ON CONSTRUCTION OF APPROXIMATE SOLUTIONS OF EQUATIONS OF THE NON-LINEAR AND NON-SHALLOW CYLINDRICAL SHELLS

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Abstract. In the present paper we consider the geometrically non-linear and non-shallow cylindrical shells, when components of the deformation tensor have non-linear terms. By means of I. Vekua method two dimensional problem is obtained. Approximate solutions of I. Vekua's equations for approximations $N = 1$ are constructed. Concrete problem is solved, when the components of the external force are constants.

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The refined theory of shells is constructed by reduced the three- dimensional problems of the theory of elasticity to the two-dimensional problems. I.Vekua had obtained the equations of shallow shells [1],[2]. It means that the interior geometry of the shell does not vary in thickness. This method for non-shallow shells in case of geometrical and physical non-linear theory was generalized by T.Meunargia [3],[4].

A complete system of equations of the three-dimensional nonlinear theory of elasticity can be written as:

$$\frac{1}{\sqrt{g}}\partial_i\sqrt{g}\boldsymbol{\sigma}^i + \boldsymbol{\Phi} = 0 \quad \Big(\partial_i = \frac{\partial}{\partial x_i}\Big),$$

$$\boldsymbol{\sigma}^i = \lambda\Big(\boldsymbol{R}^j\partial_j\boldsymbol{U} + \frac{1}{2}\partial^j\boldsymbol{U}\partial_j\boldsymbol{U}\Big)\Big(\boldsymbol{R}^i + \partial^i\boldsymbol{U})\Big)$$
$$+\mu\Big(\boldsymbol{R}^i\partial^j\boldsymbol{U} + \boldsymbol{R}^j\partial^i\boldsymbol{U} + \partial^i\boldsymbol{U}\partial^j\boldsymbol{U}\Big)\Big(\boldsymbol{R}_j + \partial_j\boldsymbol{U})\Big),$$

where g is the discriminant of the metric tensor of the space, $\boldsymbol{\sigma}^i$ are contravariant stress vectors, $\boldsymbol{\Phi}$ is an external force, λ and μ are Lame's constants, $\boldsymbol{R}_i$ and $\boldsymbol{R}^i$ are covariant and contravariant base vectors of the space and $\boldsymbol{U}$ is the displacement vector.

In the present paper we consider the system of equilibrium equations of the two-dimensional geometrically non-linear and non-shallow cylindrical shells which is obtained from the three-dimensional problems of the theory of elasticity for isotropic and homogeneous shell by the method of I. Vekua.

The displacement vector $\boldsymbol{U}(x^1, x^2, x^3)$ is expressed by the following formula [1]

$$\boldsymbol{U}(x^1, x^2, x^3) = \mathbf{u}(x^1, x^2) + \frac{x^3}{h}\mathbf{v}(x^1, x^2).$$

Here $\mathbf{u}(x^1,x^2)$ and $\mathbf{v}(x^1,x^2)$ are the vector fields on the middle surface $x^3=0$, $2h$ is the thickness of the shell, x^3 is a thickness coordinate ($-h\le x^3\le h$), x^1 and x^2 are isometric coordinates on the cylindrical surface.

The system of equilibrium equations of the two-dimensional geometrically non-linear and non-shallow cylindrical shells may be written in the following form (approximation $N=1$):

$$\begin{aligned}
&\partial_1 \overset{(0)}{\sigma}_{11}+\partial_2 \overset{(0)}{\sigma}_{21}+\varepsilon\, \overset{(0)}{\sigma}_{13}+\overset{(0)}{F}_1=0,\\
&\partial_1 \overset{(0)}{\sigma}_{12}+\partial_2 \overset{(0)}{\sigma}_{22}+\overset{(0)}{F}_2=0,\\
&\partial_1 \overset{(0)}{\sigma}_{13}+\partial_2 \overset{(0)}{\sigma}_{23}-\varepsilon\, \overset{(0)}{\sigma}_{11}+\overset{(0)}{F}_3=0,
\end{aligned}\tag{1}$$

