ON FUNDAMENTAL AND SINGULAR SOLUTIONS OF THE SYSTEM OF EQUATIONS OF THE EQUILIBRIUM OF THE PLANE TERMOELASTICITY THEORY WITH MICROTEMPERATURES

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Abstract: The present paper is devoted to the two-dimensional linear equilibrium theory of thermoelasticity for materials with inner structure whose particles, in addition to the classical displacement and temperature fields, possess microtemperatures. The fundamental and singular solutions for a governing system of this theory are constructed. The representation of the Galerkin type solution is obtained.

Key words: Thermoelasticity with microtemperatures, fundamental solution, singular solution, Galerkin type solution

MSC 2000: 74F05, 35E05, 35Q74

1. Introduction

The linear theory of thermoelasticity for materials with inner structure whose particles, in addition to the classical displacement and temperature fields, possess microtemperatures was constructed by Iesan and Quintanilla [1]. The fundamental solutions of the equations of the three-dimensional (3D) theory of thermoelasticity with microtemperatures were constructed by Svanadze [2]. The representations of the Galerkin type and general solutions of the system in this theory were obtained by Scalia, Svanadze and Tracinà [3]. The 3D linear theory of thermoelasticity for microstretch elastic materials with microtemperatures was constructed by Iesan [4], where the uniqueness and existence theorems in the dynamical case for isotropic materials are proved.

In the present paper we consider the two-dimensional (2D) linear equilibrium theory of thermoelasticity with microtemperatures and generalize some results of the classical 2D theory of thermoelasticity. The fundamental and singular solutions are constructed. The representation of the Galerkin type solution is also obtained.

2. Basic Equations

We consider an isotropic elastic material with microtemperatures. Let $D^+(D^-)$ be a bounded (respectively, an unbounded) domain of the real Euclidean 2D space E_2 bounded by the contour S. $\overline{D^+} := D^+ \bigcup S$, $D^- := E_2 \backslash \overline{D^+}$.

Let $\mathbf{x} := (x_1.x_2) \in E_2, \quad \partial\mathbf{x} := \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}\right).$

The basic homogeneous (i.e., body forces are neglected) system of the theory of thermoelasticity with microtemperatures has the form [1]

$$\mu\Delta\mathbf{u} + (\lambda+\mu) graddiv\mathbf{u} - \beta grad\theta = 0, \tag{1}$$

$$k_6\Delta\mathbf{w} + (k_4+k_5) graddiv\mathbf{w} - k_3 grad\theta - k_2\mathbf{w} = 0, \tag{2}$$

$$k\Delta\theta + k_1 div\mathbf{w} = 0, \tag{3}$$

where $\mathbf{u} := (u_1, u_2)^T$ is the displacement vector, $\mathbf{w} := (w_1, w_2)^T$ is the microtemperature vector, θ is the temperature measured from the constant absolute temperature T_0 $(T_0 > 0)$ by the natural state (i.e. by the state of the absence of loads), λ, μ, β, k, k_j, $j = 1, ..., 6$, are constitutive coefficients, Δ is the 2D Laplace operator. The superscript "T" denotes transposition.

We introduce the matrix differential operator

$$\mathbf{A}(\partial\mathbf{x}) :=\| A_{lj}(\partial\mathbf{x}) \|_{5x5},$$

where

$$\begin{aligned}
&A_{\alpha\gamma} := \mu\delta_{\alpha\gamma}\Delta + (\lambda+\mu)\frac{\partial^2}{\partial x_\alpha \partial x_\gamma},\\
&A_{\alpha+2;\gamma+2} := \delta_{\alpha\gamma}(k_6\Delta - k_2) + (k_4+k_5)\frac{\partial^2}{\partial x_\alpha \partial x_\gamma},\\
&A_{\alpha,\gamma+2} := A_{\alpha+2,\gamma} = 0, \quad A_{\alpha 5} := -\beta\frac{\partial}{\partial x_\alpha}, \quad A_{\alpha+2;5} := -k_3\frac{\partial}{\partial x_\alpha},\\
&A_{5\gamma} := 0, \quad A_{5;\gamma+2} := k_1\frac{\partial}{\partial x_\gamma}, \quad A_{55} := k\Delta, \quad \alpha,\gamma = 1,2,
\end{aligned}$$

