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A Diophantine System Concerning Sums of Cubes

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Abstract

We study the Diophantine system

$$\begin{cases} x_1 + \cdots + x_n = a, \\ x_1^3 + \cdots + x_n^3 = b, \end{cases}$$

where $a, b \in \mathbb{Q}, ab \neq 0, n \geq 4$, and prove, using the theory of elliptic curves, that it has infinitely many rational parametric solutions depending on $n-3$ free parameters. Moreover, this Diophantine system has infinitely many positive rational solutions with no common element for $n = 4$, which partially answers a question in our earlier paper.

1 Introduction

Ren and Yang [10] considered the positive integer solutions of the Diophantine chains

$$\begin{cases} \sum_{j=1}^{n} x_{1j} = \sum_{j=1}^{n} x_{2j} = \cdots = \sum_{j=1}^{n} x_{kj} = a, \\ \sum_{j=1}^{n} x_{1j}^3 = \sum_{j=1}^{n} x_{2j}^3 = \cdots = \sum_{j=1}^{n} x_{kj}^3 = b, \\ n \geq 2, \ k \geq 2, \end{cases} \tag{1}$$

where a, b are positive integers and determined by k n-tuples $(x_{i1}, x_{i2}, \ldots, x_{in}), i = 1, \ldots, k$.

For $n = 2, k = 2$, Eq. (1) has no nontrivial integer solutions [12], so we consider $n \geq 3$. For $n = 3, k = 2$, Eq. (1) reduces to

$$\begin{cases} x_1 + x_2 + x_3 = y_1 + y_2 + y_3, \\ x_1^3 + x_2^3 + x_3^3 = y_1^3 + y_2^3 + y_3^3. \end{cases} \tag{2}$$

Systems like (2) has been investigated by many authors, at least since 1915 [7, p. 713]; see [1, 2, 3, 4, 5, 8]. Eq. (2) is interesting because it reveals the relation between all of the nontrivial zeros of weight-1 $6j$ Racah coefficients and all of its non-negative integer solutions. More recently, Moreland and Zieve [9] showed that "for triples (a, b, c) of pairwise distinct rational numbers such that for every permutation (A, B, C) of (a, b, c), the conditions $(A + B)(A - B)^3 \neq (B + C)(B - C)^3$ and $AB^2 + BC^2 + CA^2 \neq A^3 + B^3 + C^3$ hold, then the Diophantine system

$$\begin{cases} x + y + z = a + b + c, \\ x^3 + y^3 + z^3 = a^3 + b^3 + c^3 \end{cases}$$

has infinitely many rational solutions (x, y, z)." This gives a complete answer to Question 5 in an earlier paper of the author [10].

For $n = 3, k \geq 3$, Choudhry [5] proved that Eq. (1) has a parametric solution in rational numbers, but the solutions are not all positive. There are arbitrarily long Diophantine chains of the form Eq. (1) with $n = 3$.

For $n \geq 3$, Ren and Yang [10] obtained a special result of Eq. (1) with $(x_1, x_2, \ldots, x_{n-3}) = (1, 2, \ldots, n-3)$, which leads to Eq. (1) has infinitely many coprime positive integer solutions for $n \geq 3$.

Now we study the case of Eq. (1) for $n \geq 4$ with the greatest possible generality. For convenience, let us consider the non-zero rational solutions of the Diophantine system

$$\begin{cases} x_1 + \cdots + x_n = a, \\ x_1^3 + \cdots + x_n^3 = b, \end{cases} \tag{3}$$

where $a, b \in \mathbb{Q}, ab \neq 0, n \geq 4$.