$$\begin{aligned}
&\partial_1 \overset{(1)}{\sigma}_{11}+\partial_2 \overset{(1)}{\sigma}_{21}-\tfrac{3}{h}\overset{(0)}{\sigma}_{31}+\varepsilon\, \overset{(1)}{\sigma}_{13}+\overset{(1)}{F}_1=0,\\
&\partial_1 \overset{(1)}{\sigma}_{12}+\partial_2 \overset{(1)}{\sigma}_{22}-\tfrac{3}{h}\overset{(0)}{\sigma}_{32}+\overset{(1)}{F}_2=0,\\
&\partial_1 \overset{(1)}{\sigma}_{13}+\partial_2 \overset{(1)}{\sigma}_{23}-3\,\overset{(0)}{\sigma}_{33}-\varepsilon\, \overset{(1)}{\sigma}_{11}+\overset{(1)}{F}_3=0,
\end{aligned}\tag{2}$$

where

$$\overset{(m)}{\boldsymbol{F}}=\overset{(m)}{\boldsymbol{\Phi}}+\frac{2m+1}{2h}\left[(1+\varepsilon)\overset{(+)}{\boldsymbol{\sigma}}_3-(-1)^m(1-\varepsilon)\overset{(-)}{\boldsymbol{\sigma}}_3\right],$$

$$\left(\overset{(m)}{\sigma}_{ij},\ \overset{(m)}{\boldsymbol{\Phi}}\right)=\frac{2m+1}{2h}\int_{-h}^{h}\left(1+\frac{x_3}{R}\right)(\sigma_{ij},\,\boldsymbol{\Phi})P_m\left(\frac{x_3}{h}\right)dx_3.$$

$$\boldsymbol{\sigma}_3(x_1,x_2,\pm h)=\overset{(\pm)}{\boldsymbol{\sigma}}_3.$$

Here P_m are Legendre polynomials of order m, $\varepsilon=\frac{h}{R_0}$ is a small parameter, R_0 is the radius of the middle surface of the cylinder.

Let us construct the solutions of the form [5]

$$u_i=\sum_{k=1}^{\infty}\overset{k}{u}_i\varepsilon^k,\qquad v_i=\sum_{k=1}^{\infty}\overset{k}{v}_i\varepsilon^k\quad(i=1,2,3),\tag{3}$$

where u_i and v_i are the components of the vectors $\mathbf{u}$ and $\mathbf{v}$ respectively.

Formal substitution of (3) into (2) and (1) shows that series (3) will satisfy equations (1), (2) if the following equations are fulfilled:

$$\begin{aligned}
\mu\Delta \overset{k}{u}_1 &+ (\lambda+\mu)\partial_1\overset{k}{\theta}+\lambda\partial_1\overset{k}{v}_3=\overset{k}{X}_1,\\
\mu\Delta \overset{k}{u}_2 &+ (\lambda+\mu)\partial_2\overset{k}{\theta}+\lambda\partial_2\overset{k}{v}_3=\overset{k}{X}_2,\\
\mu\Delta \overset{k}{v}_3 &- 3\left[\lambda\overset{k}{\theta}+(\lambda+2\mu)\overset{k}{v}_3\right]=\overset{k}{X}_3,
\end{aligned}\tag{4}$$

$$\begin{aligned} \mu\Delta \overset{k}{v}_1 &+ (\lambda+\mu)\partial_1 \overset{k}{\Theta} - 3\mu(\partial_1 \overset{k}{u}_3 + \overset{k}{v}_1) = \overset{k}{X}_4, \\ \mu\Delta \overset{k}{v}_2 &+ (\lambda+\mu)\partial_2 \overset{k}{\Theta} - 3\mu(\partial_2 \overset{k}{u}_3 + \overset{k}{v}_2) = \overset{k}{X}_5, \\ \mu\Delta \overset{k}{u}_3 &+ \mu \overset{k}{\Theta} = \overset{k}{X}_6, \quad (k=1,2,...), \end{aligned} \tag{5}$$

$\overset{k}{X}_p$ $(p=1,...,6)$ are the components of external force and well-known quantity, defined by functions $\overset{0}{u}_i, ..., \overset{k-1}{u}_i, \overset{0}{v}_j, ..., \overset{k-1}{v}_j$.