$\delta_{\alpha\gamma}$ is the Kronecker delta. Then the system (1)-(3) can be rewritten as

$$\mathbf{A}(\partial\mathbf{x})\mathbf{U} = 0, \tag{4}$$

where

$$\mathbf{U} := (u_1, u_2, w_1, w_2, \theta)^T.$$

The matrix $\widetilde{\mathbf{A}}(\partial\mathbf{x}) :=\| \widetilde{A}_{lj}(\partial\mathbf{x}) \|_{5x5}:= \mathbf{A}^T(-\partial\mathbf{x})$, where $\widetilde{A}_{lj}(\partial\mathbf{x}) := A_{jl}(-\partial\mathbf{x})$, will be called the associated operator to the differential operator $\mathbf{A}(\partial\mathbf{x})$. Thus, the homogeneous associated system to the system (4) will be the following system

$$\begin{aligned}
&\mu\Delta\mathbf{u} + (\lambda+\mu) graddiv\mathbf{u} = 0,\\
&k_6\Delta\mathbf{w} + (k_4+k_5) graddiv\mathbf{w} - k_1 grad\theta - k_2\mathbf{w} = 0,\\
&k\Delta\theta + k_3 div\mathbf{w} + \beta div\mathbf{u} = 0.
\end{aligned}$$

We assume that $\mu(\lambda+2\mu)kk_6k_7 \neq 0$, where $k_7 := k_4+k_5+k_6$. Obviously, if the last condition is satisfied, then $\mathbf{A}(\partial\mathbf{x})$ is the elliptic differential operator [2].

3. Matrix of Fundamental Solutions

In order to investigate boundary value problems (BVPs) of the theory of thermoelasticity with microtemperatures by potential method it is necessary to construct a matrix of fundamental solutions to the governing system (4). Several methods are known for constructing the matrix of fundamental solutions of the systems of differential equations of the theory of elasticity and thermoelasticity (see e.g., [2,5]).

We introduce the matrix differential operator $\mathbf{B}(\partial\mathbf{x})$ consisting of cofactors of elements of the transposed matrix $\mathbf{A}^T$ divided on $\mu(\lambda+\mu)kk_6k_7 \neq 0$:

$$\mathbf{B}(\partial\mathbf{x}) :=\| B_{lj}(\partial\mathbf{x}) \|_{5x5},$$

where

$$B_{\alpha\gamma} := B^*_{11}\delta_{\alpha\gamma} - B^*_{12}\xi_\alpha\xi_\gamma, \quad B_{\alpha+2,\gamma+2} := B^*_{33}\delta_{\alpha\gamma} - B^*_{34}\xi_\alpha\xi_\gamma,$$

$$B_{1\gamma+2} := B^*_{13}\xi_1\xi_\gamma, \quad B_{2\gamma+2} := B^*_{13}\xi_1\xi_\gamma, \quad B_{\alpha5} := B^*_{15}\xi_\alpha,$$

$$B_{3\gamma} \equiv B_{4\gamma} \equiv B_{5\gamma} \equiv 0, \quad B_{5\gamma+2} := B^*_{33}\xi_\alpha, \quad \xi_\alpha := \frac{\partial}{\partial x_\alpha}, \quad \alpha,\gamma = 1,2,$$

$$B^*_{11} := \frac{1}{\mu}\Delta\Delta(\Delta - s_1^2)(\Delta - s_2^2), \quad B^*_{12} := \frac{\lambda+\mu}{a\mu}\Delta(\Delta - s_1^2)(\Delta - s_2^2),$$

$$B^*_{13} := -\frac{\beta k_1\Delta(\Delta - s_2^2)}{akk_7}, \quad B^*_{15} := \frac{\beta\Delta(\Delta - s_2^2)(k_7\Delta - k_2)}{akk_7}, \quad B_{55} \equiv B^*_{55},$$

$$B^*_{33} := \frac{1}{k_6}\Delta(\Delta - s_1^2)\Delta\Delta, \quad B^*_{34} := \frac{1}{k_6k_7}[(k_4+k_5)\Delta - k_7s_1^2 + k_2]\Delta\Delta,$$

$$B^*_{35} := \frac{k_3}{kk_7}\Delta\Delta(\Delta - s_2^2), \quad B^*_{53} := -\frac{k_1}{kk_7}\Delta\Delta(\Delta - s_2^2), \quad s_2^2 := \frac{k_2}{k_6},$$

$$B^*_{55} := \frac{1}{kk_7}\Delta\Delta(\Delta - s_2^2)(k_7\Delta - k_2), \quad s_1^2 := \frac{kk_2 - k_1k_3}{kk_7}, \quad a := \lambda + 2\mu.$$

