CONTACT PROBLEMS TREATED WITH BOUNDARY ELEMENT METHODS

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Contact problems have many important applications in different fields of mechanics, such as earthquake engineering, civil engineering and machine dynamics. For the solution of contact problems, the boundary element method is particularly suitable, since the nonlinear contact and friction boundary conditions are formulated on the boundary of the bodies only, whereas the field equations are linear.

For ease of presentation we consider first the quasistatic contact problem for an elastic body and a rigid obstacle, using the Signorini condition as boundary condition for the contact and modelling the friction by a generalized Coulomb law. Rewriting these conditions in variational form, we get for the contact problem a weak formulation in terms of a variational inequality. Then we use penalization to approximate the contact condition and, in addition, smoothing of the friction law. This leads to an approximation of the variational inequality by a nonlinear variational equation. Under some smallness restrictions on the friction coeficient it is possible to prove existence of a solution of the corresponding contact problem [2]. A boundary integral formulation of the variational equation is then presented, which is discretized by a symmetric boundary element method described in [3].

In the resulting boundary integral equation the only volume integral originates from the original right hand side of the problem, the external forces, and can be calculated in a very efficient way with the ficticious domain method.

As an extension of the above method we consider the case, when the contact areas are very small and the transmitted force densities are big, generating plastic deformations. For the corresponding analysis we use an elasto-plastic model with hardening. The elasto-plastic material law can be formulated as an evolution problem for the displacement field, the plastic stresses and the hardening parameters, involving a nonlinear monotone differential operator acting on the displacements. Similar to the previous case, the resulting variational equation and boundary integral equation formulation are basically the same as in the purely elastic case, but with an additional volume integral over the plastic stresses, which again can be calculated by employing the ficticious domain method [1].

REFERENCES

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