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COMPLETENESS THEOREM FOR A FIRST ORDER LINEAR-TIME LOGIC

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ABSTRACT. We describe a first order temporal logic over the natural numbers time. It is well known that the corresponding set of all valid formulas is not recursively enumerable, and that there is no finitistic axiomatization. We present an infinitary axiomatization which is sound and complete with respect to the considered logic.

1. Introduction

Temporal logics provide formalisms for describing the way that systems change over time. Since the Prior's work [**15**] temporal logics have been extensively studied. At their simplest, languages of temporal logics are classical languages extended by temporal operators G, the strong future operator, and H, the strong past operator. Then, the future operator F and the past operator P abbreviate $\neg G \neg$ and $\neg H \neg$, respectively. In [**10**] two new binary temporal operators, the since operator S, and the until operator U were introduced and the resulting logics were shown to be more expressive than the logics without them. Even in the case of propositional temporal logics, one's cosmology has important consequences: one can assume that time is linear or branching, with or without last moment, discrete or dense etc. One among many good overviews of the subject can be found in [**6**].

The interest of theoretical computer scientists in temporal logics has grown because reasoning about time has been shown to be a useful tool in describing behavior of an agent's knowledge base, for specification and verification of programs, hardware, protocols in distributed systems etc., [**7**]. Although the literature contains many versions of temporal logics, the research has been particularly focused on PLTL, propositional linear temporal logics over discrete time with a first moment, but without the last one, i.e., over natural numbers, with the future temporal operators next ($\bigcirc$) and U as the basic ones [**12**]. A sound and complete axiomatic

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system for PLTL was given in [**8**], while its first order extension, FOLTL, was presented in [**13**]. There are many complete axiomatizations of different first order temporal logics. For example, some kinds of such logics with F and P operators over various classes of time flows were axiomatized in [**9**], while axiomatic systems for the first order temporal logics with since and until over linear time and rationals were given in [**16**]. In the case of FOLTL (and similarly when the flow of time is isomorphic to reals or integers) the set of valid formulas is not recursively enumerable, and there is no recursive axiomatization of the logic [**1, 3, 9, 11, 18**]. Nevertheless, since it is a very natural logic, it is useful to find any kind of proof systems. Some results about logics with F and P over various classes of flows that include natural numbers were given in [**5**]. In [**1, 2, 4**] a translation of FOLTL formulas into classical formulas with explicit time parameters was considered, and alternative notions of nonstandard completeness (for example, domains that correspond to time parameters need not be countable) were proposed.

In this paper we present an infinitary axiomatization for FOLTL and prove its completeness. The term 'infinitary' concerns the meta language only, i.e., the object language is countable, and formulas are finite, while only proofs are allowed to be infinite. A similar logic is axiomatized in [**17**], while the corresponding completeness is showed using an algebraic method. In our paper Deduction theorem is proved and the approach of Henkin is followed, similarly as in [**14**]. To the best of our knowledge such an approach has not been published so far. The presented ideas can be easily restricted to the propositional case and used in proving the corresponding extended completeness theorem. Since compactness does not hold for the propositional temporal logic over the natural numbers time, extended completeness can not be proved using finitary means. In this paper we concern a strong, reflexive version of the until operator, which means that if $\alpha U \beta$ holds in a time instant, β must eventually hold, and that the future includes the present. All the presented results can be proved without such assumptions with essentially no change.

2. Syntax and semantics

We consider a first order language L which contains classical connectives $\neg$ and $\wedge$, an unary temporal operator $\bigcirc$ (next), a binary temporal operator U (until), quantifier $\forall$, and for every integer $k \geq 0$, k-ary relation symbols $P_0^k, P_1^k, \dots$, and k-ary function symbols $F_0^k, F_1^k, \dots$ The function symbols of arity 0 are called constant symbols. Terms, formulas, and sentences are defined as usual. The other classical connectives and quantifier $\exists$ can be defined as abbreviations. The temporal operators F (sometime) and G (always) are defined as $\top U \alpha$ and $\neg F \neg \alpha$. An example of a formula is $\bigcirc P_1^1(F_6^0) \rightarrow (\forall x)(\exists x)(P_0^2(y,x)\ U\ P_0^2(F_0^0, y))$. If $T = \{\alpha_1, \alpha_2, \dots\}$ is a set of formulas, then $\bigcirc T$ denotes $\{\bigcirc \alpha_1, \bigcirc \alpha_2, \dots\}$.

