leq K} |K_{j,k}(\xi)| \leq \sum_{0 \leq j \leq k} 2^{(j+k)l} |G^{(j,k)}((2^j, 2^k) \circ \xi)|$$

with $G^{(j,k)}$ defined by $(G^{(j,k)})^\wedge = (\mu^{(j-k)})^\wedge h$. Since $j - k \leq 0$, as in Lemma 7 in [3] we obtain that $(G^{(j,k)})^\wedge \in S(\mathbb{R}^3)$ with each seminorm bounded on j, k , it follows that the same holds for $G^{(j,k)}$. Now

$$\begin{aligned} \sum_{0 \leq j \leq k} 2^{(j+k)l} |G^{(j,k)}((2^j, 2^k) \circ \xi)| \\ \leq \sum_{j,k,h \geq 0} 2^{ja+ka+ha} |G^{(j,k,h)}(2^j \xi_1, 2^k \xi_2, 2^h \xi_3)| \end{aligned}$$

with $a = \frac{l}{l+1}$, $G^{(j,k,h)} = G^{(j,k)}$ for $h = l(j+k)$ and $G^{(j,k,h)} = 0$ otherwise. It is well known that from the uniform boundedness properties of $G^{(j,k,h)}$ it follows that

$$\sum_{j,k,h \geq 0} 2^{ja+ka+ha} |G^{(j,k,h)}(2^j \xi_1, 2^k \xi_2, 2^h \xi_3)| \leq \frac{c}{|\xi_1|^a |\xi_2|^a |\xi_3|^a},$$



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so

$$\sum_{0 \leq j \leq k \leq K} |K_{j,k}(\xi)| \leq \frac{c}{|\xi_1|^{\frac{l}{l+1}} |\xi_2|^{\frac{l}{l+1}} |\xi_3|^{\frac{l}{l+1}}},$$

so $\sum_{0 \leq j \leq k \leq K} T_{j,k} R_{j,k}$ convolves $L^p(\mathbb{R}^3)$ into $L^q(\mathbb{R}^3)$ for $\frac{1}{q} = \frac{1}{p} - \frac{1}{l+1}$ with bounds independent of K . □

Lemma 2.6. *There exists a constant $C > 0$, independent of K , such that*

$$\left\| \sum_{1 \leq j \leq k \leq K} T_{j,k} (I - P_{j,k}) (I - \mathfrak{Q}_{j,k}) \right\|_{\frac{2l+2}{2l+1}, \frac{2l+2}{2l-1}} \leq C.$$

Proof. The kernel $H_{j,k}$ of

$$\sum_{1 \leq j \leq k \leq K} T_{j,k} (I - P_{j,k}) (I - \mathfrak{Q}_{j,k})$$

satisfies

$$\sum_{1 \leq j \leq k \leq K} |H_{j,k}(\xi)| \leq \sum_{0 \leq j \leq k} 2^{(j+k)l} |g^{(j,k)}((2^j, 2^k) \circ \xi)|$$

with $g^{(j,k)}$ defined by $(g^{(j,k)})^\wedge = (\mu^{(j-k)})^\wedge (1-h)(1-m_0)$.

Observe that, from Lemma 7 in [3], we have $(\mu^{(j-k)})^\wedge (1-h)(1-m_0) \in S(\mathbb{R}^3)$ with each seminorm bounded on j, k . From this fact the proof follows as in the previous lemma. □



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Proof of the theorem. We have just observed that it is enough to prove (2.1). Since we can suppose $f \geq 0$, by (2.3), we need only check that there exists $C > 0$, independent of K such that

$$\left\| \sum_{0 \leq j \leq k \leq K} T_{j,k} \right\| \leq C,$$

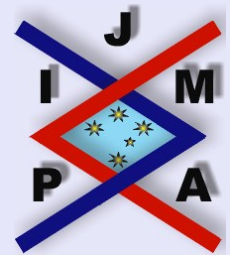
where $T_{j,k}$ are defined by (2.2). For a constant $c_0 > 0$, we define $\mathfrak{Q}'_{j,k} = \sum_{|i-j| \leq c_0} \mathfrak{Q}_{i,k}$. So $\mathfrak{Q}'_{j,k}$ have the same properties as $\mathfrak{Q}_{j,k}$ and $\mathfrak{Q}'_{j,k} \circ \mathfrak{Q}_{j,k} = \mathfrak{Q}_{j,k}$ thus we have that (2.5) holds for $\mathfrak{Q}'_{j,k}$. Then, for $1 < p < \infty$ and

$$F = \{f_{j,k}\}_{j,k \geq 0} \in L^p(l^2), \quad \left\| \sum_{j,k \geq 0} \mathfrak{Q}'_{j,k} f_{j,k} \right\|_p \leq c_p \|F\|_{L^p(l^2)}.$$

We decompose

$$\begin{aligned} & \sum_{0 \leq j \leq k \leq K} T_{j,k} f \\ &= \sum_{0 \leq j \leq k \leq K} T_{j,k} (I - P_{j,k}) (I - \mathfrak{Q}'_{j,k}) f + \sum_{0 \leq j \leq k \leq K} T_{j,k} P_{j,k} f \\ & \quad + \sum_{0 \leq j \leq k \leq K} T_{j,k} \mathfrak{Q}'_{j,k} (I - P_{j,k}) f. \end{aligned}$$

Now, proceeding as in [1], the theorem follows from Proposition 2.3, Lemmas 2.5 and 2.6 and the remarks in [8, p. 85] concerning the multiparameter maximal function. \square



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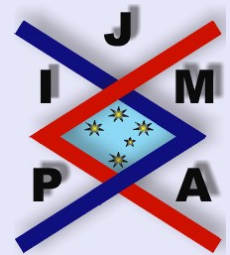
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