Substituting the vector $\mathbf{U}(\mathbf{x}) = \mathbf{B}(\partial\mathbf{x})\mathbf{\Psi}$ into (4), where $\mathbf{\Psi}$ is a five-component vector function, we get

$$\Delta\Delta\Delta(\Delta - s_1^2)(\Delta - s_2^2)\mathbf{\Psi} = 0.$$

Whence, applying the method developed in [5], after some calculations, the vector $\Delta\Psi$ can be represented as

$$\Delta\mathbf{\Psi} = -\frac{r^2(\ln r - 1)}{4s_1^2s_2^2} + \frac{K_0(s_1r) + \ln r}{s_1^4(s_1^2 - s_2^2)} - \frac{K_0(s_2r) + \ln r}{s_2^4(s_1^2 - s_2^2)}. \tag{5}$$

where $K_0(s_\alpha r)$ is the modified Hankel function of the first kind and zero order,

$$K_0(s_\alpha r) := -I_0(s_\alpha r)\left(\ln\frac{s_\alpha r}{2}+C\right)+2\sum_{k=1}^{\infty}\frac{1}{(k!)^2}\left(\frac{s_\alpha r}{2}\right)^{2k}\left(\frac{1}{k}+\frac{1}{k-1}+...+1\right),$$

$$I_0(s_\alpha r) := \sum_{k=0}^{\infty}\frac{1}{(k!)^2}\left(\frac{s_\alpha r}{2}\right)^{2k}, \quad r^2 := (x_1-y_1)^2+(x_2-y_2)^2, \quad \alpha=1,2.$$

As all the components of $\mathbf{B}$ contain the operator Δ, Substituting (5) into $\mathbf{U}=\mathbf{B\Psi}$, we obtain the matrix of fundamental solutions for the equation (4) which we denote by $\mathbf{\Gamma}(\mathbf{x}\text{-}\mathbf{y})$

$$\mathbf{\Gamma}(\mathbf{x}\text{-}\mathbf{y}) :=\| \Gamma_{kj}(\mathbf{x}\text{-}\mathbf{y}) \|_{5x5},$$

where

$$\Gamma_{\alpha\gamma}(\mathbf{x}\text{-}\mathbf{y}) := -\delta_{\alpha\gamma}\frac{\ln r}{\mu}+\frac{\lambda+\mu}{a\mu}\frac{\partial^2\Psi_{11}}{\partial x_\alpha, \partial x_\gamma}, \quad \Psi_{11} := \frac{r^2(\ln r-1)}{4},$$

$$\Gamma_{\alpha+2,\gamma+2}(\mathbf{x}\text{-}\mathbf{y}) := \delta_{\alpha\gamma}\frac{K_0(s_2 r)}{k_6}-\frac{\partial^2\Psi_{33}}{\partial x_\alpha, \partial x_\gamma},$$

$$\Psi_{33}(\mathbf{x}\text{-}\mathbf{y}) := \frac{K_0(s_2 r)+\ln r}{k_6 s_2^2}-\frac{\Psi_{35}}{k_7}, \quad \Psi_{35}(\mathbf{x}\text{-}\mathbf{y}) := \frac{K_0(s_1 r)+\ln r}{s_1^2},$$

$$\Gamma_{1,\gamma+2}(\mathbf{x}\text{-}\mathbf{y}) := -\frac{\beta k_1}{akk_7}\frac{\partial^2\Psi_{13}}{\partial x_1\partial x_\gamma}, \quad \Psi_{13}(\mathbf{x}\text{-}\mathbf{y}) := \frac{\Psi_{35}+\Psi_{11}}{s_1^2},$$

$$\Gamma_{2,\gamma+2}(\mathbf{x}\text{-}\mathbf{y}) := -\frac{\beta k_1}{akk_7}\frac{\partial^2\Psi_{13}}{\partial x_2\partial x_\gamma}, \quad \Gamma_{\alpha 5}(\mathbf{x}\text{-}\mathbf{y}) := \frac{\beta}{akk_7}\frac{\partial\Psi_{15}}{\partial x_\alpha}, \quad \alpha,\gamma=1,2,$$