Using the theory of elliptic curves, we prove the following theorems:

Theorem 1. *For $n \geq 4$, the Diophantine system (3) has infinitely many rational parametric solutions depending on $n - 3$ free parameters.*

Theorem 2. *For $n = 4$, the Diophantine system (3) has infinitely many positive rational solutions.*

From these two theorems, we have

Corollary 3. *For $n \geq 4$ and every positive integer k, there are infinitely many primitive sets of k n-tuples of polynomials in $\mathbb{Z}[t_1, t_2, \ldots, t_{n-3}]$ with the same sum and the same sum of cubes.*

Corollary 4. *For $n = 4$ and every positive integer k, there are infinitely many primitive sets of k 4-tuples of positive integers with the same sum and the same sum of cubes.*

2 The proofs of the theorems

In this section, we give the proofs of our theorems, which are related to the rational points of some elliptic curves. The proof of Theorem 1 is inspired by the method of [13].

Proof. In view of the homogeneity of Eq. (3), we let $a, b \in \mathbb{Z}, ab \neq 0$. First, we prove it for $n = 4$ and then deduce the solution of Eq. (3) for all $n \geq 5$. In the following Diophantine system

$$x_1 + x_2 + x_3 + x_4 = a, x_1^3 + x_2^3 + x_3^3 + x_4^3 = b, \tag{4}$$

eliminating x_4 from the first equation and letting $x_3 = tx_2$, we get

$$\begin{aligned} &3(tx_2 + x_2 - a)x_1^2 + 3(tx_2 + x_2 - a)^2 x_1 + 3t(t+1)x_2^3 \\ &- 3a(t+1)^2 x_2^2 + 3a^2(t+1)x_2 + b - a^3 = 0. \end{aligned} \tag{5}$$

To prove Theorem 1 for $n = 4$, it is enough to show that the set of $x_2 \in \mathbb{Q}(t)$, such that Eq. (5) has a solution (with respect to x_1), is infinite. Then we need to show that there are infinitely many $x_2 \in \mathbb{Q}(t)$ such that the discriminant of Eq. (5) is a square, which leads to the problem of finding infinitely many rational parametric solutions on the following curve

$$\begin{aligned} C: \ y^2 =& 9(t^2-1)^2 x_2^4 + 36at(t+1)x_2^3 \\ &- 18a^2(t+1)^2 x_2^2 + 12(a^3 - b)(t+1)x_2 - 3a(a^3 - 4b). \end{aligned}$$

The discriminant of C is

$$\begin{aligned} \Delta(t) =& -5038848(t+1)^4\big((-b+a^3)t^2 + (-2b-a^3)t - b + a^3\big)^2 \\ &\big((9b^2 + a^6 - 10a^3b)t^4 + (-36b^2 + 14a^3b - 2a^6)t^3 + (54b^2 - 24a^3b + 3a^6)t^2 \\ &+ (-36b^2 + 14a^3b - 2a^6)t + 9b^2 + a^6 - 10a^3b\big), \end{aligned}$$

and is non-zero as an element of $\mathbb{Q}(t)$. Then C is smooth.

By [6, Prop. 7.2.1, p. 476], we can transform the curve C into a family of elliptic curves

$$\begin{aligned} E: \ Y^2 =& X^3 - 18a^2(1+t)^2X^2 \\ &+ 108a(1+t)^2((a^3 - 4b)t^2 + (2a^3 + 4b)t + a^3 - 4b)X \\ &- 648(1+t)^2((a^6 - 8ba^3 - 2b^2)t^4 + (-8ba^3 + 4b^2 + 4a^6)t^2 + a^6 - 8ba^3 - 2b^2), \end{aligned}$$

by the inverse birational map $\phi: \ (x_2, y) \longrightarrow (X, Y)$. Because the coordinates of this map are quite complicated, we omit these equations.