The general solutions of systems (1) end (2) are written in the following form

$$\begin{aligned} 2\mu \overset{k}{u}_+ &= \frac{5\lambda+6\mu}{3\lambda+2\mu} \overset{k}{\varphi}(z) - z\overline{\overset{k}{\varphi}{}'(z)} - \overline{\overset{k}{\psi}(z)} - \frac{\lambda}{6(\lambda+\mu)} \frac{\partial \overset{k}{\chi}(z,\bar{z})}{\partial \bar{z}} + \widehat{\overset{k}{u}}_+, \\ 2\mu \overset{k}{v}_3 &= -\frac{2\lambda}{3\lambda+2\mu}\left(\overset{k}{\varphi}{}'(z) + \overline{\overset{k}{\varphi}{}'(z)}\right) + \overset{k}{\chi}(z,\bar{z}) + \widehat{\overset{k}{v}}_3, \\ 2\mu \overset{k}{v}_+ &= \frac{4(\lambda+2\mu)}{3\mu}\overline{\overset{k}{f}{}''(z)} + z\overline{\overset{k}{f}{}'(z)} + \overset{k}{f}(z) - 2\overline{\overset{k}{g}{}'(z)} + i\frac{\partial \overset{k}{w}(z,\bar{z})}{\partial \bar{z}} + \widehat{\overset{k}{v}}_+, \\ 2\mu \overset{k}{u}_3 &= -\frac{1}{2}\left(\bar{z}\overset{k}{f}(z) + z\overline{\overset{k}{f}(z)}\right) + \overset{k}{g}(z) + \overline{\overset{k}{g}(z)} + \widehat{\overset{k}{u}}_3, \end{aligned}$$

$$\left(\overset{k}{u}_+ = \overset{k}{u}_1 + i\overset{k}{u}_2, \quad \overset{k}{v}_+ = \overset{k}{v}_1 + i\overset{k}{v}_2, \quad z = x^1 + ix^2, \right.$$
$$\left. \frac{\partial}{\partial \bar{z}} = \frac{1}{2}\left(\frac{\partial}{\partial x^1} + i\frac{\partial}{\partial x^2}\right), \quad \frac{\partial}{\partial z} = \frac{1}{2}\left(\frac{\partial}{\partial x^1} - i\frac{\partial}{\partial x^2}\right)\right),$$

where $\overset{k}{\varphi}(z)$,$\overset{k}{\psi}(z)$,$\overset{k}{f}(z)$ and $\overset{k}{g}(z)$ are any analytic functions of z, $\overset{k}{\chi}(z,\bar{z})$ and $\overset{k}{w}(z,\bar{z})$ are the general solutions of the following Helmholtz's equations, respectively:

$$\begin{aligned} \Delta \overset{k}{\chi} &- \eta^2 \overset{k}{\chi} = 0 \quad \left(\eta^2 = \frac{12(\lambda+\mu)}{\lambda+2\mu}\right), \\ \Delta \overset{k}{w} &- \gamma^2 \overset{k}{w} = 0 \quad \left(\gamma^2 = 3\right). \end{aligned}$$

Here $\widehat{\overset{k}{u}}_+$, $\widehat{\overset{k}{v}}_3$ and $\widehat{\overset{k}{v}}_+$, $\widehat{\overset{k}{u}}_3$ are particular solutions of the non-homogeneous equations (1) and (2), respectively.

