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Kneser's theorem for upper Banach density

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Résumé. Supposons que A soit un ensemble d'entiers non négatifs avec densité de Banach supérieure α (voir définition plus bas) et que la densité de Banach supérieure de $A+A$ soit inférieure à 2α. Nous caractérisons la structure de $A+A$ en démontrant la proposition suivante: il existe un entier positif g et un ensemble W qui est l'union des $[2\alpha g-1]$ suites arithmétiques[1] avec la même différence g tels que $A+A \subseteq W$ et si $[a_n, b_n]$ est, pour chaque n, un intervalle d'entiers tel que $b_n - a_n \to \infty$ et la densité relative de A dans $[a_n, b_n]$ approche α, il existe alors un intervalle $[c_n, d_n] \subseteq [a_n, b_n]$ pour chaque n tel que $(d_n - c_n)/(b_n - a_n) \to 1$ et $(A+A) \cap [2c_n, 2d_n] = W \cap [2c_n, 2d_n]$.

Abstract. Suppose A is a set of non-negative integers with upper Banach density α (see definition below) and the upper Banach density of $A+A$ is less than 2α. We characterize the structure of $A+A$ by showing the following: There is a positive integer g and a set W, which is the union of $\lceil 2\alpha g-1 \rceil$ arithmetic sequences[1] with the same difference g such that $A+A \subseteq W$ and if $[a_n, b_n]$ for each n is an interval of integers such that $b_n - a_n \to \infty$ and the relative density of A in $[a_n, b_n]$ approaches α, then there is an interval $[c_n, d_n] \subseteq [a_n, b_n]$ for each n such that $(d_n - c_n)/(b_n - a_n) \to 1$ and $(A+A) \cap [2c_n, 2d_n] = W \cap [2c_n, 2d_n]$.

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Mots clefs. Upper Banach density, inverse problem, nonstandard analysis.

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[1]We call a set of the form $a + d\mathbb{N}$ an arithmetic sequence of difference d and call a set of the form $\{a, a+d, a+2d, \dots, a+kd\}$ an arithmetic progression of difference d. So an arithmetic progression is finite and an arithmetic sequence is infinite.

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