$$\Psi_{15}(\mathbf{x}\text{-}\mathbf{y}) := -\frac{k_2}{s_1^2}\Psi_{11}+\left(k_7-\frac{k_2}{s_1^2}\right)\Psi_{35}, \quad \Gamma_{\alpha+2,5}(\mathbf{x}\text{-}\mathbf{y}) := \frac{k_3}{kk_7}\frac{\partial\Psi_{35}}{\partial x_\alpha},$$

$$\Gamma_{5,\gamma+2}(\mathbf{x}\text{-}\mathbf{y}) := -\frac{k_1}{kk_7}\frac{\partial\Psi_{35}}{\partial x_\gamma}, \quad \Gamma_{55}(\mathbf{x}\text{-}\mathbf{y}) := \frac{K_0(s_1 r)}{k}-\frac{k_2}{kk_7}\Psi_{35}, \quad s_2^2 := \frac{k_2}{k_6}>0,$$

$$\Gamma_{31}\equiv\Gamma_{32}\equiv\Gamma_{41}\equiv\Gamma_{42}\equiv\Gamma_{51}\equiv\Gamma_{52}\equiv 0, \quad s_1^2 := \frac{kk_2-k_1k_3}{kk_7}>0.$$

We can easily prove the following

Theorem 1 *The elements of the matrix* $\mathbf{\Gamma}(\mathbf{x}\text{-}\mathbf{y})$ *has a logarithmic singularity as* $\mathbf{x}\to\mathbf{y}$ *and each column of the matrix* $\mathbf{\Gamma}(\mathbf{x}\text{-}\mathbf{y})$, *considered as a vector, is a solution of the system* (4) *at every point* $\mathbf{x}$ *if* $\mathbf{x}\neq\mathbf{y}$.

According to the method developed in [5], we construct the matrix $\widetilde{\mathbf{\Gamma}}(\mathbf{x}) := \mathbf{\Gamma}^T(-\mathbf{x})$ and the following basic properties of $\widetilde{\mathbf{\Gamma}}(\mathbf{x})$ may be easily verified:

Theorem 2 *Each column of the matrix* $\widetilde{\mathbf{\Gamma}}(\mathbf{x}\text{-}\mathbf{y})$, *considered as a vector, satisfies the associated system* $\widetilde{\boldsymbol{A}}(\partial\mathbf{x})\widetilde{\mathbf{\Gamma}}(\mathbf{x}\text{-}\mathbf{y})=0$, *at every point* $\mathbf{x}$ if $\mathbf{x}\neq\mathbf{y}$ *and the elements of the matrix* $\widetilde{\mathbf{\Gamma}}(\mathbf{x}\text{-}\mathbf{y})$ *have a logarithmic singularity as* $\mathbf{x}\to\mathbf{y}$.

4. Matrix of Singular Solutions

In solving BVPs of the theory of thermoelasticity with microtemperatures by the method of potential theory, besides the matrix of fundamental solutions, some other matrices of singular solutions to equation (4) are of a great importance. Using the matrix of fundamental solutions, we construct the so-called singular matrices of solutions by means of elementary functions.

We introduce the special generalized stress vector $\overset{\boldsymbol{\tau}}{\mathbf{R}}(\partial\mathbf{x},\mathbf{n})\mathbf{U}$, which acts on the element of the arc with the unit normal $\mathbf{n}=(n_1,n_2)$, where

$$\overset{\boldsymbol{\tau}}{\mathbf{R}}(\partial\mathbf{x},\mathbf{n}):=\|\overset{\tau}{\mathrm{R}}_{lj}\|_{5x5},$$

