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MAGNETIC AND SLANT CURVES IN KENMOTSU MANIFOLDS

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Abstract. Motivated by the recent studies of the magnetic curves in quasi-Sasakian, Sasakian, and Cosymplectic manifolds, in this article we investigate the magnetic trajectories with respect to contact magnetic fields in Kenmotsu manifolds. Moreover, we study the slant curves, torsion and curvature in Kenmotsu manifolds.

1 Introduction

The notion of magnetic curves in arbitrary Riemannian manifolds were introduced and studied by several authors [1, 3, 4, 6, 21]. Suppose (M, g) be a Riemannian manifold, a closed 2-form F on M is called the magnetic field. The endomorphism field ϕ corresponding to F metrically stated as the Lorentz force of F. The Newton's equation also known the Lorentz equation and is defined as $\nabla_{\beta'}\beta' = q\varphi\beta'$, where q is a real constant and ∇ is the Levi-Civita connection. A curve which satisfies the Lorentz equation is said to be a magnetic trajectory [10].

In [1], Adachi investigated the trajectories of charged particles and magnetic field corresponding to the Kahler form on a complex projective space. Adachi also studied the similarities between trajectories and geodesics on Kahler manifolds of negative curvature in [2]. Barros et al. studied the magnetic flow associated with a Killing magnetic field in 3-dimensional space [3, 4]. In [6], Cabrerizo studied the Landau–Hall problem in the two dimensional and three dimensional unit spheres and shown that the magnetic flowlines are helices with the Killing vector fields in $\mathbb{S}^3$. In [7], contact magnetic field on 3-dimensional Riemannian manifolds has been studied and it was established that metric g is adapted to the almost contact structure, with an application to magnetic fields.

Calin et al. [8] studied slant curves with proper mean curvature vector field in three-dimensional f-Kenmotsu manifolds and hyperbolic space $\mathbb{H}^3$ related with natural homogeneous normal almost contact metric structure. Furthermore, in [17], Inoguchi and Lee studied the slant curves in a 3-dimensional almost f-Kenmotsu

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manifold and proved that an almost f-Kenmotsu manifold is f-Kenmotsu manifold if and only if it is normal. Inoguchi also obtained the necessary and sufficient condition for a non-geodesic slant curve in 3-dimensional almost f-Kenmotsu manifold to have proper mean curvature vector field.

In [9], Cho et al. studied Lancret type problems for slant curves in Sasakian 3-manifolds and shown that a non geodesic curve is a slant curve iff its ratio of curvature (κ) and torsion (τ) is constant. In [10], Druta-Romaniuc et al. studied magnetic curves corresponding to the Killing magnetic fields in E^3.

In [11, 12], Druta-Romaniuc et al. studied the magnetic curves corresponding to the contact magnetic field on Sasakian and Cosymplectic manifolds. In [18], Inoguchi et al. studied the magnetic trajectories of the contact magnetic fields in 3-dimensional quasi-Sasakian manifolds and defined a family of linear connections with respect to the Okumura type connections. Moreover, in [13], Guvenc investigated the slant magnetic curves in S-manifolds and constructed the slant normal magnetic curves in $\mathbb{R}^{2n+s}(-3s)$.

Ikawa [14], studied the motion of charged particles in Sasakian manifolds and defined a Sasaki-Kahler submersion. Moreover, in [15, 16], Ikawa investigated the motion of charged particles from the geometric view point as well as in two-step nilpotent Lie groups. By using dynamical systems, Kalinin [20] investigated the trajectories of the charge particles of magnetic fields on Kahler manifolds of constant holomorphic sectional curvature.

In [21], Munteanu and Nistor studied the magnetic trajectories of charge particles under the action of Killing magnetic fields in $\mathbb{S}^2\times\mathbb{R}$. In [22], Ozdemir et al. introduced a new kind of magnetic curve namely T-magnetic curves, N-magnetic curves and B-magnetic curves in a three dimensional semi-Riemannian manifolds. Moreover, Ozdemir et al. also obtained some examples of these curves.