An easy calculation shows that the point

$$P = \Big(18a^2(t^4+1)/(t-1)^2, 36\big((a^3-b)t^6+(2b+a^3)t^5 + (b-a^3)t^4+(4a^3-4b)t^3+(b-a^3)t^2+(2b+a^3)t-b+a^3\big)/(t-1)^3 \Big)$$

lies on E. To prove that the group $E(\mathbb{Q}(t))$ is infinite, it is enough to find a point on E with infinite order. By the group law of the elliptic curves, we can get $[2]P$. Let $[2]P_2$ be the point of specialization at $t=2$ of $[2]P$. The X-coordinate of $[2]P_2$ is

$$\frac{18a^2(-567b^2+2322ba^3+80937a^6)}{(-9b+111a^3)^2}.$$

Let E_2 be the specialization of E at $t=2$, i.e.,

$$E_2: \ Y^2 = X^3 - 162a^2X^2 + 972a(9a^3-12b)X - 192456a^6 + 979776ba^3 + 104976b^2.$$

There are two cases we need to discuss.

1. For $b = 37a^3/3$, the curve E_2 becomes

$$Y^2 = X^3 - 162a^2X^2 - 135108a^4X + 27859464a^6.$$

Now $[2]P_2$ is the point at infinity on E_2, and we need find a point of infinite order. Let $Y' = Y/a^3, X' = X/a^2$. We have an elliptic curve

$$E_2': \ Y'^2 = X'^3 - 162X'^2 - 135108X' + 27859464.$$

It is easy to show that $Q = (234, -432)$ is a point of infinite order on E_2'. Then there are infinitely many rational points on E_2' and E.

2. For $b \neq 37a^3/3$, when the numerator of the $X-$coordinate of $[2]P_2$ is divided by the denominator with respect to b, the remainder equals

$$r = 69984a^5(-3b+43a^3).$$

1. For $a \neq 0$ and $b \neq 43a^3/3$, we see that r is not zero. By the Nagell-Lutz theorem ([11, p. 56]), $[2]P_2$ is a point of infinite order on E_2. Thus P is a point of infinite order on E.

2. For $a \neq 0$ and $b = 43a^3/3$, the curve E_2 becomes

$$Y^2 = X^3 - 162a^2X^2 - 158436a^4X + 35417736a^6.$$

Let $Y' = Y/a^3, X' = X/a^2$. We have an elliptic curve

$$E_2': \ Y'^2 = X'^3 - 162X'^2 - 158436X' + 35417736.$$

It is easy to show that $R = (306, -648)$ is a point of infinite order on E'_2. Then there are infinitely many rational points on E'_2 and E.

In summary, for $a, b \in \mathbb{Z}, ab \neq 0$, there are infinitely many rational points on E. By the birational map ϕ, we can get infinitely many rational solutions of Eqs. (5) and (4). This completes the proof of Theorem 1 for $n = 4$.

Next, we will deal with Eq. (3) for $n \geq 5$. Let $x'_5, x'_6, \ldots, x'_n$ be rational parameters and set

$$a' = \sum_{i=5}^{n} x'_i,\ \ b' = \sum_{i=5}^{n} x'^3_i.$$

From the proof of the previous part, we know that Eq. (4) has infinitely many rational solutions

$$(x'_{1j}, x'_{2j}, x'_{3j}, x'_{4j}), j \geq 1,$$

depending on one parameter t for $A = a - a'$ and $B = b - b'$. This leads to the conclusion that for each $j \geq 1$, the n-tuple of the following form

$$x_1 = x'_{1j}, x_2 = x'_{2j}, x_3 = x'_{3j}, x_4 = x'_{4j}, x_i = x'_i, i \geq 5$$

satisfies Eq. (3). □

Example 5. For $n = 4$, from the point $[2]P$, we get

$$\begin{aligned}
x_1 =& -\frac{q(t)}{3a^2t(t+1)(t^2-t+1)(t-1)^2p(t)},\\
x_2 =& \frac{ah(t)}{(t+1)(t-1)^2p(t)},\\
x_3 =& tx_2,\\
x_4 =& a - x_1 - x_2 - x_3 = \frac{s(t)}{3a^2t(t+1)(t^2-t+1)(t-1)^2p(t)},
\end{aligned}$$

where $q(t)$ and $s(t)$ have degree 13 as a polynomial of $\mathbb{Q}(t)$, $h(t)$ has degree 8, and $p(t)$ has degree 6.

