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APPLICATION OF THE QUASIASYMPTOTIC BOUNDEDNESS OF DISTRIBUTIONS ON WAVELET TRANSFORM

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Abstract. We analyze the boundedness of the wavelet transform $\mathcal{W}_g f$ of the quasiasymptotically bounded distribution f. Assuming that the distribution $f \in \mathcal{S}'(\mathbb{R})$ is quasiasymptotically or r-quasiasymptotically bounded at a point or at infinity related to a continuous and positive function, we obtain results for the localization of its wavelet transform.

1. Introduction

Dependence of the localization properties of the continuous wavelet transform $\mathcal{W}_g f$ from the localization of the analyzing function $f \in L^2(\mathbb{R})$ and the wavelet $g \in L^2(\mathbb{R})$ in time and frequency space, as well as the opposite dependence is analyzed by Holschneider [**2**]. Working in the Fourier space, Pathak proved some Abelian theorems for the behaviour of the wavelet transform of L^2 functions and tempered distributions [**3**]. Contrary to the approaches in [**2, 3**], that are based on classical estimations, we use the theory of asymptotic behaviour of distributions to the asymptotic analysis of the continuous wavelet transform. Several Abelian and Tauberian theorems for the wavelet transform are proved in [**5, 6, 7**] using the quasiasymptotics and the S-asymptotics of distributions. We refer to [**12, 4, 1, 9, 10, 11**] and references therein for the definitions, properties and application of these kinds of asymptotics of distributions.

In this article, we analyze the boundedness of the wavelet transform $\mathcal{W}_g f$ of the quasiasymptotically bounded distribution f. Assuming that the distribution $f \in \mathcal{S}'(\mathbb{R})$ is quasiasymptotically bounded at 0 or infinity (respectively, at $b_0 \in \mathbb{R}$) related to a continuous and positive function, we obtain novel results for the localization of its wavelet transform $\mathcal{W}_g f(b,a)$ (respectively, $\mathcal{W}_g f(b_0,a)$), Theorem 3.1

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and Theorem 3.2. Additionally, we define the notion of r-quasiasymptotic boundedness of distributions from the space $\tilde{\mathcal{S}}'_r(\mathbb{R})$, $r < 0$, and assuming that $f \in \tilde{\mathcal{S}}'_r(\mathbb{R})$ is r-quasiasymptotically bounded we prove the results for the boundedness of its wavelet transform, Theorem 3.3 and Theorem 3.4.

2. Preliminaries and notations

The domain of functions considered in this article is $\mathbb{R}$. Therefore, we omit the suffix and write C^∞ instead of $C^\infty(\mathbb{R})$, $\mathcal{S}$ instead $\mathcal{S}(\mathbb{R})$, and so on. The space of infinitely differentiable functions (smooth functions) is denoted by C^∞. The space of rapidly decreasing smooth functions defined on the real line, supplied with the usual topology is denoted by $\mathcal{S}$. Its strong dual is the well known space of tempered distributions $\mathcal{S}'$. We refer to [**8, 13**] for the properties of these spaces.

Let $\tilde{\mathcal{S}}_r$, $r < 0$ be the space of functions $\varphi \in C^\infty$ for which all norms

$$\|\varphi\|_{r,p} = \sup_{x\in\mathbb{R},\, \alpha\leqslant p} (1 + |x|^2)^{r/2}\, |\varphi^{(\alpha)}(x)|, \quad p = 0, 1, 2, \dots \tag{2.1}$$

are finite. The topology of a projective limit of the decreasing sequence of spaces

$$\tilde{\mathcal{S}}_{r,0} \supset \tilde{\mathcal{S}}_{r,1} \supset \cdots \supset \tilde{\mathcal{S}}_{r,p} \supset \cdots,$$

where $\tilde{\mathcal{S}}_{r,p}$ is the completion of $\mathcal{S}$ in the norm $\|\cdot\|_{r,p}$ defined by (2.1), is introduced in $\tilde{\mathcal{S}}_r$. Each embedding $\tilde{\mathcal{S}}_{r,p+1} \subset \tilde{\mathcal{S}}_{r,p}$, $p = 0, 1, 2, \dots$ is continuous.