The corresponding models are tuples of the form $\langle W, D, I \rangle$ where

- $W = w_0, w_1, \dots$ is an ω-sequence of *time instants*,
- D is a non empty domain, and
- I associates an interpretation $I(w_i)$ with every $w_i \in W$ such that for all j and k:

- $I(w_i)(F_j^k)$ is a function from D^k to D,
- for all m, $I(w_i)(F_j^k) = I(w_m)(F_j^k)$, and
- $I(w_i)(P_j^k)$ is a k-ary relation on D.

Note that we use fixed domain models with rigid function symbols.

Let $\langle W, D, I\rangle$ be a model. A variable valuation v assigns some element of the domain to every time instant w_i and every variable x, i.e., $v(w_i)(x) \in D$. If $w_i \in W$, $d \in D$, and v is a valuation, then $v[d/x]_{w_i}$ is a valuation identical to v with the exception that $v[d/x]_{w_i}(w_i)(x) = d$. The value of a term t in a time instant w_i with respect to v (denoted by $I(w_i)(t)_v$) is:

- if t is a variable x, then $I(w_i)(x)_v = v(w_i)(x)$, and
- if $t = F_j^k(t_1, \dots, t_k)$, then $I(w_i)(t)_v = I(w_i)(F_j^k)(I(w_i)(t_1)_v, \dots, I(w_i)(t_k)_v)$.

The truth value of a formula α in a time instant w_i with respect to v (denoted by $I(w_i)(\alpha)_v$) is (we omit the usual conditions for the propositional connectives):

- if $\alpha = P_j^k(t_1, \dots, t_k)$, then $I(w_i)(\alpha)_v =$ true if $\langle I(w_i)(t_1)_v, \dots, I(w_i)(t_k)_v\rangle \in I(w_i)(P_j^k)$, otherwise $I(w_i)(\alpha)_v =$ false,
- if $\alpha = \bigcirc\beta$, then $I(w_i)(\alpha)_v =$ true if $I(w_{i+1})(\beta)_v =$ true, otherwise $I(w_i)(\alpha)_v$ = false,
- if $\alpha = \beta U \gamma$, then $I(w_i)(\alpha)_v =$ true if there is an integer $j \geq 0$ such that $I(w_{i+j})(\gamma)_v =$ true, and for every k such that $0 \leq k < j$, $I(w_{i+k})(\beta)_v =$ true, otherwise $I(w_i)(\alpha)_v =$ false,
- if $\alpha = (\forall x)\beta$, then $I(w_i)(\alpha)_v =$ true if for every $d \in D$, $I(w_i)(\beta)_{v[d/x]_{w_i}} =$ true, otherwise $I(w_i)(\alpha)_v =$ false.

We write $(M, w_i) \models \alpha$ if for every valuation v, $I(w_i)(\alpha)_v =$ true in the model M. A sentence α is satisfiable if there is a time instant w_i in a model M such that $(M, w_i) \models \alpha$. A set T of sentences is satisfiable if there is a time instant w_i in a model M such that for every $\alpha \in T$, $(M, w_i) \models \alpha$.

In the above definition the future includes the present, so that:

- $(M, w_i) \models F\alpha$ if there is $j \geq 0$ such that $(M, w_{i+j}) \models \alpha$, and
- $(M, w_i) \models G\alpha$ if for every $j \geq 0$, $(M, w_{i+j}) \models \alpha$.

