

APPROXIMATE SOLUTIONS TO SOME NON-AUTONOMOUS DIFFERENTIAL EQUATIONS FOR GROWTH PHENOMENA

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Abstract. Growth modeling is widely used in various fields of applied sciences. The purpose of this paper is to develop analytic approximate solutions to some non-autonomous differential equations used in population growth. We demonstrate that when the carrying capacity varies with time, an approximate solution to the generalized Turner model and any particular case of this model can be produced without expensive calculations.

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