PUBLICATIONS DE L'INSTITUT MATHÉMATIQUE
Nouvelle série, tome 100(114) (2016), 271–278 DOI: 10.2298/PIM1614271K

INFINITELY MANY WEAK SOLUTIONS FOR SOME ELLIPTIC PROBLEMS IN $\mathbb{R}^N$

Mehdi Khodabakhshi, Abdolmohammad Aminpour, and Mohamad Reza Heidari Tavani

Abstract. We investigate the existence of infinitely many weak solutions to some elliptic problems involving the p-Laplacian in $\mathbb{R}^N$ by using variational method and critical point theory.

1. Introduction

We are going to establish infinitely many weak solutions to the following elliptic problem:

$$\begin{aligned} &-\Delta_p u + |u|^{p-2}u = \lambda f(x,u), \quad \text{in } \mathbb{R}^N, \\ &u \in W^{1,p}(\mathbb{R}^N) \end{aligned} \tag{1.1}$$

where $N > 1$, $\Delta_p u := \operatorname{div}(|\nabla u|^{p-2}\nabla u)$ denotes the p-Laplace operator with $p > N$, and $f : \mathbb{R}^N \times \mathbb{R} \to \mathbb{R}$ be an L^1-Carathéodory function, λ is a positive real parameter. Problems on unbounded domains have been intensively studied. For instance, we mention the works [**1**–**3**, **9**–**12**]. In [**8**] Condito and Molica Bisci considered problem (1.1) by taking $f(x,u) := \alpha(x)f(u)$, where $f\colon \mathbb{R} \to \mathbb{R}$ is a continuous function, $\alpha \in L^1(\mathbb{R}^{\mathbb{N}}) \cap L^\infty(\mathbb{R}^{\mathbb{N}})$ is a nonnegative (not identically zero) map. It should be mentioned the different cases of problem (1.1) and even in general have been considered on the variety of bounded domains. We just refer in the large literature on the subject, the papers [**5**, **6**, **13**–**15**]. For instance, in [**14**] the authors have studied the following elliptic problem of Kirchhoff type:

$$\begin{aligned} &-\Big(a + b\int_\Omega \nabla u|^p dx\Big)\Delta_p u + \alpha(x)|u|^{p-2}u = \lambda h(x)f(x) \quad \text{in } \Omega, \\ &u|_{\partial\Omega} = 0, \end{aligned}$$

where a and b are two non-negative constant, $\alpha \in L^\infty(\Omega)$ with $\operatorname{ess\,inf}_{x\in\Omega}\alpha(x) \geqslant 0$, λ is a positive parameter, $f\colon \mathbb{R} \to \mathbb{R}$ is a continuous function and finally, $h \in L^\infty(\Omega)$

2010 *Mathematics Subject Classification:* 35J20.
Key words and phrases: p-Laplace operator; variational methods; critical point theory.
Communicated by Stevan Pilipović.

with ess inf $x \in \Omega h(x) > 0$. Recall that a function $f\colon \mathbb{R}^N \times \mathbb{R} \to \mathbb{R}$ is said to be an L^1-Carathéodory function, if (C_1) the function $x \mapsto f(x,t)$ is measurable for every $t \in \mathbb{R}$ (C_2) the function $t \mapsto f(x,t)$ is continuous for almost every $x \in \mathbb{R}^{\mathbb{N}}$ (C_3) for every $\varrho > 0$ there exists a function $l_\varrho(x) \in L^1(\mathbb{R}^{\mathbb{N}})$ such that

$$\sup_{|t| \leqslant \varrho} |f(x,t)| \leqslant l_\varrho(x),$$

for a.e. $x \in \mathbb{R}^{\mathbb{N}}$.

2. Preliminaries

Let $N > 1$, denote by X the space $W^{1,p}(\mathbb{R}^N)$ endowed with the norm

$$\|u\| := \left(\int_{\mathbb{R}^{\mathbb{N}}} |\nabla u(x)|^p \, dx + \int_{\mathbb{R}^{\mathbb{N}}} |u(x)|^p dx \right)^{1/p}.$$