From the above example, it seems too difficult to prove that these rational parametric solutions are positive, so we need a new idea to prove Theorem 2.

Proof. In the proof of Theorem 1, for $n = 4$ we get the curve

$$\begin{aligned}
C:\ y^2 =& 9(t^2-1)^2x_2^4 + 36at(t+1)x_2^3\\
&- 18a^2(t+1)^2x_2^2 + 12(a^3-b)(t+1)x_2 - 3a(a^3-4b).
\end{aligned}$$

The discriminant of C is

$$\begin{aligned}
\Delta(t) =& -5038848(t+1)^4\big((-b+a^3)t^2 + (-2b-a^3)t - b + a^3\big)^2\\
&\big((9b^2+a^6-10a^3b)t^4 + (-36b^2+14a^3b-2a^6)t^3 + (54b^2-24a^3b+3a^6)t^2\\
&+ (-36b^2+14a^3b-2a^6)t + 9b^2 + a^6 - 10a^3b\big).
\end{aligned}$$

Let us consider $\Delta(t) = 0$, so that C has multiple roots. Put

$$(-b + a^3)t^2 + (-2b - a^3)t - b + a^3 = 0,$$

and solving for t, we get

$$t = \frac{2b + a^3 \pm \sqrt{12ba^3 - 3a^6}}{-b + a^3}.$$

In order to make t be a rational number, take

$$12ba^3 - 3a^6 = c^2,$$

where c is a rational parameter. Then we have

$$b = \frac{3a^6 + c^2}{12a^3}, \ t = \frac{3a^3 + c}{3a^3 - c}, \ or \ \frac{3a^3 - c}{3a^3 + c}.$$

According to the symmetry of t, consider

$$t = \frac{3a^3 + c}{3a^3 - c}.$$

Let

$$Y_1 = Y + \frac{6atX}{t-1} + 36(a^3 - b)(t-1)(t+1)^2,$$

we get

$$E' : \ Y_1^2 = X^3 - 18a^2(t+1)^2X^2 - 108a(t+1)^2((a^3 - 4b)t^2 + (4b + 2a^3)t + a^3 - 4b)X \\ - 648(t+1)^2((a^6 - 8ba^3 - 2b^2)t^4 + (-8ba^3 + 4b^2 + 4a^6)t^2 + a^6 - 8ba^3 - 2b^2).$$

Substituting

$$b = \frac{3a^6 + c^2}{12a^3}, \ t = \frac{3a^3 + c}{3a^3 - c}$$

into E', we get

$$Y_1^2 = \frac{((3a^3 - c)^2X + 72a^2c^2)((3a^3 - c)^2X - 36a^2(c^2 + 9a^6))^2}{(3a^3 - c)^6}.$$

To get infinitely many solutions of (Y_1, X), put

$$(3a^3 - c)^2X + 72a^2c^2 = d^2,$$

which leads to

$$X = \frac{d^2 - 72a^2c^2}{(3a^3 - c)^2}.$$

Then

$$Y = -\frac{d(d+12ca)(27a^7+9ac^2-dc)}{c(3a^3-c)^3}.$$

Tracing back, we get

$$\begin{aligned}
x_1 =& \frac{(-3a^3+c)d^2+(54a^7+18ac^2)d+108a^2(3a^3+c)(3a^6+c^2)}{72a^3(dc+27a^7+9c^2a)},\\
x_2 =& \frac{d(d+12ca)(3a^3-c)}{72a^3(dc+27a^7+9c^2a)},\\
x_3 =& \frac{d(d+12ca)(3a^3+c)}{72a^3(dc+27a^7+9c^2a)},\\
x_4 =& \frac{(-3a^3-c)d^2+(-54a^7-18ac^2)d+108a^2(3a^3-c)(3a^6+c^2)}{72a^3(dc+27a^7+9c^2a)}.
\end{aligned}$$