2 Preliminaries

In order to fix our notations as well as to make this paper self contained, in this section, we recall some fundamentals of Kenmotsu manifolds and properties of the Frenet curves and magnetic curves.

2.1 Kenmotsu manifolds

A $(2m+1)$-dimensional manifold M is said to admit an almost contact structure if there exist a $(1,1)$-tensor field φ, a vector field ξ, a 1-form η satisfying

$$\begin{cases}\varphi\xi = 0,\ \varphi^2 X = -X + \eta(X)\,\xi \\ \eta(\xi) = 1,\ \eta(\varphi X) = 0\end{cases} \tag{2.1}$$

An almost contact structure together with $(2m+1)$-dimensional manifold is called an almost contact manifold. If Riemannian metric g satisfies

$$g(\varphi X, \varphi Y) = g(X, Y) - \eta(X)\eta(Y) \tag{2.2}$$

for all X and Y in $\mathcal{X}(M^{2m+1})$, then $(M, \varphi, \xi, \eta, g)$ is called an almost contact metric manifold. With respect to g, η is the dual form of ξ,

i.e. $g(X, \xi) = \eta(X)$ for any $X \in \mathcal{X}(M^{2m+1})$.

Moreover, if

$$(\nabla_X \varphi) Y = -g(X, \varphi Y)\xi - \eta(Y)\varphi X \tag{2.3}$$

and

$$\nabla_X \xi = X - \eta(X)\xi \tag{2.4}$$

where ∇ is a Levi-Civita Connection on M, then the structure $(M, \varphi, \xi, \eta, g)$ is said to be a Kenmotsu manifold [5, 19].

The fundamental 2-form, Ω of the almost contact metric manifold is given by

$$\Omega(X, Y) = g(\varphi X, Y) \tag{2.5}$$

for all X and Y in $\mathcal{X}(M^{2m+1})$.

An almost contact metric manifold M is called a contact metric manifold if $\Omega = d\eta$. The exterior derivative $d\eta$ is defined by

$$d\eta(X, Y) = \frac{1}{2}\left(X\eta(Y) - Y\eta(X) - \eta([X, Y])\right) \tag{2.6}$$

for all X and Y in $\mathcal{X}(M^{2m+1})$ [11].

On a contact metric manifold, η is contact form that is $\eta \wedge (d\eta)^n \neq 0$ on M.

2.2 Frenet curves

Let (M^3, g) be a Riemannian 3-manifold and ∇ be a Levi-Civita connection defined on it. Suppose $\beta : I \to M^3$ be a Frenet curve parametrized by the arc length with Frenet frame field (T, N, B), where T, N, B respectively denote the tangent, principal normal, and binormal vector fields . These three vector fields T, N, B are mutually orthogonal at each point on β. We have Frenet-Serret equations:

$$\begin{cases} \nabla_T T = \kappa N \\ \nabla_T N = -\kappa T + \tau B \\ \nabla_T B = -\tau N \end{cases} \tag{2.7}$$

where κ and τ respectively denote the curvature and torsion of β.

A unit speed curve β is said to be a Frenet curve of osculating order r (where $r \geq 1$), if there exist an orthonormal set of vector fields $\{\beta' = T, E_1, E_2, ..., E_{r-1}\}$ along β such that

$$\begin{cases} \nabla_T T = \kappa_1 E_1 \\ \nabla_T E_1 = -\kappa_1 T \ + \kappa_2 E_2 \\ \nabla_T E_j = -\kappa_j E_{j-1} + \kappa_{j+1} E_{j+1} \text{ for } j = 2, 3, \ldots, r-2 \\ \nabla_T E_{r-1} = -\kappa_{r-1} E_{r-2} \end{cases} \tag{2.8}$$

where $\kappa_1, \kappa_2, \ldots, \kappa_{r-1}$ are positive C^∞ functions of the arc length parameter (s). Furthermore, κ_j is called the j-th curvature of β [5].

A Frenet curve is said to be geodesic in (M, g) if its osculating order is 1. A circle is a Frenet curve if its osculating order is two and the curvature κ_1 is constant. A helix of order r if all the curvatures $\kappa_1, \kappa_2, \ldots, \kappa_{r-1}$ are constants.