Let $p > N$, we recall continuous embedding $W^{1,p}(\mathbb{R}^{\mathbb{N}}) \hookrightarrow L^\infty(\mathbb{R}^{\mathbb{N}})$, and one has

$$\|u\|_\infty \leqslant \frac{2p}{p-N} \|u\|,$$

for every $u \in W^{1,p}(\mathbb{R}^{\mathbb{N}})$ (see [**7**, Morrey's theorem]). Let us define $F(x,\xi) = \int_0^\xi f(x,t)\,dt$, for every $(x,\xi) \in \mathbb{R}^{\mathbb{N}} \times \mathbb{R}$, and introduce the functional $I_\lambda\colon X \to \mathbb{R}$ associated with (1.1),

$$I_\lambda(u) := \Phi(u) - \lambda \Psi(u), \qquad (\forall u \in X)$$

where

$$\Phi(u) = \frac{1}{p}\|u\|^p, \quad \Psi(u) = \int_{\mathbb{R}^{\mathbb{N}}} F(x, u(x))\, dx$$

for every $u \in X$. Now, in [**8**], it is shown that Φ is a Gâteaux differentiable and sequentially weakly lower semicontinuous and coercive in X whose Gâteaux derivative is given by

$$\Phi'(u)(v) = \int_{\mathbb{R}^{\mathbb{N}}} |\nabla u(x)|^{p-2} \nabla u(x).\nabla v(x)\, dx + \int_{\mathbb{R}^{\mathbb{N}}} |u(x)|^{p-2} u(x) v(x)\, dx,$$

In this section, we summarize the for every $v \in X$. On the other hand, standard arguments show that Ψ is a well defined, Gâteaux differentiable and sequentially weakly upper semicontinuous functional whose Gâteaux derivative is given by

$$\Psi'(u)(v) = \int_{\mathbb{R}^{\mathbb{N}}} f(x, u(x))\, dx,$$

for every $v \in X$. Fixing the real parameterλ, a function $u\colon \mathbb{R}^{\mathbb{N}} \to \mathbb{R}$ is said to be a weak solution of (1.1) if $u \in X$ and

$$\int_{\mathbb{R}^{\mathbb{N}}} |\nabla u(x)|^{p-2} \nabla u(x) \nabla v(x)\, dx + \int_{\mathbb{R}^{\mathbb{N}}} |u(x)|^{p-2} u(x) v(x)\, dx - \lambda \int_{\mathbb{R}^{\mathbb{N}}} f(x, u(x)) v(x)\, dx = 0,$$

for every $v \in X$. Hence, the critical points of I_λ are exactly the weak solutions of (1.1). Our main tool to investigate the existence of infinitely many weak solutions to the problem (1.1) is the following critical points theorem.

THEOREM 2.1. [**4**, Theorem 2.1] *Let X be a reflexive real Banach space, and let $\Phi, \Psi \colon X \to \mathbb{R}$ be two Gâteaux differentiable functionals such that Φ is strongly continuous, sequentially weakly lower semicontinuous, coercive and Ψ is sequentially weakly upper semicontinuous. For every $r > \inf_X \Phi$, put*

$$\varphi(r) := \inf_{\varphi(v)<r} \frac{(\sup_{\Phi(u)<r} \Psi(u)) - \Psi(v)}{r - \Phi(v)}$$

and

$$\gamma := \liminf_{r\to+\infty} \varphi(r), \quad \delta := \liminf_{r\to(\inf_X \Phi)^+} \varphi(r).$$

then, one has

(a) *For every $r > \inf_X \Phi$ and $\lambda \in \left]0, 1/\Phi(r)\right[$, the restriction of the functional $I_\lambda = \Phi - \lambda\Psi$ to $\Phi^{-1}\left]-\infty, r\right[$ admits a global minimum, which is a critical point (local minimum) of I_λ in X.*

(b) *If $\gamma < +\infty$, then for each $\lambda \in \left]0, \frac{1}{\gamma}\right[$, the following alternative holds: either*
  - (b_1) *I_λ possesses a global minimum, or*
  - (b_2) *there is a sequence $\{u_n\}$ of critical points (local minima) of I_λ such that $\lim_{n\to\infty} \Phi(u_n) = +\infty$.*

(c) *If $\delta < +\infty$, then for each $\lambda \in \left]0, \frac{1}{\delta}\right[$, the following alternative holds: either*
  - (c_1) *there is a global minimum of Φ which is a local minimum of I_λ, or*
  - (c_2) *there is a sequence $\{u_n\}$ of pairwise distinct critical points (local minima) of I_λ which weakly converges to a global minimum of Φ, with $\lim_{n\to\infty} \Phi(u_n) = \inf_X \Phi$.*