$$\begin{aligned}
&\overset{\tau}{\mathrm{R}}_{\alpha\gamma}:=\delta_{\alpha\gamma}\mu\frac{\partial}{\partial\mathbf{n}}+(\lambda+\mu)n_\alpha\frac{\partial}{\partial x_\gamma}+\tau_1\mathcal{M}_{\alpha\gamma},\\
&\overset{\tau}{\mathrm{R}}_{\alpha,\gamma+2}\equiv\overset{\tau}{\mathrm{R}}_{\alpha+2,\gamma}\equiv\overset{\tau}{\mathrm{R}}_{\alpha+2,5}\\
&\equiv\overset{\tau}{\mathrm{R}}_{5\gamma}\equiv 0,\quad \overset{\tau}{\mathrm{R}}_{\alpha 5}:=-\beta n_\alpha,\\
&\overset{\tau}{\mathrm{R}}_{\alpha+2;\gamma+2}:=\delta_{\alpha\gamma}k_6\frac{\partial}{\partial\mathbf{n}}+(k_4+k_5)n_\alpha\frac{\partial}{\partial x_\gamma}+\tau_2\mathcal{M}_{\alpha\gamma},\\
&\overset{\tau}{\mathrm{R}}_{5,\gamma+2}:=k_1n_\gamma,\quad \overset{\tau}{\mathrm{R}}_{55}:=k\frac{\partial}{\partial\mathbf{n}},\quad \mathcal{M}_{\alpha\gamma}:=n_\gamma\frac{\partial}{\partial x_\alpha}-n_\alpha\frac{\partial}{\partial x_\gamma},\ \alpha,\gamma=1,2,
\end{aligned}\tag{6}$$

here $\boldsymbol{\tau}:=(\tau_1,\tau_2)$, τ_α, $\alpha=1,2$, are the arbitrary numbers. If $\tau_1=\mu$, $\tau_2=k_5$, we denote the obtained operator by $\boldsymbol{P}(\partial\mathbf{x},\mathbf{n})$. The operator, which we get from $\overset{\boldsymbol{\tau}}{\mathbf{R}}(\partial\mathbf{x},\mathbf{n})$ for $\tau_1=\dfrac{\mu(\lambda+\mu)}{\lambda+3\mu}$, $\tau_2=\dfrac{k_6(k_4+k_5)}{k_4+k_5+2k_6}$, we denote by $\mathbf{N}(\partial\mathbf{x},\mathbf{n})$ and the vector $\mathbf{N}(\partial\mathbf{x},\mathbf{n})\mathbf{U}$ will be called the pseudostress vector.

Applying the operator $\overset{\boldsymbol{\tau}}{\mathbf{R}}(\partial\mathbf{x},\mathbf{n})$ to the matrix $\mathbf{\Gamma}(\mathbf{x}\text{-}\mathbf{y})$, we construct the so-called singular matrix of solutions

$$\overset{\boldsymbol{\tau}}{\mathbf{R}}(\partial\mathbf{x},\mathbf{n})\mathbf{\Gamma}(\mathbf{x}\text{-}\mathbf{y}):=\|\overset{\tau}{\mathrm{M}}_{lj}(\partial\mathbf{x})\|_{5\times5},$$

where

$$\overset{\tau}{\mathrm{M}}_{\gamma\gamma}(\partial\mathbf{x}):=-\frac{\partial\ln r}{\partial\mathbf{n}}+(-1)^{\gamma+1}\frac{(\lambda+\mu)(\tau_1+\mu)}{a\mu}\frac{\partial}{\partial s}\frac{\partial^2\Psi_{11}}{\partial x_1\partial x_2},$$

$$\overset{\tau}{\mathrm{M}}_{12}(\partial\mathbf{x}):=\frac{\partial}{\partial s}\left[-\frac{\tau_1}{\mu}\ln r+\frac{(\lambda+\mu)(\tau_1+\mu)}{a\mu}\frac{\partial^2\Psi_{11}}{\partial x_2^2}\right],$$

$$\overset{\tau}{\mathrm{M}}_{21}(\partial\mathbf{x}) := \frac{\partial}{\partial s}\left[\frac{\tau_1}{\mu}\ln r - \frac{(\lambda+\mu)(\tau_1+\mu)}{a\mu}\frac{\partial^2\Psi_{11}}{\partial x_1^2}\right],$$

$$\overset{\tau}{\mathrm{M}}_{1,\gamma+2}(\partial\mathbf{x}) := -\frac{\beta k_1(\mu+\tau_1)}{kak_7}\frac{\partial}{\partial s}\frac{\partial^2\Psi_{13}}{\partial x_\gamma\partial x_2},$$
$$\overset{\tau}{\mathrm{M}}_{2,\gamma+2}(\partial\mathbf{x}) := \frac{\beta k_1(\mu+\tau_1)}{kak_7}\frac{\partial}{\partial s}\frac{\partial^2\Psi_{13}}{\partial x_1\partial x_\gamma},$$