Suppose β be a Frenet curve of osculating order r on M^{2m+1}, where $(M^{2m+1}, \varphi, \xi, \eta, g)$ denotes the almost contact metric manifold. A Frenet curve of osculating order two is said to be φ-curve if $\{T, E_1, \xi\}$ spans a φ-invariant space. A curve of osculating order $r \geq 3$ is said to be φ-curve if $\{T, E_1, E_2, \ldots, E_{r-1}\}$ is φ-invariant. Moreover, a φ-helix of order r is said to be a φ-curve of osculating order r if $\kappa_1, \kappa_2, \ldots, \kappa_{r-1}$ are constants [11, 12].

The angle between the tangent to β and the reeb vector field ξ is known as the contact angle θ of β.

i.e. $$cos\,\theta(s) = g(\beta'(s), \xi)$$

where s denotes the arc-length (parameter) of β. We call $\beta(s)$ a slant curve if the contact angle θ is constant. Legendre curves are the curves of contact angle $\frac{\pi}{2}$ and a curve of contact angle 0 is called a Reeb flow.

2.3 Magnetic Curves

The trajectories of charged particles moving on a Riemannian manifold (M, g) under the action of a magnetic field F is known as Magnetic curves. In 3-dimensional oriented Riemannian manifold (M^3, g), a divergence free vector field defined as a magnetic field. A closed 2-form F on M is called the magnetic field. The Lorentz force of a magnetic field F on (M, g) is an $(1,1)$-tensor field ϕ is defined by

$$g\,(\phi X, Y) = F(X, Y) \tag{2.9}$$

for each X and Y in $\mathcal{X}(M)$.

A regular curve β will be a magnetic curve with F, if it satisfies the Lorentz equation (also known as Newton's equation)

$$\nabla_{\beta'} \beta' = \phi(\beta') \tag{2.10}$$

where ∇ is the Levi-Civita connection of g. When Lorentz force vanishes, we have

$$\nabla_{\beta'}\beta^{'} = 0$$

If $\nabla F = 0$, then a magnetic field is known as uniform. The magnetic trajectories are of constant speed. The magnetic curve is known as normal magnetic curve, if it is parametrized by the arc length (s) [11].

3 Magnetic curves in Kenmotsu manifolds

Let M^{2m+1} be a contact metric manifold and Ω be the fundamental 2-form defined by (2.5). Since $\Omega = d\eta$, then magnetic field on M^{2m+1} can be define by

$$q\Omega\,(X,Y) = F_q(X,Y), \tag{3.1}$$

where q is a real constant and $X,\ Y \in \mathcal{X}(M^{2m+1})$. F_q is known as the contact magnetic field with the strength q. The contact magnetic field vanishes and magnetic curves are the geodesics on M^{2m+1} if $q = 0$. Now, we assume $q \neq 0$.

By combining equations (2.5) and (2.9), the equation of Lorentz force ϕ_q related to the contact magnetic field F_q is given by

$$\phi_q = q\varphi \tag{3.2}$$

Now, the Lorentz equation (2.10) is given by

$$\nabla_{\beta'}\beta^{'} = q\varphi\beta^{'} \tag{3.3}$$

where β is a Frenet curve parametrized by arc length (s) and the solution of the above equation is known as the normal magnetic curve.

A classification of the normal magnetic curves related with contact magnetic field F_q on Kenmotsu manifold is specified in the following result.

Theorem 1. *Let $(M^{2m+1},\varphi,\xi,\eta,g)$ be a Kenmotsu manifold and F_q be the contact magnetic field for $q \neq 0$. Then β is a normal magnetic curve corresponding to F_q, if β belongs to the following cases:*

1. *geodesics obtained as integral curve of ξ.*
2. *For non-geodesic φ-circle of curvature $\kappa_1 = \sqrt{q^2 - sin^2\theta}$ for $|q| > 1$ and having the constant angle $\theta = arc\ cot\frac{1}{|q|}$.*
3. *Legendre φ-curves in M^{2m+1} with curvatures $\kappa_1 = |q|$ and $\kappa_2 = 1$, for $\theta = \frac{\pi}{2}$.*
4. *φ-helices of order 3 with axis ξ having curvatures $\kappa_1 = |q|$ and $\kappa_2 = |sgn\,(q)\ \ sin\,\theta +\ q\ cos\,\theta|$, for $\theta \neq \frac{\pi}{2}$*

Proof. When the magnetic curve β is a geodesic, by the Lorentz equation we have $\varphi\beta^{'} = 0$, then $\beta^{'}$ is parallel to ξ. As $\beta^{'}$ and ξ are both unit vector fields, then $\beta^{'} = \pm\xi$, which implies that β is an integral curve of ξ.

