

## A SURVEY ON DILATIONS OF PROJECTIVE ISOMETRIC REPRESENTATIONS

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**Abstract.** In this paper we present Laca-Raeburn's dilation theory of projective isometric representations of a semigroup to projective isometric representations of a group [4] and Murphy's proof of a dilation theorem more general than that proved by Laca and Raeburn. Murphy applied the theory which involves positive definite kernels and their Kolmogorov decompositions to obtain the Laca-Raeburn dilation theorem [6].

We also present Heo's dilation theorems for projective representations, which generalize Stinespring dilation theorem for covariant completely positive maps and generalize to Hilbert  $C^*$ -modules the Naimark-Sz-Nagy characterization of positive definite functions on groups [2].

In the last part of the paper it is given the dilation theory obtained in [6] in the case of unitary operator-valued multipliers [3].

[Full text](#)

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