We solve the problem when the middle surface of the body after developing on the plane, is the circle with the radius R. Let's consider the concrete problem, when the components of the external force are constant $X_1 = X_2 = 0, X_3 = q$. Boundary conditions are

$$u_r + iu_\vartheta = 0, \quad |z| = R, \qquad v_3 = 0 \quad |z| = R, \tag{6}$$

$$v_r + iv_\vartheta = 0, \quad |z| = R, \qquad u_3 = 0 \quad |z| = R, \tag{7}$$

This problem for the approximation $k = 1$ is a well known case in the theory of elasticity for which we have

$$\begin{aligned}
2\mu \overset{1}{u}_{+} &= \left(\frac{2(\lambda+2\mu)}{3\lambda+2\mu}a_1 + \frac{\lambda}{12(\lambda+\mu)}q\right) z - \frac{\lambda\eta}{12(\lambda+\mu)}\alpha_0 I_1(\eta r)e^{i\theta},\\
2\mu \overset{1}{v}_{3} &= \alpha_0 I_0(\eta r) - \frac{\lambda+2\mu}{6(\lambda+\mu)}q - \frac{4\lambda}{3\lambda+2\mu}a_1,\\
2\mu \overset{1}{v}_{+} &= -\frac{3\mu R^2}{8(\lambda+2\mu)}qz + \frac{3\mu R^2}{8(\lambda+2\mu)}qz^2\bar{z},\\
2\mu \overset{1}{u}_{3} &= -\left(1+\frac{3\mu R^2}{16(\lambda+2\mu)}\right)\frac{R^2 q}{2} + \left(1+\frac{3\mu R^2}{8(\lambda+2\mu)}\right)\frac{qz\bar{z}}{2}\\
&\quad -\frac{3\mu}{32(\lambda+2\mu)}qz^2\bar{z}^2,
\end{aligned}$$

where

$$\begin{aligned}
a_1 &= \frac{-\frac{\lambda R}{12(\lambda+2\mu)} + \frac{\lambda(\lambda+2\mu)\eta I_1(\eta R)}{72(\lambda+\mu)^2 I_0(\eta R)}}{\frac{2(\lambda+2\mu)R}{3\lambda+2\mu} - \frac{\lambda^2\eta I_1(\eta R)}{3(\lambda+\mu)(3\lambda+2\mu)I_0(\eta R)}}q,\\
\alpha_0 &= \left[\frac{\lambda+2\mu}{6(\lambda+\mu)} - \frac{-\frac{\lambda^2 R}{3(\lambda+2\mu)} + \frac{\lambda^2(\lambda+2\mu)\eta I_1(\eta R)}{18(\lambda+\mu)^2 I_0(\eta R)}}{2(\lambda+2\mu)R - \frac{\lambda^2\eta I_1(\eta R)}{3(\lambda+\mu)}}\right]\frac{q}{I_0(\eta R)}.
\end{aligned}$$

The system of equilibrium equations, for the approximation $k = 2$, are:

$$\begin{aligned}
\mu\Delta \overset{2}{v}_{+} + 2(\lambda+\mu)\partial_{\bar{z}}\overset{2}{\Theta} - 3\mu(2\partial_{\bar{z}}\overset{2}{u}_{3} + \overset{2}{v}_{+}) &= A_1 + A_2 z\bar{z} + A_3 z^2\bar{z}^2 \\
&\quad + A_4(z+\bar{z}) + A_5(I_1(\eta r)e^{i\vartheta} + I_{-1}(\eta r)e^{-i\vartheta}),
\end{aligned} \tag{8}$$

$$\mu\Delta \overset{2}{u}_{3} + \mu\overset{2}{\Theta} = B_1 + B_2 z\bar{z} + B_3 z^2\bar{z}^2 + B_4(z^2+\bar{z}^2) + B_5(z^3\bar{z} + \bar{z}^3 z), \tag{9}$$

