

STABILITY FOR PERIODIC EVOLUTION FAMILIES OF BOUNDED LINEAR OPERATORS

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Abstract The long time behavior for solutions of evolution periodic equations are reviewed.

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2010 Mathematics Subject Classification: 42A16; 45A05; 47A10; 47A35; 47D06; 47G10; 93D20.

Keywords: Periodic evolution families; uniform exponential stability; boundedness; evolution semigroup; almost periodic functions.

This work is based on the author's PhD thesis.

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Surveys in Mathematics and its Applications **10** (2015), 61 – 93

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Surveys in Mathematics and its Applications **10** (2015), 61 – 93
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