To prove $x_i > 0, i = 1, 2, 3, 4$, assume that $a > 0, c > 0, d > 0$. Then we have

$$72a^3(dc+27a^7+9c^2a) > 0, x_3 > 0,$$

so we just need to consider the numerators of x_1, x_2, x_4. Moreover, set $3a^3 - c > 0$, we have $x_2 > 0$, and the discriminants of

$$(-3a^3+c)d^2+(54a^7+18ac^2)d+108a^2(3a^3+c)(3a^6+c^2)$$

and

$$(-3a^3-c)d^2+(-54a^7-18ac^2)d+108a^2(3a^3-c)(3a^6+c^2)$$

are $108(-c^2+45a^6)(3a^6+c^2)a^2 > 0$. We see that the intervals of d such that $x_1 > 0, x_4 > 0$ are given by

$$\left(\frac{3(9a^6+3c^2-\sqrt{\delta})a}{3a^3-c}, \frac{3(9a^6+3c^2+\sqrt{\delta})a}{3a^3-c}\right)$$

and

$$\left(\frac{3(-9a^6-3c^2-\sqrt{\delta})a}{3a^3+c}, \frac{3(-9a^6-3c^2+\sqrt{\delta})a}{3a^3+c}\right),$$

respectively, where $\delta = 405a^{12} + 126a^6c^2 - 3c^4$. It is easy to show that

$$\frac{3(-9a^6-3c^2+\sqrt{\delta})a}{3a^3+c} > 0, \frac{3(9a^6+3c^2-\sqrt{\delta})a}{3a^3-c} < 0,$$

and

$$\frac{3(9a^6+3c^2+\sqrt{\delta})a}{3a^3-c} > \frac{3(-9a^6-3c^2+\sqrt{\delta})a}{3a^3+c}.$$

Hence if

$$d \in \left(0, \frac{3(-9a^6-3c^2+\sqrt{\delta})a}{3a^3+c}\right),$$

we have $x_1, x_4 > 0$. This completes the proof of Theorem 2. □

Example 6. If we take $a = c = 1$, then $t = 2$, $b = 1/3$, and

$$x_1 = \frac{-d^2 + 36d + 864}{36(d + 36)}, x_2 = \frac{d(d + 12)}{36(d + 36)}, x_3 = \frac{d(d + 12)}{18(d + 36)}, x_4 = \frac{-d^2 - 18d + 216}{18(d + 36)},$$

where $d \in (0, -9+3\sqrt{33} \approx 8.233687940)$ and d is a rational number. Taking $d = 1, 2, 3, 4, 5, 6, 7, 8$, we get eight 4-tuples of positive rational solutions with the same sum 1 and the same sums of cubes 1/3, which are as follows:

$$\begin{aligned}(x_1, x_2, x_3, x_4) = &\left(\frac{899}{1332}, \frac{13}{1332}, \frac{13}{666}, \frac{197}{666}\right), \left(\frac{233}{342}, \frac{7}{342}, \frac{7}{171}, \frac{44}{171}\right), \\ &\left(\frac{107}{156}, \frac{5}{156}, \frac{5}{78}, \frac{17}{78}\right), \left(\frac{31}{45}, \frac{2}{45}, \frac{4}{45}, \frac{8}{45}\right), \left(\frac{1019}{1476}, \frac{85}{1476}, \frac{85}{738}, \frac{101}{738}\right), \\ &\left(\frac{29}{42}, \frac{1}{14}, \frac{1}{7}, \frac{2}{21}\right), \left(\frac{1067}{1548}, \frac{133}{1548}, \frac{133}{774}, \frac{41}{774}\right), \left(\frac{68}{99}, \frac{10}{99}, \frac{20}{99}, \frac{1}{99}\right).\end{aligned}$$

3 The proofs of the corollaries

In this section, we give the proofs of the corollaries and two examples.