where

$$\begin{aligned}
A_1 &= -\frac{3\lambda}{2\mu}\left(1+\frac{3\mu R^2}{16(\lambda+2\mu)}\right)\frac{R^2 q}{2}, \qquad A_2 = \frac{3\lambda}{4\mu}\left(1+\frac{3\mu R^2}{8(\lambda+2\mu)}\right)q,\\
A_3 &= -\frac{9\lambda q}{64(\lambda+2\mu)}, \quad A_4 = \frac{3\mu R^2 q}{8(\lambda+2\mu)}, \quad A_5 = -\frac{3\mu q}{8(\lambda+2\mu)} - \frac{\lambda+2\mu}{2\mu}\alpha_0,\\
B_1 &= \frac{9\mu R^2 q^2}{128(\lambda+2\mu)} + \frac{3\mu R^2 q}{8(\lambda+2\mu)} + \frac{3\lambda}{4\mu}\left(1+\frac{3\mu R^2}{16(\lambda+2\mu)}\right)R^2 q,\\
B_2 &= \left(1+\frac{3\mu R^2}{8(\lambda+2\mu)}\right)\left(\frac{3\lambda q}{2\mu} - \frac{3(3\lambda+10\mu)q^2}{8(\lambda+2\mu)}\right)\\
&\quad -\frac{27\mu^2 R^2 q^2}{128(\lambda+2\mu)} + \frac{3\mu q}{4(\lambda+2\mu)},\\
B_3 &= \frac{9\mu^2 q^2}{16(\lambda+2\mu)^2} - \frac{27\mu(\lambda+\mu)q^2}{128(\lambda+2\mu)^2} - \frac{9\lambda q}{64(\lambda+2\mu)^2},
\end{aligned}$$

$$
\begin{aligned}
B_4 &= -\frac{9q^2}{32}\left(1+\frac{3\mu R^2 q}{8(\lambda+2\mu)}\right)-\frac{9\mu^2R^2q^2}{128(\lambda+2\mu)^2},\\
B_5 &= -\frac{9\mu q}{128(\lambda+2\mu)}.
\end{aligned}
$$

The general solutions of systems (8) end (9) are written in the following form

$$
\begin{aligned}
2\mu \overset{2}{v}_+ &= \frac{4(\lambda+2\mu)}{3\mu}\overline{\overset{2}{f}{}''(z)}+z\overline{\overset{2}{f}{}'(z)}+\overset{2}{f}(z)-2\overline{\overset{2}{g}{}'(z)}+i\frac{\partial \overset{2}{w}(z,\bar z)}{\partial \bar z}+\\
&+ N_0+N_1z+N_2\bar z+N_3z^2+N_4\bar z^2+N_5z\bar z+N_6z^2\bar z+\\
&+ N_7z^3\bar z^2+N_8I_0(\eta r)+N_9I_{-1}(\eta r)e^{-i\vartheta}+N_{10}I_3(\eta r)e^{3i\vartheta},\\
2\mu \overset{2}{u}_3 &= -\frac{1}{2}\left(\bar z\,\overset{2}{f}(z)+z\overline{\overset{2}{f}(z)}\right)+\overset{2}{g}(z)+\overline{\overset{2}{g}(z)}+M_0(z^2\bar z+\bar z^2 z)+\\
&+ M_1(z^3\bar z+\bar z^3 z)+M_2z^2\bar z^2+M_3z^3\bar z^3+M_4z^4\bar z^4+M_5I_0(\eta r)+\\
&+ M_6(I_2(\eta r)e^{2i\vartheta}+I_{-2}(\eta r)e^{-2i\vartheta}),
\end{aligned}
$$

where

$$
\begin{aligned}
M_0 &= -\frac{\mu A_1}{16(\lambda+2\mu)},\\
M_1 &= \frac{\mu}{24(\lambda+2\mu)}\left(\frac{2(\lambda+\mu)}{\mu}B_4-\frac{A_4}{2}\right),\\
M_2 &= \frac{\mu}{16(\lambda+2\mu)}\left(\frac{2(\lambda+\mu)}{\mu}B_2+\frac{3B_1}{2}-A_4\right),\\
M_3 &= \frac{\mu}{72(\lambda+2\mu)}\left(\frac{4(\lambda+\mu)}{\mu}B_3-\frac{B_2}{2}\right),\\
M_4 &= -\frac{\mu B_3}{384(\lambda+2\mu)}, \qquad M_5=-\frac{\mu A_5}{12(\lambda+\mu)}, \qquad M_6=-\frac{\mu A_5}{24(\lambda+\mu)},\\
N_0 &= -\frac{A_1}{3}, \qquad N_1=B_1, \qquad N_2=\frac{A_4}{3}-\frac{4(\lambda+\mu)}{3\mu}B_2+\frac{64}{9}B_5,\\
N_3 &= -\frac{A_2}{6}-\frac{8A_3}{9}-2M_0, \qquad N_4=\frac{4B_5}{9},\\
N_5 &= -\frac{A_2}{3}-\frac{16A_3}{9}-4M_0, \qquad N_6=\frac{B_2}{2}-4M_4, \qquad N_7=\frac{B_3}{3}-6M_6,\\
N_8 &= \frac{(\lambda+2\mu)\eta A_5}{6(3\lambda+2\mu)}, \qquad N_9=\frac{(\lambda+2\mu)\eta A_5}{6(3\lambda+2\mu)}-2M_6,\\
N_{10} &= \frac{(\lambda+2\mu)\eta A_5}{6(\lambda+2\mu)}-2M_6.
\end{aligned}
$$