3. Main results

Fix $D > 0$ such that

$$\kappa := \frac{1}{m(D)\left(\frac{2p}{p-N}\right)^p\left(\frac{\sigma(N,p)}{D^p} + g(p,N)\right)}, \tag{3.1}$$

where $\sigma(N,p) := 2^{p-N}(2^N - 1)$, as well as

$$g(p,N) := \frac{1 + 2^{N+p}N\int_{\frac{1}{2}}^{1} t^{N-1}(1-t)^p dt}{2^N}.$$

We also note that $m(D) := D^N \frac{\pi^{\frac{N}{2}}}{\Gamma(1+\frac{N}{2})}$, where Γ is the Gamma function defined by

$$\Gamma(t) := \int_0^{+\infty} z^{t-1}e^{-z}dz, \quad (\forall t > 0).$$

THEOREM 3.1. *Let $f \colon \mathbb{R}^N \times \mathbb{R} \to \mathbb{R}$ be an L^1-Carathéodory function, and assume that*

i) *$F(x,t) \geqslant 0$ for every $(x,t) \in \mathbb{R}^{\mathbb{N}} \times \mathbb{R}^+$;*

ii) *$A := \liminf_{\varrho\to+\infty} \frac{\|l_\varrho\|_1}{\varrho^{p-1}} < \kappa B$,* *where* $B := \limsup_{\varrho\to+\infty} \frac{1}{\varrho^p}\int_{B(0,\frac{D}{2})} F(x,\varrho)\,dx$ *and* $l_\varrho \in L^1(\mathbb{R}^{\mathbb{N}})$ *that satisfies* (c_3) *condition on* $f(x,t)$ *for every* $\varrho > 0$ *and* κ *is given by* (3.1). *Then for every*

$$\lambda \in \left]\frac{m(D)}{B}\Big(\frac{\sigma(N,p)}{pD^p} + \frac{g(p,N)}{p}\Big), \frac{1}{p\big(\frac{2p}{p-N}\big)^p A}\right[$$

the problem (1.1) *admits a sequence of many weak solutions which is unbounded in* X.

Proof. Fix λ as in our conclusion. Our aim is to apply Theorem 2.1, part (b) with $X = W^{1,p}(\mathbb{R}^N)$, and Φ, Ψ are the functionals introduced in Section 2. As seen before, the functionals Φ and Ψ satisfy all regularity assumptions requested in Theorem 2.1. Now, we look on the existence of critical points of the functional I_λ in X. To this end, we take $\{\varrho_n\}$ to be a real sequence such that $\lim_{n\to\infty}\varrho_n = +\infty$ and

$$\lim_{n\to\infty}\frac{\|l_{\varrho_n}\|_1}{\varrho_n^{p-1}} = A.$$

Set $r_n := \varrho_n^p/p\big(\frac{2p}{p-N}\big)^p$, for every $n\in\mathbb{N}$. By relation (1.1), one has $\|v\|_\infty \leqslant \varrho_n$ for all $v\in X$ such that $\|v\|^p < pr_n$, we have the following inequalities:

$$\begin{aligned}\varphi(r_n) &= \inf_{\|u\|^p<pr_n}\frac{\sup_{\|v\|^p<pr_n}\int_{\mathbb{R}^{\mathbb{N}}}F(x,v(x))\,dx - \int_{\mathbb{R}^{\mathbb{N}}}F(x,u(x))\,dx}{r_n - \frac{\|u\|^p}{p}}\\ &\leqslant \frac{\sup_{\|v\|^p<pr_n}\int_{\mathbb{R}^{\mathbb{N}}}F(x,v(x))\,dx}{r_n} \leqslant p\Big(\frac{2p}{p-N}\Big)^p\frac{\|l_{\varrho_n}\|_1}{\varrho_n^{p-1}},\end{aligned}$$

for every $n\in\mathbb{N}$. Hence, it follows that

$$\gamma \leqslant \liminf_{n\to\infty}\Phi(r_n) \leqslant p\Big(\frac{2p}{p-N}\Big)^p A < +\infty,$$

since condition ii) shows $A < +\infty$. Now, we prove that the functional I_λ is unbounded from below. For our goal, let $\{d_n\}$ be a real sequence such that $\lim_{n\to\infty} d_n = +\infty$ and

$$\lim_{n\to+\infty}\frac{1}{d_n^p}\int_{B(0,\frac{D}{2})}F(x,d_n)\,dx = B. \tag{3.2}$$