As a consequence, let β is a non-geodesic magnetic curve of osculating order $r > 1$. We have

$$g(q\varphi T, \xi) = g(\nabla_T T, \xi) = 0$$
$$= \frac{d}{ds} g\,(T, \xi) - g(T, \nabla_T \xi)$$

By using (2.4), we get that $\frac{d}{ds} g\,(T, \xi)$ vanishes.

As a result, $0 \le \theta \le \pi$ is a constant angle between T and ξ, we have

$$\eta\,(T) = \cos\theta \tag{3.4}$$

By clubbing the Lorentz equation and first Frenet formula, we have

$$\kappa_1 E_1 = q\varphi T \tag{3.5}$$

and the first curvature is given by

$$\kappa_1 = |q| \sqrt{1 - cos^2\theta} \tag{3.6}$$

from (3.5) and (3.6), we have

$$\varphi T = sgn\,(q) \sqrt{1 - cos^2\theta}\; E_1 \tag{3.7}$$

where $sgn\,(q)$ denotes the real number.

If second curvature vanishes i.e. $\kappa_2 = 0$, then β is a Frenet curve of osculating order two and since κ_1 is a constant, β becomes a circle.

From equation (3.7), we get

$$\eta\,(\varphi T) = 0 = sgn\,(q) \sqrt{1 - cos^2\theta}\; \eta\,(E_1)$$

that is,

$$\eta\,(E_1) = 0$$

Taking covariant derivative of the above equation with respect to T, we obtain

$$\sin\theta(-\,|q|\cos\theta\; + \sqrt{1 - cos^2\theta}) = 0$$

Since β is non-geodesic, we get

$$cot\,\theta = \frac{1}{|q|}$$

For $|q| > 1$, equation (3.6) gives

$$\kappa_1 = \sqrt{q^2 - sin^2\theta}$$

For $\kappa_2 \neq 0$, from (2.1) and (3.4), we get

$$\varphi^2 T = -T + \cos\theta\ \xi \tag{3.8}$$

Now,

$$\nabla_T \varphi T = (\nabla_T \varphi)T + \varphi(\nabla_T T)$$

$$\nabla_T \varphi T = sgn\,(q)\ \ sin\,\theta\ \xi - sgn\,(q)\ \ sin\,\theta\ cos\,\theta\ T - qT + qcos\,\theta\ \xi \tag{3.9}$$

Now, taking covariant derivative of the (3.7) with respect to T and making use of second Frenet formula, yields

$$\nabla_T \varphi T = sgn\,(q)\ \ sin\,\theta\ (-\,|q|\ \ sin\,\theta\ T + \kappa_2 E_2) \tag{3.10}$$

From (3.9) and (3.10), we obtain

$$sgn\,(q)\ \ sin\,\theta\,\xi - sgn\,(q)\ \ sin\,\theta\ cos\,\theta\ T - qT + qcos\,\theta\ \xi = \sin\theta(-qsin\,\theta\ T + sgn\,(q)\,\kappa_2 E_2)$$

A straightforward computation yields,

$$(sgn\,(q)\,sin\,\theta\ \ +\ \ q\ cos\,\theta)\,(\xi - \cos\theta\ T) = sgn\,(q)\sin\theta\,\kappa_2 E_2 \tag{3.11}$$