Boundary conditions are

$$
\overset{2}{v}_r+i\overset{2}{v}_\vartheta=0, \qquad \overset{2}{u}_3=0, \quad |\, z\,|=R. \tag{10}
$$

Let us introduce the functions $\overset{2}{f}(z)$, $\overset{2}{g}(z)$and $\overset{2}{w}(z,\bar{z})$ by the series

$$\overset{2}{f}(z)=\sum_{n=1}^{\infty}c_n z^n,\quad \overset{2}{g}(z)=\sum_{n=0}^{\infty}d_n z^n,\quad \overset{2}{w}(z,\bar{z})=\sum_{-\infty}^{\infty}\beta_n I_n(\gamma r)e^{in\theta}. \tag{11}$$

where $I_n(\eta r)$ are Bessel's modifications functions.

By substituting (11) into (10) we obtain

$$\begin{aligned}
c_1 &= -N_1-N_6R^2-N_7R^4,\\
c_2 &= -\frac{N_0+N_5R^2+N_8I_0(\eta R)+\frac{I_0(\gamma R)}{I_2(\gamma R)}N_3R^2+2M_0R^2}{\left(\frac{I_0(\gamma R)}{I_2(\gamma R)}+1\right)R^2+\frac{8(\lambda+2\mu)}{\mu}},\\
c_3 &= -\frac{N_2R+N_9I_{-1}(\eta R)+\frac{I_1(\gamma R)}{I_3(\gamma R)}N_{10}I_3(\eta R)+4M_1R^3}{\left(\frac{I_1(\gamma R)}{I_3(\gamma R)}+1\right)R^3+\frac{8(\lambda+2\mu)}{\mu}R},\\
c_4 &= \frac{N_4}{\left(\frac{I_2(\gamma R)}{I_4(\gamma R)}+1\right)R^2+\frac{8(\lambda+2\mu)}{\mu}},\\
d_0 &= -\frac{1}{4}\left(N_1R^2+N_6R^4+N_7R^6\right)-M_2R^4-M_3R^5-M_4R^8-M_5I_0(\eta R),\\
d_1 &= \frac{R^2}{2}c_2-M_0R^2,\quad d_2=\frac{R^2}{2}c_3-\frac{1}{R^2}\left(M_1R^4+M_6I_3(\eta R)\right),\quad d_3=\frac{R^2}{2}c_4,\\
\beta_1 &= \frac{2i}{\gamma I_3(\gamma R)}\left(N_3R^2-R^2c_2\right),\\
\beta_2 &= \frac{2i}{\gamma I_3(\gamma R)}\left(N_{10}I_3(\eta R)-R^3c_3\right).
\end{aligned}$$

The system of equilibrium equations for $\overset{2}{u}_+$ and $\overset{2}{v}_3$ are:

$$\begin{gathered}
\mu\Delta\overset{k}{u}_+ +2(\lambda+\mu)\partial_{\bar{z}}\overset{2}{\vartheta}+2\lambda\partial_{\bar{z}}\overset{2}{v}_3=C_1\bar{z}+C_2z+C_3z\bar{z}^2+C_4\bar{z}z^2\\
\mu\Delta\overset{2}{u}_3+\mu\overset{2}{\Theta}=D_1+D_2z\bar{z}+D_3z^2\bar{z}^2+D_4(z^2+\bar{z}^2),
\end{gathered} \tag{12}$$