Let $\{v_n\}$ be a sequence in X which is defined by

$$v_n(x) := \begin{cases} 0 & x\in\mathbb{R}^{\mathbb{N}}\setminus B(0,D)\\ d_n & x\in B(0,\frac{D}{2})\\ \frac{2d_n}{D}(D-|x|) & x\in B(0,D)\setminus B(0,\frac{D}{2}),\end{cases}$$

One can compute that

$$\|v_n\|^p = d_n^p m(D)\Big(\frac{\sigma(N,p)}{D^P} + g(p,N)\Big),$$

see for instance, the paper [**8**]. At this point, by using condition i), we infer

$$\int_{\mathbb{R}^{\mathbb{N}}}F(x,v_n(x))\,dx \geqslant \int_{B(0,\frac{D}{2})}F(x,d_n)\,dx,$$

for every $n \in \mathbb{N}$. Then, we have

$$I_\lambda(v_n) \leqslant \frac{d_n^p m(D)}{P}\Big(\frac{\sigma(N,p)}{D^p} + g(p,N)\Big) - \lambda \int_{B(0,\frac{D}{2})} F(x,d_n)\,dx,$$

for every $n \in \mathbb{N}$. If $B < +\infty$, let

$$\epsilon \in \Big]\frac{m(D)}{\lambda B}\Big(\frac{\sigma(N,p)}{pD^P} + \frac{g(p,N)}{p}\Big), 1\Big[$$

By (3.2) there exists N_ϵ such that

$$\int_{B(0,\frac{D}{2})} F(x,d_n)\,dx > \epsilon B d_n^p, \quad (\forall n > N_\epsilon).$$

Consequently, one has

$$\begin{aligned} I_\lambda(v_n) &\leqslant \frac{d_n^p m(D)}{p}\Big(\frac{\sigma(N,p)}{D^p} + g(p,N)\Big) - \lambda\epsilon B d_n^p, \\ &= d_n^p\Big(m(D)\Big(\frac{\sigma(N,p)}{pD^p} + \frac{g(p,N)}{p}\Big) - \lambda\epsilon B\Big), \end{aligned}$$

for every $n > N_\epsilon$. Thus, it follows that

$$\lim_{n\to\infty} I_\lambda(v_n) = -\infty.$$

If $B = +\infty$, let us consider

$$M > \frac{m(D)}{\lambda p}\Big(\frac{\sigma(N,p)}{D^p} + g(p,N)\Big).$$

By (3.2) there exists $N(M)$ such that

$$\int_{B(0,\frac{D}{2})} F(x,d_n)\,dx > M d_n^p, \qquad (\forall n > N(M)).$$

Hence, we have

$$\begin{aligned} I_\lambda(v_n) &\leqslant \frac{d_n^p m(D)}{p}\Big(\frac{\sigma(N,p)}{D^p} - g(p,N)\Big) - \lambda M d_n^p \\ &= d_n^p\Big(m(D)\Big(\frac{\sigma(N,p)}{pD^p} + \frac{g(p,N)}{p}\Big) - \lambda M\Big), \end{aligned}$$

for every $n > N(M)$. Taking into account the choice of M, also in this case, one has

$$\lim_{n\to\infty} I_\lambda(v_n) = -\infty.$$

Thanks to Theorem 2.1, the functional I_λ admits an unbounded sequence $\{u_n\} \subset X$ of critical points. In conclusion, problem (1.1) admits a sequence of many weak solutions which is unbounded in $W^{1,p}(\mathbb{R}^{\mathbb{N}})$. □

The following result is a consequence of Theorem 3.1.

Corollary 3.1. *Let $f : \mathbb{R}^{\mathbb{N}} \times \mathbb{R} \to \mathbb{R}$ be an L^1-Carathéodory function. Assume that condition i) of Theorem* 3.1 *holds. Further, require that*

ii$_1$) $\limsup_{\varrho\to+\infty} \frac{1}{\varrho^p}\int_{B(0,\frac{D}{2})} F(x,\varrho)\,dx > \frac{m(D)}{p}\big(\frac{\sigma(N,p)}{D^p} + g(p,N)\big)$*;*

ii$_2$) $\liminf_{\varrho\to+\infty} \frac{\|l_\varrho\|_1}{\varrho^{p-1}} < p\left(\frac{2p}{p-N}\right)^{-p}$.