Then, we have

$$\kappa_2 = |sgn\,(q)\ \ sin\,\theta + qcos\,\theta|$$

Now, the ξ in terms of Frenet frame of β can be expressed by the following expression

$$\xi = (\varepsilon sgn\,(q)\,sin\,\theta\ E_2 + cos\,\theta T) \tag{3.12}$$

Where $\varepsilon = sgn(sgn\,(q)\,sin\,\theta + qcos\,\theta)$

Now, applying φ on (3.12), we get

$$\varphi E_2 = -\varepsilon E_1 cos\,\theta$$

From the equations (3.4), (3.7), (3.12), we have

$$\varphi E_1 = \varepsilon\ E_2 cos\,\theta - sgn\,(q)\ \ sin\,\theta\ T \tag{3.13}$$

By applying η to above equation, we get

$$\eta E_2 = \varepsilon sgn\,(q)\,sin\,\theta$$

When $\theta = \frac{\pi}{2}$, the equation (3.12) yields

$$E_2 = -sgn\,(q)\,\xi$$

and the curvatures are $\kappa_1 = |q|$, $\kappa_2 = 1$ and $\kappa_3 = 0$.
When $\theta \neq \frac{\pi}{2}$, taking covariant derivative of (3.13) with respect to T, we get

$$\nabla_T E_2 = (q\cos\theta \;\; - \varepsilon\ sgn\,(q)\sin\theta\ \cos\theta\ \xi - sgn\,(q)\sin\theta\ )E_1$$

and hence $\kappa_3 = 0$.

Hence, on the Kenmotsu manifold, the non-geodesic magnetic curves with the Lorentz force are Frenet curves of osculating order three with constant curvatures κ_1 and κ_2. □

Remark 2. *Since $\xi \in span\{T, E_2\}$, thus ξ can expressed as*

$$\xi = \rho E_2 + \cos\theta T \tag{3.14}$$

By taking norm on both sides, we obtain $\rho^2 = sin^2\,\theta$.
For $\theta = \frac{\pi}{2}$, we have

$$\xi = \rho E_2 \qquad and \qquad \rho^2 = 1.$$

Proposition 3. *If β is a non-geodesic Legendre φ-curve of order three in a Kenmotsu manifold, then $\kappa_2 = 1$ and $E_2 = \pm\xi$.*

4 Slant curves in Kenmotsu manifolds

In this section, we discuss the slant curves in a Kenmotsu manifold. Let β be a smooth curve in an almost contact metric 3-manifold parametrized by arc length. The contact angle of β is given by

$$cos\,\theta(s) = g\left(\beta^{'}(s), \xi\right)$$

where $\theta\,(s) = [0, \pi]$.

Differentiating the above formula along β by operating Levi-Civita connection ∇, we get

$$\begin{aligned} -\theta^{'} sin\,\theta &= g\,(\kappa N, \xi) + g\,(T, \nabla_T \xi) \\ &= \kappa\ \eta\,(N) + 1 - cos^2\,\theta \end{aligned}$$

By the above equation, we have following result as given below:

Proposition 4. *If Frenet curve β is a slant curve in a Kenmotsu manifold, then β satisfies the following expression*

$$\eta\left(N\right)=\left(-\frac{\sin^{2}\theta}{\kappa}\right) \tag{4.1}$$

By using the Frenet Frame field $\{T, N, B\}$, ξ can be written as

$$\xi=\left(\cos\theta\right)\,T+\left(-\frac{\sin^{2}\theta}{\kappa}\right)N+\eta\left(B\right)B$$

Since ξ is unitary vector field, the above equation gives

$$\eta\left(B\right)=\frac{\sin\theta}{\kappa}\sqrt{\kappa^{2}-sin^{2}\theta} \tag{4.2}$$

Remark 5. *The expression of ξ for a slant curve β in the Frenet Frame field is given by*

$$\xi=\left(\cos\theta\right)\,T+\left(-\frac{\sin^{2}\theta}{\kappa}\right)N+\left(\frac{\sin\theta}{\kappa}\sqrt{\kappa^{2}-sin^{2}\theta}\right)B \tag{4.3}$$