The strong dual of $\tilde{\mathcal{S}}_r$, $r < 0$ is denoted by $\tilde{\mathcal{S}}'_r$. It is the inductive limit of the increasing sequence of spaces

$$\tilde{\mathcal{S}}'_{r,0} \subset \tilde{\mathcal{S}}'_{r,1} \subset \cdots \subset \tilde{\mathcal{S}}'_{r,p} \subset \cdots,$$

where $\tilde{\mathcal{S}}'_{r,p}$ is the dual to the space $\tilde{\mathcal{S}}_{r,p}$, $p = 0, 1, 2, \dots$.

We also need the definition of the spaces of highly localized function over the real line which are introduced in [**2**]. By S_+ is denoted the set of functions s for which $\operatorname{supp}\hat{s} \subset [0, \infty)$ and for every localized exponent $\alpha > 0$

$$\|s\|_\alpha = \sup_{x\in\mathbb{R}} k_\alpha^{-1}(x)|s(x)| + \sup_{\omega\geqslant 0} \phi_\alpha^{-1}(\omega)|\hat{s}(\omega)| < \infty,$$

where $k_\alpha(x) = (1+|x|^2)^{-\alpha/2}$, $\phi_\alpha(\omega) = \omega^\alpha(1+\omega)^{-\alpha-1}$ and $\hat{s}$ is the Fourier transform of s. The image of S_+ under the parity operator is denoted by S_-, that is $s \in S_-$ if and only if $s(-x) \in S_+$. The direct sum of S_+ and S_- is denoted by S_0: $S_0 = S_+ \oplus S_-$. It is proved that the spaces S_+, S_- and S_0 are closed subspaces of $\mathcal{S}$; S_+ (respectively, S_-) consists of those functions in $\mathcal{S}$ whose Fourier transforms are supported by the positive (respectively, negative) frequencies only, and S_0 consists of functions from $\mathcal{S}$ for which all the moments vanish [**2**, Theorem 19.1.3].

We refer to [**2**] for the definition and properties of the wavelet transform over the spaces L^2 and $\mathcal{S}'$. The *wavelet transform* of $f \in L^2$ with respect to the wavelet $g \in L^2$ is defined by

$$\mathcal{W}_g f(b, a) := \int_{-\infty}^{+\infty} f(t)\, \frac{1}{a}\, \bar{g}\Big(\frac{t-b}{a}\Big)\, dt, \quad b \in \mathbb{R},\ a > 0\,.$$

The function g is usually called *mother wavelet* or *analyzing wavelet*. The functions $g_{b,a}(\cdot) = \frac{1}{a} g\big(\frac{\cdot - b}{a}\big)$, $b \in \mathbb{R}$, $a > 0$ which are obtained from the wavelet g by the operations of dilation and translation are called *wavelets*. The wavelet transform is a continuous linear transform from $L^2(\mathbb{R})$ into the space

$$L^2\Big(\mathbb{R}^2, \frac{da\,db}{a^2}\Big) = \Big\{ F(b,a) : \int_0^{+\infty} \frac{da}{a^2} \int_{-\infty}^{+\infty} |F(b,a)|^2 db < \infty \Big\},$$

which is a Hilbert space with the inner product

$$\langle F, G\rangle = \int_0^{+\infty} \frac{da}{a^2} \int_{-\infty}^{+\infty} F(b,a)\, \bar{G}(b,a)\, db, \quad F, G \in L^2\Big(\mathbb{R}^2, \frac{da\,db}{a^2}\Big).$$

The wavelet transform of the distribution $f \in \mathcal{S}'$ with respect to the wavelet $g \in S_0$ is the C^∞ function over $\{(b,a) \mid b \in \mathbb{R},\ a > 0\}$ given by

$$\mathcal{W}_g f(b,a) := \langle f(t), \bar{g}_{b,a}(t)\rangle, \quad t \in \mathbb{R} \tag{2.2}$$

(see [**2**, Chapter 1, Section 25]). For every $a > 0$, $\mathcal{W}_g f(\cdot, a)$ is a smooth function of polynomial growth since $\mathcal{W}_g f(b,a) = (f * \breve{\bar{g}}_a)(b)$, $b \in \mathbb{R}$, where $\breve{\bar{g}}_a(t) = \bar{g}(-t/a)/a$, $t \in \mathbb{R}$ is a rapidly decreasing function.

The wavelet transform of the distribution $f \in \tilde{\mathcal{S}}'_r$, $r < 0$ with respect to the wavelet $g \in S_0$ is also given by formula (2.2).

We will give a definition of quasiasymptotic boundedness of distributions from $\mathcal{S}'$ at a point and at infinity.