$$\overset{\tau}{\mathrm{M}}_{15}(\partial\mathbf{x}) := \frac{\beta(\mu+\tau_1)}{kak_7}\frac{\partial}{\partial s}\frac{\partial\Psi_{15}}{\partial x_2}, \quad \overset{\tau}{\mathrm{M}}_{25}(\partial\mathbf{x}) := -\frac{\beta(\mu+\tau_1)}{kak_7}\frac{\partial}{\partial s}\frac{\partial\Psi_{15}}{\partial x_1},$$

$$\overset{\tau}{\mathrm{M}}_{35}(\partial\mathbf{x}) := \frac{k_3}{kk_7}\left[k_7 n_1 K_0(s_1 r) + (k_6+\tau_2)\frac{\partial}{\partial s}\frac{\partial\Psi_{35}}{\partial x_2}\right], \ \frac{\partial}{\partial s} := n_2\frac{\partial}{\partial x_1} - n_1\frac{\partial}{\partial x_2},$$

$$\overset{\tau}{\mathrm{M}}_{45}(\partial\mathbf{x}) := \frac{k_3}{kk_7}\left[k_7 n_2 K_0(s_1 r) - (k_6+\tau_2)\frac{\partial}{\partial s}\frac{\partial\Psi_{35}}{\partial x_1}\right],$$

$$\overset{\tau}{\mathrm{M}}_{53}(\partial\mathbf{x}) := -\frac{k_1}{k_2}\frac{\partial}{\partial s}\frac{\partial}{\partial x_2}(K_0(s_2 r) + \ln r),$$
$$\overset{\tau}{\mathrm{M}}_{54}(\partial\mathbf{x}) := \frac{k_1}{k_2}\frac{\partial}{\partial s}\frac{\partial}{\partial x_1}(K_0(s_2 r) + \ln r),$$

$$\overset{\tau}{\mathrm{M}}_{3\gamma}(\partial\mathbf{x}) \equiv \overset{\tau}{\mathrm{M}}_{4\gamma}(\partial\mathbf{x}) \equiv \overset{\tau}{\mathrm{M}}_{5\gamma}(\partial\mathbf{x}) \equiv 0, \quad \overset{\tau}{\mathrm{M}}_{55}(\partial\mathbf{x}) := \frac{\partial \ln r}{\partial\mathbf{n}}, \quad \gamma = 1,2,$$

$$\overset{\tau}{\mathrm{M}}_{33}(\partial\mathbf{x}) := \frac{\partial K_0(s_2 r)}{\partial\mathbf{n}} - (\tau_2+k_6)\frac{\partial}{\partial s}\frac{\partial^2\Psi_{33}}{\partial x_1\partial x_2} + n_1\frac{\partial}{\partial x_1}[K_0(s_1 r) - K_0(s_2 r)],$$
$$\overset{\tau}{\mathrm{M}}_{44}(\partial\mathbf{x}) := \frac{\partial K_0(s_2 r)}{\partial\mathbf{n}} + (\tau_2+k_6)\frac{\partial}{\partial s}\frac{\partial^2\Psi_{33}}{\partial x_1\partial x_2} + n_2\frac{\partial}{\partial x_2}[K_0(s_1 r) - K_0(s_2 r)],$$

$$\overset{\tau}{\mathrm{M}}_{43}(\partial\mathbf{x}) := -\frac{\tau_2}{k_6}\frac{\partial K_0(s_2 r)}{\partial s} + (\tau_2+k_6)\frac{\partial}{\partial s}\frac{\partial^2\Psi_{33}}{\partial x_1^2} + n_2\frac{\partial}{\partial x_1}[K_0(s_1 r) - K_0(s_2 r)],$$

$$\overset{\tau}{\mathrm{M}}_{34}(\partial\mathbf{x}) := \frac{\tau_2}{k_6}\frac{\partial K_0(s_2 r)}{\partial s} - (\tau_2+k_6)\frac{\partial}{\partial s}\frac{\partial^2\Psi_{33}}{\partial x_2^2} + n_1\frac{\partial}{\partial x_2}[K_0(s_1 r) - K_0(s_2 r)].$$

We prove the following theorem.

