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CONSIDERATIONS ON SOME ALGEBRAIC PROPERTIES OF FEYNMAN INTEGRALS

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Abstract. Some algebraic properties of integrals over configuration spaces are investigated in order to better understand quantization and the Connes-Kreimer algebraic approach to renormalization.

In order to isolate the mathematical-physics interface to quantum field theory independent from the specifics of the various implementations, the sigma model of Kontsevich is investigated in more detail. Due to the convergence of the configuration space integrals, the model allows to study the Feynman rules independently, from an axiomatic point of view, avoiding the intricacies of renormalization, unavoidable within the traditional quantum field theory.

As an application, a combinatorial approach to constructing the coefficients of formality morphisms is suggested, as an alternative to the analytical approach used by Kontsevich. These coefficients are "Feynman integrals", although not quite typical since they do converge.

A second example of "Feynman integrals", defined as state-sum model, is investigated. Integration is understood here as formal categorical integration, or better as a duality structure on the corresponding category. The connection with a related TQFT is mentioned, supplementing the Feynman path integral interpretation of Kontsevich formula.

A categorical formulation for the Feynman path integral quantization is sketched, towards Feynman Processes, i.e. representations of dg-categories with duality, thought of as complexified Markov processes.

Full text

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