Definition 2.1. Let $f \in \mathcal{S}'$ and $c(\varepsilon)$, $\varepsilon \in (0,a)$ (respectively, $c(k)$, $k \in (a,\infty)$), $a > 0$ be a continuous positive function. We say that f is *quasiasymptotically bounded* at x_0 (respectively, at infinity) related to $c(\varepsilon)$ (respectively, $c(k)$) if there exist $p \in \mathbb{N}_0$ and $M > 0$ such that

$$\Big|\Big\langle \frac{f(x_0 + \varepsilon x)}{c(\varepsilon)}, \varphi(x) \Big\rangle\Big| \leqslant M\|\varphi\|_p, \quad 0 < \varepsilon < 1$$

$$\Big(\text{respectively, } \Big|\Big\langle \frac{f(kx)}{c(k)}, \varphi(x) \Big\rangle\Big| \leqslant M\|\varphi\|_p, \quad k > 1\Big),$$

for every $\varphi \in \mathcal{S}$.

We will also define quasiasymptotic boundedness of distributions from the space $\tilde{\mathcal{S}}'_r$, $r < 0$, which we will call r-quasiasymptotic boundedness.

Definition 2.2. Let $f \in \tilde{\mathcal{S}}'_r$, $r < 0$ and $c(\varepsilon)$, $\varepsilon \in (0,a)$ (respectively, $c(k)$, $k \in (a,\infty)$), $a > 0$ be a continuous positive function. We say that f is *r-quasiasymptotically bounded* at x_0 (respectively, at infinity) related to $c(\varepsilon)$ (respectively, $c(k)$) if there exist $p \in \mathbb{N}_0$ and $M > 0$ such that

$$\Big|\Big\langle \frac{f(x_0 + \varepsilon x)}{c(\varepsilon)}, \varphi(x) \Big\rangle\Big| \leqslant M\|\varphi\|_{r,p}, \quad 0 < \varepsilon < 1$$

$$\Big(\text{respectively, } \Big|\Big\langle \frac{f(kx)}{c(k)}, \varphi(x) \Big\rangle\Big| \leqslant M\|\varphi\|_{r,p}, \quad k > 1\Big),$$

for every $\varphi \in \tilde{\mathcal{S}}_r$.

3. Main results

In the following theorems we consider the boundedness of the wavelet transform $\mathcal{W}_g f(b,a)$ (respectively, $\mathcal{W}_g f(b_0,a)$) assuming that distribution $f\in\mathcal{S}'$ is quasiasymptotically bounded at 0 (respectively, at b_0) related to the continuous and positive function.

THEOREM 3.1. *Let $f\in\mathcal{S}'$ and $c(\varepsilon)$, $0<\varepsilon<\varepsilon'$ be a continuous positive function. If f is quasiasymptotically bounded at 0 related to $c(\varepsilon)$, then there exist $p\in\mathbb{N}_0$ and $C>0$ such that*

$$|\mathcal{W}_g f(b,a)|\leqslant \frac{C}{a}\Big(1+\frac{1}{a}\Big)^p(1+|b|)^p,\ \ b\in\mathbb{R},\ 0<a<1,$$

for every wavelet $g\in S_0$.

PROOF. From the definition of the distributional wavelet transform we have

$$\Big|\frac{\mathcal{W}_g f(\varepsilon b,\varepsilon a)}{c(\varepsilon)}\Big|=\Big|\Big\langle\frac{f(x)}{c(\varepsilon)},\frac{1}{\varepsilon a}\,\bar{g}\Big(\frac{x-\varepsilon b}{\varepsilon a}\Big)\Big\rangle\Big|.$$

After the change of variables $x=\varepsilon t$ we obtain

$$\Big|\frac{\mathcal{W}_g f(\varepsilon b,\varepsilon a)}{c(\varepsilon)}\Big|=\Big|\Big\langle\frac{f(\varepsilon t)}{c(\varepsilon)},\frac{1}{a}\,\bar{g}\Big(\frac{t-b}{a}\Big)\Big\rangle\Big|.$$