where

$$\begin{aligned}
C_1 &= -\frac{5\lambda+9\mu}{8(\lambda+2\mu)}q-\frac{3\mu R^2q}{16(\lambda+2\mu)},\\
C_2 &= \frac{\lambda+\mu}{2\lambda}\left(1+\frac{3\mu R^2}{8(\lambda+2\mu)}\right)q-\frac{\mu}{\lambda+2\mu}q,\\
C_3 &= \frac{3\mu q}{16(\lambda+2\mu)},\qquad C_4=\frac{3(\lambda+\mu)q}{16(\lambda+2\mu)},
\end{aligned}$$

$$\begin{aligned}
D_1 &= -\frac{3\lambda}{4\mu}\left(1+\frac{3\mu R^2}{16(\lambda+2\mu)}\right)R^2q-\frac{3\lambda+2\mu}{2\mu}\Big(2+\frac{3\mu R^2 q}{8(\lambda+2\mu)}\Big)q,\\
D_2 &= -\frac{3\lambda}{4\mu}\left(1+\frac{3\mu R^2}{8(\lambda+2\mu)}\right)+\frac{3(3\lambda+2\mu)q}{8(\lambda+2\mu)},\\
D_3 &= -\frac{9\lambda q}{64(\lambda+2\mu)}, \qquad D_4=\frac{3(\lambda-6\mu)q}{32(\lambda+2\mu)}q.
\end{aligned}$$

The general solutions of systems (12) are written in the following form:

$$2\mu\overset{2}{u}_{+}=\frac{5\lambda+6\mu}{3\lambda+2\mu}\overset{2}{\varphi}(z)-z\overline{\overset{2}{\varphi}{}'(z)}-\overline{\overset{2}{\psi}(z)}-\frac{\lambda}{6(\lambda+\mu)}\frac{\partial\overset{2}{\chi}(z,\bar z)}{\partial\bar z}+K_0z \tag{13}$$
$$+K_1z^3+K_2z\bar z^2+K_3z^2\bar z+K_4z^3\bar z+K_5z^2\bar z^2+K_6z^2\bar z^3+K_7z^4\bar z+K_8z^3\bar z^2,$$

$$2\mu\overset{2}{v}_{3}=-\frac{2\lambda}{3\lambda+2\mu}\left(\overset{2}{\varphi}{}'(z)+\overline{\overset{2}{\varphi}{}'(z)}\right)+\overset{2}{\chi}(z,\bar z)+L_0+L_1(z^2+\bar z^2) \tag{14}$$
$$+L_2z\bar z+L_3(z^2\bar z+\bar z^2z)+L_4(z^3\bar z+\bar z^3z)+L_5z^2\bar z^2,$$

where

$$\begin{aligned}
L_0 &= -\frac{(\lambda+2\mu)^2}{(\lambda+\mu)^2}\left[\frac{\lambda(6C_4-C_3)}{36(\lambda+\mu)}+\frac{D_2}{18}+\frac{2(\lambda+2\mu)}{27(\lambda+\mu)D_3}\right]-\frac{\lambda(\lambda+2\mu)}{24(\lambda+\mu)^2}C_4,\\
L_1 &= -\frac{\lambda C_1}{8(\lambda+\mu)}-\frac{(\lambda+2\mu)D_4}{6(\lambda+\mu)}, \qquad L_2=-\frac{\lambda C_2}{24(\lambda+\mu)}-\frac{(\lambda+2\mu)D_2}{6(\lambda+\mu)},\\
L_3 &= -\frac{\lambda(\lambda+2\mu)}{2(\lambda+\mu)^2}C_3, \qquad L_4=-\frac{\lambda C_3}{2(\lambda+\mu)^2},\\
L_5 &= -\frac{\lambda C_4}{48(\lambda+\mu)}-\frac{(\lambda+2\mu)D_3}{6(\lambda+\mu)},\\
K_0 &= -\frac{\lambda L_0}{2(\lambda+\mu)}, \qquad K_1=-\frac{(\lambda+\mu)C_1+4\lambda L_1}{24(\lambda+2\mu)},\\
,K_2 &= \frac{(\lambda+2\mu)C_1+4\lambda L_1}{8(\lambda+2\mu)},\\
K_3 &= \frac{C_2}{4}-\frac{(\lambda+\mu)C_2+4\lambda L_2}{16(\lambda+2\mu)}, \qquad K_4=K_5=-\frac{4\lambda L_3}{24(\lambda+2\mu)},\\
K_6 &= \frac{(\lambda+3\mu)C_3-6\lambda L_4}{24(\lambda+2\mu)}, \qquad K_7=-\frac{(\lambda+\mu)C_3+6\lambda L_4}{48(\lambda+2\mu)},\\
K_8 &= \frac{C_4}{12}-\frac{(\lambda+\mu)C_4+8\lambda L_5}{48(\lambda+2\mu)}.
\end{aligned}$$