Theorem 3 *Every column of the matrix* $\left[\overset{\tau}{\mathbf{R}}(\partial\mathbf{y},\mathbf{n})\mathbf{\Gamma}(\mathbf{y}\text{-}\mathbf{x})\right]^T$, *considered as a vector, is a solution of the system* $\widetilde{\mathbf{A}}(\partial\mathbf{x}) = 0$ *at any point* $\mathbf{x}$ if $\mathbf{x} \neq \mathbf{y}$ *and the elements of the matrix* $\left[\overset{\tau}{\mathbf{R}}(\partial\mathbf{y},\mathbf{n})\mathbf{\Gamma}(\mathbf{y}\text{-}\mathbf{x})\right]^T$ *contain a singular part, which is integrable in the sense of the Cauchy principal value.*

Let

$$\tilde{\boldsymbol{R}}^{\tau}(\partial\mathbf{x},\mathbf{n}) := \begin{pmatrix} \overset{\tau}{\mathrm{R}}_{11} & \overset{\tau}{\mathrm{R}}_{12} & 0 & 0 & 0 \\ \overset{\tau}{\mathrm{R}}_{21} & R_{22} & 0 & 0 & 0 \\ 0 & 0 & \overset{\tau}{\mathrm{R}}_{33} & \overset{\tau}{\mathrm{R}}_{34} & 0 \\ 0 & 0 & \overset{\tau}{\mathrm{R}}_{43} & \overset{\tau}{\mathrm{R}}_{44} & 0 \\ 0 & 0 & k_3 n_1 & k_3 n_2 & \overset{\tau}{\mathrm{R}}_{55} \end{pmatrix},$$

where $\overset{\tau}{\mathrm{R}}_{\alpha\gamma}$, $\overset{\tau}{\mathrm{R}}_{\alpha+2,\gamma+2}$, $\overset{\tau}{\mathrm{R}}_{55}$, $\alpha,\gamma = 1,2$, are given by (6), then

$$\tilde{\boldsymbol{R}}^{\tau}(\partial\mathbf{x},\mathbf{n})\tilde{\mathbf{\Gamma}}(\mathbf{x}\text{-}\mathbf{y}) = \|\overset{\tau}{\widetilde{\mathrm{M}}}_{lj}(\partial\mathbf{x})\|_{5\times5},$$

here

$$\overset{\tau}{\widetilde{\mathrm{M}}}_{\alpha\gamma}(\partial\mathbf{x}) := \overset{\tau}{\mathrm{M}}_{\alpha\gamma}(\partial\mathbf{x}), \quad \overset{\tau}{\widetilde{\mathrm{M}}}_{\alpha+2,\gamma+2}(\partial\mathbf{x}) := \overset{\tau}{\mathrm{M}}_{\alpha+2,\gamma+2}(\partial\mathbf{x}),$$

$$\overset{\tau}{\widetilde{\mathrm{M}}}_{55}(\partial\mathbf{x}) := \overset{\tau}{\mathrm{M}}_{55}(\partial\mathbf{x}), \quad \overset{\tau}{\widetilde{\mathrm{M}}}_{\alpha3}(\partial\mathbf{x}) \equiv \overset{\tau}{\widetilde{\mathrm{M}}}_{\alpha4}(\partial\mathbf{x}) \equiv \overset{\tau}{\widetilde{\mathrm{M}}}_{\alpha5}(\partial\mathbf{x}) \equiv 0,$$

$$\overset{\tau}{\widetilde{\mathrm{M}}}_{3\gamma}(\partial\mathbf{x}) := -\frac{k_1\beta}{akk_7}\left[k_7 n_1 \frac{\partial\Psi_{35}}{\partial x_\gamma} + (k_6+\tau_2)\frac{\partial}{\partial s}\frac{\partial^2\Psi_{13}}{\partial x_\gamma \partial x_2}\right],$$

$$\overset{\tau}{\widetilde{\mathrm{M}}}_{4\gamma}(\partial\mathbf{x}) := -\frac{k_1\beta}{akk_7}\left[k_7 n_2 \frac{\partial\Psi_{35}}{\partial x_\gamma} - (k_6+\tau_2)\frac{\partial}{\partial s}\frac{\partial^2\Psi_{13}}{\partial x_\gamma \partial x_1}\right],$$