Since f is quasiasymptotically bounded at 0 related to $c(\varepsilon)$, and $g\in\mathcal{S}$ it follows that there exist $p\in\mathbb{N}_0$ and $M_1>0$ such that

$$\Big|\frac{\mathcal{W}_g f(\varepsilon b,\varepsilon a)}{c(\varepsilon)}\Big|\leqslant M_1\Big\|\frac{1}{a}\,\bar{g}\Big(\frac{x-b}{a}\Big)\Big\|_p, \tag{3.1}$$

for every $0<\varepsilon<1$. In the following, we will use the condition $0<a<1$, as well as the following elementary inequalities

$$1+|x+y|\leqslant(1+|x|)(1+|y|),\ \ 1+|xy|\leqslant(1+|x|)(1+|y|).$$

$$\begin{aligned}\Big\|\frac{1}{a}\,\bar{g}\Big(\frac{x-b}{a}\Big)\Big\|_p &= \sup_{x\in\mathbb{R},\,\alpha\leqslant p}(1+|x|^2)^{p/2}\Big|\Big(\frac{1}{a}\,\bar{g}\Big(\frac{x-b}{a}\Big)\Big)^{(\alpha)}\Big| \\ &= \sup_{x\in\mathbb{R},\,\alpha\leqslant p}(1+|x|^2)^{p/2}\frac{1}{a^{\alpha+1}}\Big|\bar{g}^{(\alpha)}\Big(\frac{x-b}{a}\Big)\Big| \\ &\leqslant \frac{M_2}{a^{p+1}}\sup_{x\in\mathbb{R}}(1+|x|^2)^{p/2}\Big(1+\Big|\frac{x-b}{a}\Big|\Big)^{-p} \\ &\leqslant \frac{M_2}{a^{p+1}}\sup_{x\in\mathbb{R}}((1+|x|)^2)^{p/2}\Big(1+\Big|\frac{x-b}{a}\Big|\Big)^{-p} \\ &= \frac{M_2}{a^{p+1}}\sup_{x\in\mathbb{R}}(1+|x|)^{p}\Big(1+\Big|\frac{x-b}{a}\Big|\Big)^{-p} \\ &\leqslant \frac{M_2}{a^{p+1}}\sup_{x\in\mathbb{R}}(1+|x-b|)^{p}(1+|b|)^p\Big(1+\Big|\frac{x-b}{a}\Big|\Big)^{-p} \\ &= \frac{M_2}{a^{p+1}}(1+|b|)^p\sup_{x\in\mathbb{R}}\Big(1+a\Big|\frac{x-b}{a}\Big|\Big)^{p}\Big(1+\Big|\frac{x-b}{a}\Big|\Big)^{-p}\end{aligned} \tag{3.2}$$

$$\begin{aligned}&\leqslant \frac{M_2}{a^{p+1}}(1+|b|)^p \sup_{x\in\mathbb{R}}\Big(1+\Big|\frac{x-b}{a}\Big|\Big)^p(1+a)^p\Big(1+\Big|\frac{x-b}{a}\Big|\Big)^{-p}\\ &= M_2\frac{(1+a)^p}{a^{p+1}}(1+|b|)^p,\end{aligned}$$

where M_2 is a positive constant.

From (3.1) and (3.2) it follows the estimation

$$\Big|\frac{\mathcal{W}_g f(\varepsilon b,\varepsilon a)}{c(\varepsilon)}\Big| \leqslant M_1M_2\frac{(1+a)^p}{a^{p+1}}(1+|b|)^p, \tag{3.3}$$

for $0<\varepsilon<1$, $0<a<1$ and $b\in\mathbb{R}$. We choose ε_0 such that $0<\varepsilon_0<1$ and put $\varepsilon_0 b=s$ and $\varepsilon_0 a=m$ $(0<m<1)$ in (3.3). So, we obtain

$$\begin{aligned}\big|\mathcal{W}_g f(s,m)\big| &\leqslant \frac{M_1M_2}{\varepsilon_0^{p-1}m^{p+1}}(\varepsilon_0+m)^p(\varepsilon_0+|s|)^p c(\varepsilon_0)\\ &\leqslant M_1M_2\frac{(1+m)^p}{m^{p+1}}(1+|s|)^p\frac{(1+\varepsilon_0)^p}{\varepsilon_0^{p-1}}\,c(\varepsilon_0).\end{aligned}$$

Since $c(\varepsilon)$ is a positive function it follows that there exists constant $C>0$ such that

$$\big|\mathcal{W}_g f(s,m)\big| \leqslant C\frac{(1+m)^p}{m^{p+1}}(1+|s|)^p,\ \ s\in\mathbb{R},\ 0<m<1. \qquad \square$$

The proof of the next theorem is similar to the proof of Theorem 3.1.