3. A complete axiomatization

The axiomatic system contains the following axiom schemata:

1. all the axioms of the classical propositional logic
2. $(\forall x)(\alpha \to \beta) \to (\alpha \to (\forall x)\beta)$, where x is not free in α
3. $(\forall x)\alpha(x) \to \alpha(t/x)$, where $\alpha(t/x)$ is obtained by substituting all free occurrences of x in $\alpha(x)$ by the term t which is free for x in $\alpha(x)$
4. $\bigcirc(\alpha \to \beta) \to (\bigcirc\alpha \to \bigcirc\beta)$,
5. $\neg \bigcirc \alpha \leftrightarrow \bigcirc\neg\alpha$
6. $\alpha U \beta \leftrightarrow \beta \vee (\alpha \wedge \bigcirc(\alpha U \beta))$
7. $\alpha U \beta \to F\beta$
8. $(\forall x) \bigcirc \alpha(x) \to \bigcirc(\forall x)\alpha(x)$

and inference rules:

1. from $\{\alpha, \alpha \to \beta\}$ infer β
2. from α infer $(\forall x)\alpha$
3. from α infer $\bigcirc\alpha$
4. from $\{\beta \to \bigcirc^i\alpha\}$ for all $i \geq 0$, infer $\beta \to G\alpha$

The axiom 8 is a variant of well known Barcan formula. The infinitary inference rule 4 is the only such rule.

A formula α is a theorem ($\vdash \alpha$) if there is an at most countable sequence of formulas $\alpha_0, \alpha_1, \ldots, \alpha$, such that every α_i is an axiom or it is derived from the preceding formulas by an inference rule. A formula α is deducible from a set T of formulas ($T \vdash \alpha$) if there is an at most countable sequence of formulas $\alpha_0, \alpha_1, \ldots, \alpha$, such that every α_i is an axiom or a formula from T, or it is derived from the preceding formulas by an inference rule, with the exception that the inference rule 3 can be applied on theorems only. A set T of sentences is consistent if there is at least one formula which is not deducible from T, otherwise T is inconsistent. A set T of sentences is said to be maximal if for every sentence α, either $\alpha \in T$ or $\neg\alpha \in T$. A set T of sentences is saturated if it is consistent and maximal and satisfies:

- if $\neg(\forall x)\alpha(x) \in T$, then for some term t, $\neg\alpha(t) \in T$.

THEOREM 3.1 (Deduction theorem). *If T is a set of formulas, α is a sentence, and $T, \alpha \vdash \beta$, then $T \vdash \alpha \to \beta$.*

PROOF. We use the transfinite induction on the length of the inference. The cases when $\vdash \beta$ or $\beta = \alpha$ or β is obtained by the inference rules 1 and 2, are standard. Assume that $T, \alpha \vdash \beta$, where $\beta = \bigcirc\gamma$ is obtained by the inference rule 3. By definition of the derivability relation $\vdash$, we have $\vdash \gamma$. Thus, $\vdash \beta$, and $T \vdash \alpha \to \beta$. Now assume that β is of the form $\gamma \to G\gamma'$, and that $T, \alpha \vdash \beta$ is obtained by the inference rule 4 from $T, \alpha \vdash \gamma \to \bigcirc^i\gamma'$ for every $i \geq 0$. By the induction hypothesis, for $i \geq 0$, $T \vdash \alpha \to (\gamma \to \bigcirc^i\gamma')$, i.e., $T \vdash (\alpha \wedge \gamma) \to \bigcirc^i\gamma'$. By the inference rule 4, we obtain $T \vdash (\alpha \wedge \gamma) \to G\gamma'$, and $T \vdash \alpha \to (\gamma \to G\gamma')$. $\square$

LEMMA 3.1. *Let α, β be formulas.*

1. $\vdash G\alpha \leftrightarrow \alpha \wedge \bigcirc G\alpha$,
2. $\vdash G \bigcirc \alpha \leftrightarrow \bigcirc G\alpha$,
3. $\vdash (\bigcirc\alpha \to \bigcirc\beta) \to \bigcirc(\alpha \to \beta)$,
4. $\vdash \bigcirc(\alpha \wedge \beta) \leftrightarrow (\bigcirc\alpha \wedge \bigcirc\beta)$,
5. $\vdash \bigcirc(\alpha \vee \beta) \leftrightarrow (\bigcirc\alpha \vee \bigcirc\beta)$,
6. $G\alpha \vdash \bigcirc^i\alpha$ *for every* $i \geq 0$,
7. *if* $\vdash \alpha$, *then* $\vdash G\alpha$,
8. *for* $j \geq 0$, $\bigcirc^j\beta, \bigcirc^0\alpha, \ldots, \bigcirc^{j-1}\alpha \vdash \alpha U\beta$,
9. *if T is a set of formulas and* $T \vdash \alpha$, *then* $\bigcirc T \vdash \bigcirc\alpha$.