$$\overset{\tau}{\widetilde{\mathrm{M}}}_{35}(\partial\mathbf{x}) := \frac{k_1}{kk_7}\left[k_7 n_1 K_0(s_1 r) + (k_6+\tau_2)\frac{\partial}{\partial s}\frac{\partial\Psi_{35}}{\partial x_2}\right],$$

$$\overset{\tau}{\widetilde{\mathrm{M}}}_{45}(\partial\mathbf{x}) := \frac{k_1}{kk_7}\left[k_7 n_2 K_0(s_1 r) - (k_6+\tau_2)\frac{\partial}{\partial s}\frac{\partial\Psi_{35}}{\partial x_1}\right],$$

$$\overset{\tau}{\widetilde{\mathrm{M}}}_{5\gamma}(\partial\mathbf{x}) := \frac{\beta}{a}\frac{\partial}{\partial\mathbf{n}}\frac{\partial\Psi_{11}}{\partial x_\gamma}, \quad \alpha,\gamma = 1,2, \quad a := \lambda + 2\mu,$$

$$\overset{\tau}{\widetilde{\mathrm{M}}}_{53}(\partial\mathbf{x}) := -\frac{k_3}{k_2}\frac{\partial}{\partial s}\frac{\partial}{\partial x_2}\left[K_0(s_2 r) + \ln r\right],$$

$$\overset{\tau}{\widetilde{\mathrm{M}}}_{54}(\partial\mathbf{x}) := \frac{k_3}{k_2}\frac{\partial}{\partial s}\frac{\partial}{\partial x_1}\left[K_0(s_2 r) + \ln r\right].$$

We prove the following theorem.

Theorem 4 *Every column of the matrix* $\left[\tilde{\boldsymbol{R}}^{\tau}(\partial\mathbf{y},\mathbf{n})\tilde{\mathbf{\Gamma}}(\mathbf{y}\text{-}\mathbf{x})\right]^T$, *considered as a vector, is a solution of the system* $\mathbf{A}(\partial\mathbf{x})\mathbf{U} = 0$ at *any point* $\mathbf{x}$ *if* $\mathbf{x} \neq \mathbf{y}$ *and the elements of the matrix* $\left[\tilde{\boldsymbol{R}}^{\tau}(\partial\mathbf{y},\mathbf{n})\tilde{\mathbf{\Gamma}}(\mathbf{y}\text{-}\mathbf{x})\right]^T$, *contain a singular part, which is integrable in the sense of the Cauchy principal value.*

5. Galerkin Type Solution

The Galerkin type general solution (of the class $C^2(D^+)$) of the system (4) can be represented as

$$\mathbf{u}(\mathbf{x}) = a\Delta\boldsymbol{\phi}^{(1)} - (\lambda+\mu) graddiv\boldsymbol{\phi}^{(1)} - \beta\mu k_1 k_6 graddiv\boldsymbol{\phi}^{(2)}$$
$$+\beta\mu(k_7\Delta - k_2) grad\Psi,$$

$$\mathbf{w}(\mathbf{x}) = a\mu k k_7 \Delta\Delta(\Delta - s_1^2)\boldsymbol{\phi}^{(2)} - a\mu\Delta[k(k_4+k_5)\Delta + kk_3] graddiv\boldsymbol{\phi}^{(2)}$$
$$+a\mu k_3 grad\Psi,$$

$$\theta = a\mu\Delta(k_7\Delta - k_2)\Psi - a\mu k_1 k_6 \Delta(\Delta - s_2^2) div\boldsymbol{\phi}^{(2)},$$

where

$$\Delta\Delta\boldsymbol{\phi}^{(1)} = 0, \quad \Delta\Delta(\Delta - s_1^2)(\Delta - s_2^2)\boldsymbol{\phi}^{(2)} = 0, \quad \Delta\Delta(\Delta - s_1^2)\Psi = 0,$$

$$\boldsymbol{\phi}^{(1)} = (\phi_1^{(1)}, \phi_2^{(1)}), \quad \boldsymbol{\phi}^{(2)} = (\phi_1^{(2)}, \phi_2^{(2)}).$$

Acknowledgement. The designated project has been fulfilled by financial support of the Shota Rustaveli National Science Foundation (Grant #GNSF/ ST08/3-388). Any idea in this publication is possessed by the author and may not represent the opinion of Shota Rustaveli National Science Foundation itself.

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Received April, 4, 2011; Revised June 1, 2011; Accepted June, 8, 2011.