THEOREM 3.2. *Let $f\in\mathcal{S}'$ and $c(\varepsilon)$, $0<\varepsilon<\varepsilon'$ be a continuous positive function. If f is quasiasymptotically bounded at b_0, $b_0\in\mathbb{R}$ related to $c(\varepsilon)$, then there exist $p\in\mathbb{N}_0$ and $C>0$ such that*

$$|\mathcal{W}_g f(b_0,a)| \leqslant C\frac{(1+a^2)^{p/2}}{a^{p+1}},\ \ 0<a<1,$$

for every wavelet $g\in S_0$.

REMARK 3.1. An analogous result could be obtained for the boundedeness of the wavelet transform $\mathcal{W}_g f(b,a)$ in the case when distribution $f\in\mathcal{S}'$ is quasi-asymptotically bounded at infinity.

THEOREM 3.3. *Let $f\in\tilde{\mathcal{S}}'_r$, $r<0$ and $c(\varepsilon)$, $0<\varepsilon<\varepsilon'$ be a continuous positive function. If f is r-quasiasymptotically bounded at 0 with respect to $c(\varepsilon)$, then there exist $p\in\mathbb{N}_0$ and $C>0$ such that*

$$\big|\mathcal{W}_g f(b,a)\big| \leqslant C\frac{(1+a)^q}{a^{p+1}}\frac{1}{|b|^{q/2}},\ \ q=-r>0,\ b\in\mathbb{R},\ 0<a<1,$$

for every wavelet $g\in S_0$.

PROOF. As in the proof of Theorem 3.1 we have

$$\Big|\frac{\mathcal{W}_g f(\varepsilon b,\varepsilon a)}{c(\varepsilon)}\Big| = \Big|\Big\langle\frac{f(x)}{c(\varepsilon)},\frac{1}{\varepsilon a}\,\bar{g}\Big(\frac{x-\varepsilon b}{\varepsilon a}\Big)\Big\rangle\Big| = \Big|\Big\langle\frac{f(\varepsilon t)}{c(\varepsilon)},\frac{1}{a}\,\bar{g}\Big(\frac{t-b}{a}\Big)\Big\rangle\Big|.$$

Since f is r-quasiasymptotically bounded at 0 related to $c(\varepsilon)$, and $g \in S_0$ it follows that there exist $p \in \mathbb{N}_0$ and $M_1 > 0$ such that

$$\Big|\frac{\mathcal{W}_g f(\varepsilon b, \varepsilon a)}{c(\varepsilon)}\Big| \leqslant M_1 \Big\| \frac{1}{a}\,\bar{g}\Big(\frac{x-b}{a}\Big)\Big\|_{r,p} \tag{3.4}$$

for every $0 < \varepsilon < 1$. Since $0 < a < 1$ we have

$$\begin{aligned}
\Big\|\frac{1}{a}\,\bar{g}\Big(\frac{x-b}{a}\Big)\Big\|_{r,p} &= \sup_{x\in\mathbb{R},\,\alpha\leqslant p}(1+|x|^2)^{r/2}\Big|\Big(\frac{1}{a}\,\bar{g}\Big(\frac{x-b}{a}\Big)\Big)^{(\alpha)}\Big| \\
&= \sup_{x\in\mathbb{R},\,\alpha\leqslant p}(1+|x|^2)^{r/2}\frac{1}{a^{\alpha+1}}\Big|\,\bar{g}^{(\alpha)}\Big(\frac{x-b}{a}\Big)\Big| \\
&\leqslant \frac{1}{a^{p+1}}\sup_{x\in\mathbb{R},\,\alpha\leqslant p}(1+|x|^2)^{r/2}\Big|\,\bar{g}^{(\alpha)}\Big(\frac{x-b}{a}\Big)\Big|.
\end{aligned}$$

We put $r = -q$, $q > 0$ and get

$$\Big\|\frac{1}{a}\bar{g}\Big(\frac{x-b}{a}\Big)\Big\|_{r,p} \leqslant \frac{M_2}{a^{p+1}}\sup_{x\in\mathbb{R}}\frac{1}{(1+|x|^2)^{q/2}}\Big(1+\Big|\frac{x-b}{a}\Big|\Big)^{-q}, \tag{3.5}$$

where M_2 is a positive constant.