Then, the problem

$$-\Delta_p u + |u|^{p-2}u = f(x,u), \quad \text{in } \mathbb{R}^N,$$
$$u \in W^{1,p}(\mathbb{R}^N)$$

possesses a sequence of many weak solutions which is unbounded in $W^{1,p}(\mathbb{R}^{\mathbb{N}})$.

Finally, We point out the following result.

Corollary 3.2. *Let* $f\colon \mathbb{R}^{\mathbb{N}}\times\mathbb{R}\to\mathbb{R}$ *be an* L^1*-Carathéodory function. Assume that condition i) of Theorem* 3.1 *holds. Further, require that*

$$\limsup_{\varrho\to+\infty} \frac{1}{\varrho^p}\int_{B(0,\frac{D}{2})} F(x,\varrho)\,dx = \infty, \quad \liminf_{\varrho\to+\infty} \frac{\|l_\varrho\|_1}{\varrho^{p-1}} = 0,$$

Then, for every $\lambda > 0$*, the problem*

$$-\Delta_p u + |u|^{p-2}u = \lambda f(x,u), \quad \text{in } \mathbb{R}^N,$$
$$u \in W^{1,p}(\mathbb{R}^N)$$

possesses a sequence of many weak solutions which is unbounded in $W^{1,p}(\mathbb{R}^{\mathbb{N}})$.

Remark 3.1. We note that assumption ii) in Theorem 3.1 could be replaced by the following more general hypotheses:

ii$'$) There exists two sequences $\{a_n\}$ and $\{b_n\}$ such that

$$0 \leqslant a_n < \frac{(p-N)b_n}{2p\Big(\frac{\sigma(N,p)}{D^p} + g(p,N)\Big)^{\frac{1}{p}}(m(D))^{\frac{1}{p}}}$$

for every $n\in\mathbb{N}$ and $\lim_{n\to\infty} b_n = +\infty$ such that

$$\hat{A} < \kappa \limsup_{\varrho\to+\infty} \frac{1}{\varrho^p}\int_{B(0,\frac{D}{2})} F(x,\varrho)\,dx,$$

where κ is given by (3.1) and

$$\hat{A} := \lim_{n\to\infty} \frac{\big(b_n\|l_{b_n}\|_1 - \int_{B(0,\frac{D}{2})} F(x,a_n)\,dx\big)p\big(\frac{2p}{p-N}\big)^p}{b_n^p - m(D)a_n^p\big(\frac{2p}{p-N}\big)^p\big(\frac{\sigma(N,p)}{D^p} + g(p,N)\big)}.$$

For instance, in this setting by choosing

$$\lambda \in \left]\frac{m(D)}{B}\Big(\frac{\sigma(N,p)}{pD^p} + \frac{g(p,N)}{p}\Big), \frac{1}{p\Big(\frac{2p}{p-N}\Big)^p\hat{A}}\right[,$$

problem (1.1) admits a sequence of many weak solutions which is unbounded in $W^{1,p}(\mathbb{R}^{\mathbb{N}})$. Indeed, if

$$\lambda \in \left]\frac{m(D)}{B}\Big(\frac{\sigma(N,p)}{pD^p} + \frac{g(p,N)}{p}\Big), \frac{1}{p\Big(\frac{2p}{p-N}\Big)^p\hat{A}}\right[,$$

and $r_n := \frac{b_n^p}{p(\frac{2p}{p-N})^p}$ for every $n \in \mathbb{N}$, then one has

$$\begin{aligned}\varphi(r_n) &= \inf_{\|u\|^p < pr_n} \frac{\sup_{\|v\|^p < pr_n} \int_{\mathbb{R}^\mathbb{N}} F(x, v(x))\, dx - \int_{\mathbb{R}^\mathbb{N}} F(x, u(x))\, dx}{r_n - \frac{\|u\|^p}{p}} \\ &\leqslant \frac{\sup_{\|v\|^p < pr_n} \int_{\mathbb{R}^\mathbb{N}} F(x, v(x))\, dx - \int_{\mathbb{R}^\mathbb{N}} F(x, v_n(x))\, dx}{r_n - \frac{\|v_n\|^p}{p}} \\ &\leqslant \frac{\big(b_n \|l_{b_n}\|_1 - \int_{B(0,\frac{D}{2})} F(x, a_n)\, dx\big) p\big(\frac{2p}{p-N}\big)^p}{b_n^p - \big(\frac{2p}{p-N}\big)^p a_n^p m(D)\big(\frac{\sigma(N,p)}{D^p} + g(p,N)\big)},\end{aligned}$$