Proof. Take any k rational parametric solutions $(x_{i1}, \dots, x_{i,n}), i \le k$ of Eq. (3), where $x_{i5} = t_2, \dots, x_{in} = t_{n-3}, i \le k$ are parameters. Let $c = \text{lcm}_{i,j}(x_{ij}, j = 1, \dots, n, i \le k)$, and write

$$x_{ij} = \frac{y_{ij}}{c}, y_{ij} \in \mathbb{Z}[t_1, t_2, \dots, t_{n-3}],$$

with $(\text{gcd}_{i,j}(y_{ij}, c)) = 1$ and $c \in \mathbb{Z}[t_1, t_2, \dots, t_{n-3}]$, where $t_1 = t$. Then

$$\sum_{j=1}^{n} y_{ij} = ac, \sum_{j=1}^{n} y_{ij}^3 = bc^3.$$

Hence

$$\gcd_{i,j}\ (y_{ij}) = 1.$$

For two sets of solutions $\{(x_{i1}, \dots, x_{in}), i \le k\}$ and $\{(x'_{i1}, \dots, x'_{in}), i \le k\}$, if the sets of n-tuples $\{(y_{i1}, \dots, y_{in}), i \le k\}$ and $\{(y'_{i1}, \dots, y'_{in}), i \le k\}$ coincide, then $d = d'$ and the n-tuples coincide. Since there are infinitely many choices of k elements, for every k there are infinitely many primitive sets of k n-tuples of polynomials with the same sum and the same sum of cubes. This finishes the proof of Corollary 3. □

Example 7. For $n = 4$, we have the rational parametric solutions

$$\begin{aligned} x_1 =& -\frac{q(t)}{3a^2t(t+1)(t^2-t+1)(t-1)^2p(t)}, \\ x_2 =& \frac{ah(t)}{(t+1)(t-1)^2p(t)}, \\ x_3 =& \, tx_2, \\ x_4 =& \, a - x_1 - x_2 - x_3 = \frac{s(t)}{3a^2t(t+1)(t^2-t+1)(t-1)^2p(t)}. \end{aligned}$$

Multiply the least common multiple of the denominator of $x_i, i = 1, \ldots, 4$. When $a, b \in \mathbb{Z}$, we get that

$$x_1 = -q(t), x_2 = 3a^3t(t^2-t+1)h(t), x_3 = 3a^3t^2(t^2-t+1)h(t), x_4 = s(t)$$

are the 4-tuples of polynomials in $\mathbb{Z}[t]$ satisfying Eq. (3).

Proof. The proof of Corollary 4 is similar to the proof of Corollary 3, so we omit it. $\square$

Example 8. From the eight 4-tuples of positive rational solutions of Example 6, we get the following eight 4-tuples of positive integers

$$\begin{aligned} (y_1, y_2, y_3, y_4) =& (150719584015, 2179482305, 4358964610, 66055079090), \\ & (152140218230, 4570736170, 9141472340, 57460683280), \\ & (153169889565, 7157471475, 14314942950, 48670806030), \\ & (153837920236, 9925027112, 19850054224, 39700108448), \\ & (154170771755, 12860172325, 25720344650, 30561821290), \\ & (154192385490, 15950936430, 31901872860, 21267915240), \\ & (153924475705, 19186462295, 38372924590, 11829247430), \\ & (153386782640, 22556879800, 45113759600, 2255687980) \end{aligned}$$

with the same sum 223313110020 and the same sum of cubes 371211485419839924645710057 7336000.

4 A remaining question

Ren and Yang [10, Ques. 4] raised the following question:

Question 9. Are there infinitely many n-tuples of positive integers, having no common element, with the same sum and the same sum of their cubes for $n \geq 4$?

It's easy to calculate that any 4-tuples (x_1, x_2, x_3, x_4), given by the same method from Example 6, have no common element for $d \in \mathbb{Q} \bigcap (0, -9 + 3\sqrt{33})$. This gives a positive answer to Question 9 for $n = 4$. When $n \geq 5$, it seems out of our reach. However, we conjecture that the answer to Question 9 is yes.

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