Since it holds

$$\begin{aligned}
(1+|b|)^q &\leqslant (1+|b-x|+|x|)^q \leqslant (1+|x-b|)^q\,(1+|x|)^q \\
&= \Big(1+a\,\Big|\frac{x-b}{a}\Big|\Big)^q(1+|x|)^q \leqslant \Big(1+\,\Big|\frac{x-b}{a}\Big|\Big)^q(1+a)^q(1+|x|)^q,
\end{aligned}$$

we have

$$\Big(1+\Big|\frac{x-b}{a}\Big|\Big)^{-q} \leqslant \frac{(1+a)^q(1+|x|)^q}{(1+|b|)^q}. \tag{3.6}$$

From the inequalities (3.4), (3.5) and (3.6) we obtain

$$\begin{aligned}
\Big|\frac{\mathcal{W}_g f(\varepsilon b, \varepsilon a)}{c(\varepsilon)}\Big| &\leqslant M_1M_2\frac{(1+a)^q}{a^{p+1}}\frac{1}{(1+|b|)^q}\sup_{x\in\mathbb{R}}\frac{((1+|x|)^2)^{q/2}}{(1+|x|^2)^{q/2}} \\
&= M_1M_2\frac{(1+a)^q}{a^{p+1}}\frac{1}{(1+|b|)^q}\sup_{x\in\mathbb{R}}\Big(1+\frac{2|x|}{1+|x|^2}\Big)^{q/2} \\
&\leqslant M_1M_2 2^{q/2}\frac{(1+a)^q}{a^{p+1}}\frac{1}{(1+|b|)^q}.
\end{aligned}$$

If we choose $0 < \varepsilon_0 < 1$ and put $\varepsilon_0 b = s$ and $\varepsilon_0 a = m$ $(0 < m < 1)$ in the above inequality, we get

$$\Big|\frac{\mathcal{W}_g f(s,m)}{c(\varepsilon_0)}\Big| \leqslant M_1M_2 2^{q/2}\frac{\varepsilon_0^{p+1}}{m^{p+1}}\frac{(\varepsilon_0+m)^q}{(\varepsilon_0+|s|)^q}.$$

Since $c(\varepsilon)$ is a positive function and from the following inequalities

$$(\varepsilon_0+m)^q \leqslant (1+\varepsilon_0)^q(1+m)^q, \quad (\varepsilon_0+|s|)^q = ((\varepsilon_0+|s|)^2)^{q/2} \geqslant (2\varepsilon_0|s|)^{q/2},$$

we have

$$\begin{aligned}|\mathcal{W}_g f(s,m)| &\leqslant M_1 M_2 2^{q/2} \frac{\varepsilon_0^{p+1}(1+\varepsilon_0)^q}{2^{q/2}\varepsilon_0^{q/2}} c(\varepsilon_0)\, \frac{(1+m)^q}{m^{p+1}|s|^{q/2}} \\ &= C\frac{(1+m)^q}{m^{p+1}}\frac{1}{|s|^{q/2}}, \qquad \square\end{aligned}$$

where $s \in \mathbb{R}$, $0 < m < 1$ and C is a positive constant.

The result for the boundedness of the wavelet transform $\mathcal{W}_g f(b_0, a)$ for fixed $b_0 \in \mathbb{R}$ when f is r-quasiasymptotically bounded distribution at the point b_0 is given in the following theorem.

THEOREM 3.4. *Let $f \in \tilde{\mathcal{S}}'_r$, $r < 0$ and $c(\varepsilon)$, $0 < \varepsilon < \varepsilon'$ be a continuous positive function. If f is r-quasiasymptotically bounded at b_0, $b_0 \in \mathbb{R}$ related to $c(\varepsilon)$, then there exist $p \in \mathbb{N}_0$ and $C > 0$ such that*

$$|\mathcal{W}_g f(b_0,a)| \leqslant C\frac{(1+a^2)^{q/2}}{a^{p+1}}, \quad q = -r > 0, \ 0 < a < 1,$$

for every wavelet $g \in S_0$.

REMARK 3.2. An analogous result could be obtained assuming that $f \in \tilde{\mathcal{S}}'_r$, $r < 0$ is r-quasiasymptotically bounded at infinity.

REMARK 3.3. It is also possible to obtain an analogous result for the localization of the wavelet transform $\mathcal{W}_g f$, assuming that the Fourier transform of distribution $f \in \mathcal{S}'$ or $f \in \tilde{\mathcal{S}}'_r$, $r < 0$ is quasiasymptotically or r-quasiasymptotically bounded, respectively.

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