PROOF. The proofs are easy consequences of the temporal part of the axiomatization. For example, consider (9) and use the induction on the depth of the derivation of α from T. Suppose that $T \vdash (\forall x)\alpha$ is obtained from $T \vdash \alpha$ by the inference rule 2. Then we have

$T \vdash \alpha$
$\bigcirc T \vdash \bigcirc\alpha$ (by the induction hypothesis)
$\bigcirc T \vdash (\forall x) \bigcirc \alpha$ (by the inference rule 2)
$\bigcirc T \vdash \bigcirc(\forall x)\alpha$ (by Barcan formula).

The other cases follow similarly. □

THEOREM 3.2. *Let T be a consistent set of sentences in the language L and C a countably infinite set of constants such that $L \cap C = \emptyset$. Then T can be extended to a saturated set in the language $L \cup C$.*

PROOF. Let $\alpha_0, \alpha_1, \dots$ be an enumeration of all sentences in L. We define a sequence of sets T_i, $i = 0, 1, 2, \dots$ of sentences, and a set $\mathcal{T}$ such that:

1. $T_0 = T$.
2. For every $i \geq 0$,
   (a) If $T_i \cup \{\alpha_i\}$ is consistent, then $T_{i+1} = T_i \cup \{\alpha_i\}$.
   (b) Otherwise, if α_i is of the form $\gamma \to G\beta$, then $T_{i+1} = T_i \cup \{\neg\alpha_i, \gamma \to \neg \bigcirc^{j_0} \beta\}$ for some $j_0 \geq 0$ such that T_{i+1} is consistent.
   (c) Otherwise, if α_i is of the form $\neg(\forall x)\beta(x)$, then $T_{i+1} = T_i \cup \{\neg\alpha_i, \neg\beta(c)$ for some $c \in C$ such that T_{i+1} is consistent.
   (d) Otherwise, $T_{i+1} = T_i \cup \{\neg\alpha_i\}$.
3. $\mathcal{T} = \bigcup_{i=0}^{\infty} T_i$.

The sets obtained by the steps 2a, and 2d are obviously consistent. A standard proof can be used to show that the same holds for the sets obtained by the step 2c. Suppose that for some $i \geq 0$, T_i is consistent, while $T_i \cup \{\gamma \to G\beta\}$ is not. Then, $T_i \cup \{\neg(\gamma \to G\beta)\}$ is consistent. Next, suppose that for every $j \geq 0$, $T_i, \neg(\gamma \to G\beta), \gamma \to \neg \bigcirc^j \beta \vdash \perp$. By Deduction Theorem, for every $j \geq 0$, $T_i, \neg(\gamma \to G\beta) \vdash \neg(\gamma \to \neg \bigcirc^j \beta)$. Thus, for every $j \geq 0$, $T_i, \neg(\gamma \to G\beta) \vdash \gamma \to \bigcirc^j\beta$, and by the inference rule 4, $T_i, \neg(\gamma \to G\beta) \vdash \gamma \to G\beta$ which leads to a contradiction.