Boundary conditions are

$$\overset{2}{u}_{r}+i\overset{2}{u}_{\vartheta}=0, \qquad \overset{2}{v}_{3}=0, \quad |\, z\,|=R. \tag{15}$$

Let us introduce the functions $\overset{2}{\varphi}(z)$, $\overset{2}{\psi}(z)$and $\overset{2}{\chi}(z,\bar z)$ by the series

$$\overset{2}{\varphi}(z)=\sum_{n=1}^{\infty}\rho_nz^n, \quad \overset{2}{\psi}(z)=\sum_{n=0}^{\infty}\varrho_nz^n, \quad \overset{2}{\chi}(z,\bar z)=\sum_{-\infty}^{\infty}\delta_nI_n(\eta r)e^{in\theta}. \tag{16}$$

By substituting (16) into (15) we obtain:

$$
\begin{aligned}
\rho_1 &= -\frac{K_0R + K_3R^3 + \frac{\lambda\eta I_1(\eta R)(L_0+L_2R^2+L_5R^4)}{12(\lambda+2\mu)I_0(\eta R)}}{\frac{2(\lambda+2\mu)R}{3\lambda+2\mu} + \frac{\lambda^2\eta I_1(\eta R))}{3(\lambda+\mu)(3\lambda+2\mu)I_0(\eta R))}},\\
\rho_2 &= -\frac{N_0 + N_5R^2 + N_8I_0(\eta R) + \frac{I_0(\gamma R)}{I_2(\gamma R)}N_3R^2 + 2M_0R^2}{\left(\frac{I_0(\gamma R)}{I_2(\gamma R)}+1\right)R^2 + \frac{8(\lambda+2\mu)}{\mu}},\\
\rho_3 &= -\frac{N_2R + N_9I_{-1}(\eta R) + \frac{I_1(\gamma R)}{I_3(\gamma R)}N_{10}I_3(\eta R) + 4M_1R^3}{\left(\frac{I_1(\gamma R)}{I_3(\gamma R)}+1\right)R^3 + \frac{8(\lambda+2\mu)}{\mu}R},\\
\rho_4 &= \frac{N_4}{\left(\frac{I_2(\gamma R)}{I_4(\gamma R)}+1\right)R^2 + \frac{8(\lambda+2\mu)}{\mu}},\\
\varrho_0 &= -\frac{1}{4}\left(N_1R^2 + N_6R^4 + N_7R^6\right) - M_2R^4 - M_3R^5 - M_4R^8 - M_5I_0(\eta R),\\
\varrho_1 &= \frac{R^2}{2}c_2 - M_0R^2, \quad \varrho_2 = \frac{R^2}{2}c_3 - \frac{1}{R^2}\left(M_1R^4 + M_6I_3(\eta R)\right), \quad d_3 = \frac{R^2}{2}c_4,\\
\delta_1 &= \frac{2i}{\gamma I_3(\gamma R)}\left(N_3R^2 - R^2c_2\right),\\
\delta_2 &= \frac{2i}{\gamma I_3(\gamma R)}\left(N_{10}I_3(\eta R) - R^3c_3\right).
\end{aligned}
$$

The problems when the middle surface of the body after developing on the plane are the circular ring with the radiuses radiuses R_1 and R_2 will be solved.

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