by choosing

$$v_n(x) := \begin{cases} 0 & x \in \mathbb{R}^\mathbb{N} \setminus B(0, D) \\ a_n & x \in B\big(0, \frac{D}{2}\big) \\ \frac{2a_n}{D}(D - |x|) & x \in B(0, D) \setminus B\big(0, \frac{D}{2}\big), \end{cases}$$

for each $n \in \mathbb{N}$. Hence, by assumption ii′) one has $\hat{A} < +\infty$, we obtain

$$\gamma \leqslant \liminf_{n\to\infty} \varphi(r_n) \leqslant p\Big(\frac{2p}{p-N}\Big)^p \hat{A} < +\infty.$$

From now on, arguing as in the proof of Theorem 3.1, the conclusion follows.

Acknowledgement. The authors would like to thank the unknown referee for her/his valuable comments and suggestions.

References

1. T. Bartsch, M. Willem, *Infinitely many non-radial solutions of an Euclidean scalar field equation*, J. Funct. Anal. **117** (1993), 447–460.
2. H. Berestycki, P.L. Lions, *Nonlinear scalar field equations*, Arch. Ration. Mech. Anal. **82** (1983), 313–376.
3. G. Bonanno, G. Molica Bisci, *Infinitely many solutions for a boundary value problem with discontinuous nonlinearities*, Bound. Value Probl. **2009** (2009), 1–20.
4. ______, *Infinitely many solutions for a Dirichlet problem involving the p-Laplacian*, Proc. R. Soc. Edinb., Sect. A, Math. **140** (2009), 737–752.
5. G. Bonanno, G. Molica Bisci, V. Radulescu, *Existence results for gradient-type systems on the Sierpiński gasket*, Chin. Ann. Math., Ser. B **34**(2) (2013), 941–953.
6. ______, *Variational analysis for a nonlinear elliptic problem on the Sierpinski gasket*, ESAIM, Control Optim. Calc. Var. **18** (2012), 941–953.
7. H. Brezis, *Functional Analysis, Sobolev Spaces and Partial Differential Equations*, Springer, 2010.
8. P. Candito, G. Molica Bisci, *Radiality symetric weak solutions for elliptic problems in* $\mathbb{R}^N$, Differ. Integral Equ. **26** (2013), 1009–1026.
9. G. Dai, *Infinitely many solutions for a* $p(x)$*-Laplacian equation in* $\mathbb{R}^\mathbb{N}$, Nonlinear Anal., Theory Methods Appl., Ser. A, Theory Methods **71** (2009), 1133–1139.
10. F. Gazzola, V. Rădulescu, *A nonsmooth critical point theory approuch to some nonlinear elliptic equation in* $\mathbb{R}^\mathbb{N}$, Differ. Integral Equ. **13** (2000), 47–60.
11. A. Kristály, *Infinitely many solutions for a differential inclusion problem in* $\mathbb{R}^\mathbb{N}$, J. Differ. Equations **220** (2006), 511–530.
12. G. Li, S. Yan, *Eigenvalue problems for quasilinear elliptic equations on* $\mathbb{R}^\mathbb{N}$, Commun. Partial Differ. Equations **14** (1989), 1291–1314.

13. G. Molica Bisci, *Variational problems on the sphere*, Contemp. Math. **595** (2013), 273–291.
14. G. Molica Bisci, P. Pizzimenti, *Sequences of weak solutions for non-local elliptic problems with Dirichlet boundary condition*, Proc. Edinb. Math. Soc. **57** (2014), 779–809.
15. G. Molica Bisci, D. Repovs, *On sequences of solutions for discrete anisotropic equations*, Expo. Math. **32** (2014), 284–295.

(Received 04 02 2015)
(Revised 19 06 2016)

Department of Mathematics
Faculty of Mathematical and Computer Sciences
Shahid Chamran University of Ahwaz
Ahwaz
Iran
m.khodabakhshi11@gmail.com
aminpour@scu.ac.ir

Department of Mathematics
Ramhormoz Branch
Islamic Azad University
Ramhormoz
Iran
m.reza.h56@gmail.com