Consider the set $\mathcal{T}$. It is maximal by the step 2. We show by the induction that $\mathcal{T}$ is a deductively closed set which does not contain all formulas, and as a consequence that $\mathcal{T}$ is consistent. Suppose that $\mathcal{T} \vdash \alpha$. We can show that $\alpha \in \mathcal{T}$. If α is an axiom, an element of $\mathcal{T}$ or obtained by the finitary inference rules the proof is standard. For example, let α be obtained from $\mathcal{T} \vdash \beta \to \alpha$ and $\mathcal{T} \vdash \beta$ by Modus Ponens. By the induction hypothesis, $\beta \to \alpha, \beta \in \mathcal{T}$. By construction of $\mathcal{T}$, there is some $i \geq 0$ such that $\beta \to \alpha, \beta \in T_i$ and either $\alpha \in T_i$ or $\neg\alpha \in T_i$ (but not both by consistency of T_i). Actually, $\alpha \in T_i$ otherwise this leads to a contradiction by consistency of T_i.

Let $\gamma \to G\beta$ be obtained from $\mathcal{T} \vdash \gamma \to \bigcirc^j\beta$ for every $j \geq 0$ by the inference rule 4. Suppose that $\gamma \to G\beta \notin \mathcal{T}$, which is equivalent to $\neg(\gamma \to G\beta) \in \mathcal{T}$ by maximality of $\mathcal{T}$. Then there are $i, j_0 \geq 0$ such that $\neg(\gamma \to G\beta), \gamma \to \neg \bigcirc^{j_0} \beta \in T_i$. By the induction hypothesis, for every $j \geq 0$, $\gamma \to \bigcirc^j\beta \in \mathcal{T}$. So there is $i' \geq i$ such that $\gamma \to \neg \bigcirc^{j_0} \beta$, $\gamma \to \bigcirc^{j_0}\beta$, and $\neg(\gamma \to G\beta) \in T_{i'}$. So $T_{i'} \vdash \neg(\gamma \to G\beta)$ and $T_{i'} \vdash \gamma \wedge \neg G\beta$. Since $T_{i'} \vdash \gamma \to \neg \bigcirc^{j_0} \beta$ and $T_{i'} \vdash \gamma \to \bigcirc^{j_0}\beta$, we get $T_{i'} \vdash \neg \bigcirc^{j_0} \beta \wedge \bigcirc^{j_0}\beta$ which is in contradiction with consistency of $T_{i'}$.

Finally, the step 2c of the construction guarantees that $\mathcal{T}$ is saturated. □

Starting from a consistent set T and its saturated extension $\mathcal{T}$ in a language L, we define a tuple $M_T = \langle W, D, I\rangle$ as follows:

- $W = w_0, w_1, \dots, w_0 = \mathcal{T}$, and for $i \geq 0$, $w_{i+1} = \{\alpha : \bigcirc\alpha \in w_i\}$,
- D is the set of all variable-free terms in L,
- for $i \geq 0$, $I(w_i)$ is an interpretation such that:
  - for every function symbol F^k_j, $I(w_i)(F^k_j)$ is a function from D^k to D such that for all variable-free terms $t_1, \dots, t_k$ in L, $I(w_i)(F^k_j) : \langle t_1, \dots, t_k\rangle \mapsto F^k_j(t_1, \dots, t_k)$, and
  - for every relation symbol P^k_j, $I(w_i)(P^k_j) = \{\langle t_1, \dots, t_k\rangle : t_1, \dots, t_k$ are variable-free terms in $L, P^k_j(t_1, \dots, t_k) \in w_i\}$.

LEMMA 3.2. *For every $i \geq 0$, w_i is a saturated set.*

PROOF. The proof is by induction on i. By hypothesis, w_0 is saturated. Let $i \geq 0$ and w_i be saturated. Suppose that w_{i+1} is not maximal. There is a formula α such that $\{\alpha, \neg\alpha\} \cap w_{i+1} = \emptyset$. Consequently, $\{\bigcirc\alpha, \neg\bigcirc\alpha\} \cap w_i = \emptyset$ which is a contradiction with the maximality of w_i. Suppose that w_{i+1} is not consistent, i.e. $w_{i+1} \vdash \alpha \wedge \neg\alpha$, for any formula α. By theorems 3.1(9), and 3.1(4), we have $w_i \vdash \bigcirc\alpha \wedge \neg\bigcirc\alpha$ which is a contradiction with consistency of w_i. Finally, suppose that there is a sentence $\neg(\forall x)\alpha(x) \in w_{i+1}$ such that for every variable-free term t in L, $\neg\alpha(t) \notin w_{i+1}$. Thus, $\bigcirc\neg(\forall x)\alpha(x) \in w_i$, and for every t, $\bigcirc\neg\alpha(t) \notin w_i$. Using Barcan formula and Axiom 5, we obtain $\neg(\forall x)\bigcirc\alpha(x) \in w_i$, and for every term t in L, $\neg\bigcirc\alpha(t) \notin w_i$, a contradiction. □