5 The curvature and torsion

Let β be a non geodesic curve, then β cannot be an integral curve of ξ. In an almost contact metric 3-manifold M, we consider the orthonormal frame field along β as

$$e_{1}=\left(\beta^{'}\right)=T,\qquad e_{2}=\left(\frac{\varphi\beta^{'}}{sin\,\theta}\right),\qquad e_{3}=\left(\frac{\xi-\cos\theta\beta^{'}}{sin\,\theta}\right) \tag{5.1}$$

Also, the characteristic vector field ξ can be written as

$$\xi=\left(\cos\theta\right)e_{1}+\left(\sin\theta\right)e_{3} \tag{5.2}$$

Then, for a slant curve β in a Kenmotsu manifold, we have

$$\begin{cases} \nabla_{\beta'}e_{1}=\delta\sin\theta\ e_{2} \\ \nabla_{\beta'}e_{2}=\delta\sin\theta\ e_{1}-\cos\theta\ e_{2}-\delta\cos\theta\ e_{3} \\ \nabla_{\beta'}e_{3}=\sin\theta\ e_{1}-\delta\cos\theta\ e_{2}-\cos\theta\ \ e_{3} \end{cases} \tag{5.3}$$

where $\delta=\frac{g(\nabla_{\beta'}\beta^{'},\varphi\beta^{'})}{\sin^{2}\theta}$
From the equation (2.4), we have

$$\nabla_{\beta'}\xi=sin^{2}\theta e_{1}-\sin\theta\cos\theta\ e_{3}$$

On an arbitrary oriented Riemannian 3-manifold, the cross product of two vector fields X and Y in $\mathcal{X}\left(M^3\right)$ is given by

$$dv_g(X,Y,Z) = g\left(X \times Y, Z\right)$$

for any Z in $\mathcal{X}\left(M^3\right)$, where dv_g is the volume given by the metric g. On the almost contact metric 3-manifold, the cross product of two vector fields is defined by the following formula

$$X \times Y = g\left(\varphi X, Y\right)\xi - \eta\left(Y\right)\varphi X + \eta(X)\varphi(Y)$$

Since vector field X is orthogonal to ξ, the basis vectors X, φX and ξ are supposed to be counterclockwise oriented, thus

$$\varphi\beta^{'} = \xi \times \beta^{'}$$

Since, $\beta^{'} = T$, then magnetic equation can be written as

$$\nabla_{\beta'}\beta^{'} = q\left(\xi \times \beta^{'}\right) = \kappa N \tag{5.4}$$

As a result, we get

$$\kappa^2 = q^2 g\left(\xi \times \beta^{'}, \xi \times \beta^{'}\right) = q^2 sin^2\theta$$

Therefore, $\kappa = |q|\sin\theta$ i.e. β has constant curvature. From (5.1) and (5.4), we have

$$N = \frac{q}{\kappa}\varphi\beta^{'}$$

Now, the Binormal vector field B is given by

$$B = \beta^{'} \times N = \beta^{'} \times \left[\frac{q}{\kappa}\left(\xi \times \beta^{'}\right)\right] = \frac{q}{\kappa}\left[\xi - \cos\theta\beta^{'}\right]$$

Taking covariant derivative and using the equation (2.4) and (5.2), yields

$$\nabla_{\beta'}B = \frac{q}{\kappa}\left[\nabla_{\beta'}\xi - \cos\theta\ \nabla_{\beta'}\beta^{'}\right]$$

$$\nabla_{\beta'}B = \frac{q}{\kappa}\left[\beta^{'} - cos^2\theta\ e_1 - q\cos\theta\, sin\,\theta\, e_2 - \cos\theta\, sin\,\theta\, e_3\right]$$

and using the relation $\nabla_{\beta'}B = -\tau N$ with (5.1), we obtain

$$\tau = q\cos\theta$$

6 Conclusion

In this paper, magnetic curves and slant curves in Kenmotsu manifolds have been investigated. We have obtained a classification theorem of the normal magnetic curves on Kenmotsu manifolds, and a characterization result for the Frenet curve to be a slant curve in a Kenmotsu manifold. Moreover, we gave a few results on the curvature and torsion in Kenmotsu manifolds. We hope that this work will be useful in the study of magnetic and slant curves in some other manifolds e.g. in the almost Kenmotsu manifolds.

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