THEOREM 3.3 (Extended completeness theorem). *Every consistent set T of sentences is satisfiable.*

PROOF. By Theorem 3.2, T can be extended to a saturated set $\mathcal{T}$. We can define a model $M_T = \langle W, D, I\rangle$ as above and prove that for every sentence α and every $w_i \in W$, $\alpha \in w_i$ iff $(M, w_i) \models \alpha$. If α is an atomic formula, by the definition of $I(w_i)$, $(M, w_i) \models \alpha$ iff $\alpha \in w_i$. The cases when formulas are negations and conjunctions can be proved as usual. If $\alpha = (\forall x)\beta \in w_i$, then, by Axiom 3, $\beta(t) \in w_i$ for every $t \in D$. By the induction hypothesis $(M, w_i) \models \beta(t)$ for every $t \in D$, and $(M, w_i) \models (\forall x)\beta$. If $\alpha \notin w_i$, there is some $t \in D$ such that $(M, w_i) \models \neg\beta(t)$, because w_i is saturated. It follows that $(M, w_i) \not\models (\forall x)\beta$. If $\alpha = \bigcirc\beta$, we have $w_i \models \alpha$ iff $w_{i+1} \models \beta$ iff $\beta \in w_{i+1}$ iff $\alpha \in w_i$ (by construction of w_{i+1}). Finally, let $\alpha = \beta U\gamma$. Suppose that $w_i \models \beta U\gamma$. There is some $j \geq 0$ such that $w_{i+j} \models \gamma$ and for every k, $0 \leq k < j$, $w_{i+k} \models \beta$. By the induction hypothesis, $\gamma \in w_{i+j}$, and $\beta \in w_{i+k}$, for $j \geq 0$, $0 \leq k < j$. By construction of M_T, $\bigcirc^j\gamma \in w_i$, $\bigcirc^k\beta \in w_i$, for $j \geq 0$, $0 \leq k < j$. It follows from Lemma 3.1(8) that $\beta U\gamma \in w_i$. For the other direction, assume that $\beta U\gamma \in w_i$. It follows from Axiom 7 that $F\gamma \in w_i$, i.e. that $\neg F\gamma = G\neg\gamma \notin w_i$. By construction of the model M_T, for some $j \geq 0$, $\bigcirc^j\gamma \in w_i$, i.e., $\gamma \in w_{i+j}$. Let $j_0 = \min\{j : \bigcirc^j\gamma \in w_i\}$. If $j_0 = 0$, $\gamma \in w_i$, and by the induction hypothesis $w_i \models \gamma$. It follows that $w_i \models \beta U\gamma$. Thus, suppose that $j_0 > 0$. For every j such that $0 \leq j < j_0$, $\bigcirc^j\gamma \notin w_i$, i.e. $\gamma \notin w_{i+j}$. From Axiom 6, Lemma 3.1(4), Lemma 3.1(5), and $\beta U\gamma \in w_i$ we have

$\gamma \vee (\beta \wedge (\bigcirc\gamma \vee (\bigcirc\beta \wedge \ldots \wedge (\bigcirc^{j_0-1}\gamma \vee (\bigcirc^{j_0-1}\beta \wedge \bigcirc^{j_0}(\beta U\gamma)\ldots) \in w_i$. It follows that for every $j < j_0$, $\bigcirc^j\beta \in w_i$, $\beta \in w_{i+j}$, $w_{i+j} \models \gamma$, and $w_i \models \beta U\gamma$.

Thus, T is satisfiable in M_T. □

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