Transformation! – A Graphing Calculator Activity to Practice Transformations of Functions[1]

Dane R. Camp, Glen Ellyn (Il, USA)

Abstract: An understanding of function transformation is essential for mastering mathematics in high school and beyond. The classroom activity presented here and the game, *Transformation!*, are designed so that students can develop fluency in working with transformations of functions. Both take advantage of the technology of the graphing calculator and the method of cooperative learning.

Kurzreferat: *Transformation! – Eine Aktivität zum Umgang mit Transformationen von Funktionen mit Unterstützung graphischer Taschenrechner.* Das Verständnis für Transformationen von Funktionen ist wesentlich für eine Beherrschung der Mathematik in der Sekundarstufe und darüberhinaus. Die hier vorgestellte Unterrichtsaktivität und das Spiel *Transformation!* sind so konzipiert, dass Schüler Fertigkeiten beim Arbeiten mit Transformationen von Funktionen entwickeln können. Beide nutzen die Technik graphischer Taschenrechner sowie die Methode des kooperativen Lernens.

ZDM-Classification: I20, U60, U70

1. Introduction

If you add to a function, you'll give it a lift,
for the graph will be moved with a vertical shift,
But if you multiply, take a close look and see,
the graph's stretched by that factor vertically,
and negating the function will cause a reflection,
across the x-axis in an up-down direction.

But if you add before the function is used, hey!
The shift's horizontal – the opposite way!
And multiplication by a factor inside reveals,
the graph's being stretched by the reciprocal's.
And negating the values before f is applied,
reflects across the y-axis – it flips side to side!

Few concepts in mathematics occur as often as that of the transformation of functions. An understanding of how and why transformations work empowers high school mathematics students in classes ranging from algebra through calculus. In an effort to practice and solidify expertise in working with transformations, I have used the game *Transformation!* in my classes. I designed the game so that it can be tailored to involve any ability level. What follows is a brief introduction to transformations of functions, which students can work on in groups. Some samples of *Transformation!,* applied to various functions, are also included. (Note: The experiments are designed for any Texas Instrument calculator but can be adapted to other brands as well.)

2. The calculator lab experiment

The calculator laboratory experiments, which precede the game, are designed for students to explore the behavior of function transformations. The students are also asked to reflect upon why the basic transformations behave the way that they do. I usually summarize the activity by using the poem above. The function used as an example is "fPart" which, on a Texas Instrument calculator, yields the fractional part of any real number. This function is used because students, and teachers for that matter, should have no preconceived notions as to how the function will behave. Also, "fPart" is a periodic function, which will naturally provide a good foundation for trigonometric functions.

3. The transformation game

Once students have had an opportunity to explore transforming functions with their calculators using the lab activity and have thought about why transformations behave the way they do, they are ready to play the transformation game. The game is designed to be played with partners, but the first time the class plays the game I often use teams. I have also used this game with classes of different ability levels by changing the functions being transformed. Quadratic functions are perhaps the easiest for students and inverse trigonometric functions are possibly the most difficult for them. Once they are acclimated to the game, *Transformation!* is a nice activity to use on those days when one needs a meaningful activity but has a limited amount of time. The advantage of having a class play the game a few times is that they will develop the ability to describe the behavior of functions without having to refer to the calculator.

4. Suggested timeline

Two class periods:

1. First class period:
   a) Have students work in groups on the calculator lab experiment.
   b) Discuss the rules of *Transformation!*
   c) Distribute a copy of the game to each student, demonstrate two examples, and allow them to finish the sheet individually for homework.
2. Second class period:
   a) Go over the homework and any questions.
   b) Have students select an opponent and play the game.

5. Activity sheets

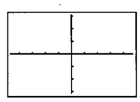
The activity sheets that follow include a calculator laboratory experiment and samples of the transformation game applied to three typical functions.

5.1 Calculator laboratory experiment

Name(s)________________

Understanding the Transformation of Functions

Directions: Complete the following experiments with your partner. Set up your calculator as follows:



1. Set $y1 = f\text{Part}(x)$ (In the MATH NUM menu).
2. Put into Dot mode.
3. Set the window to Zoom Decimal.
4. Sketch the graph in the window at the right:
5. *Predict* what the following would look like, then check on your calculator:

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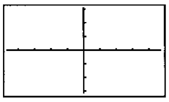
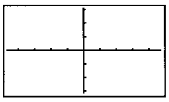
[1]Paper presented to the 8th International Conference on Geometry, Nahsholim (Israel), March 7–14, 1999

a) Sketch $y1 = f\text{Part}(x) + 2$ b) Sketch $y1 = f\text{Part}(x) - 1$

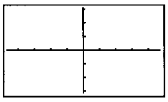
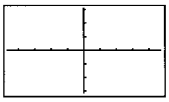



Describe *how* the graph of $y = f\text{Part}(x) + b$ differs from the graph of $y = f\text{Part}(x)$ [compare to the sketch you drew in #4 above]:

Explain *why* you think this is true:

6. *Predict* what the following would look like, then check on your calculator:

a) Sketch $y1 = \text{fPart}(x + 2)$ b) Sketch $y1 = \text{fPart}(x - 1)$

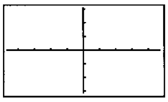
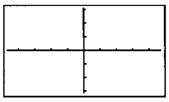



Describe *how* the graph of $y = \text{fPart}(x + a)$ differs from the graph of $y = \text{fPart}(x)$ [see #4 above]:

Explain *why* you think this is true:

7. *Predict* what the following would look like, then check on your calculator:

a) Sketch $y1 = 3\text{fPart}(x)$ b) Sketch $y1 = \frac{1}{2}\text{fPart}(x)$

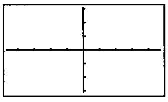
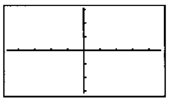



Describe *how* the graph of $y = d\text{fPart}(x)$ differs from the graph of $y = \text{fPart}(x)$ [see #4 again]:

Explain *why* you think this is true:

8. *Predict* what the following would look like, then check on your calculator:

a) Sketch $y1 = \text{fPart}(3x)$ b) Sketch $y1 = \text{fPart}(\frac{1}{2}x)$

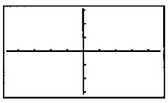
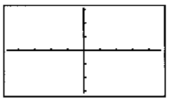



Describe *how* the graph of $y = \text{fPart}(cx)$ differs from the g of $y = \text{fPart}(x)$ [see #4 again]:

Explain *why* you think this is true:

9. *Predict* what the following would look like, then chec your calculator:

a) Sketch $y1 = -\text{fPart}(x)$ b) Sketch $y1 = \text{fPart}(-x)$

Describe *how* the graph of $y = -\text{fPart}(x)$ and $y = \text{fPart}(-x)$ differs from the graph of $y = \text{fPart}(x)$:

Explain *why* you think this is true:

(Caution: Though the graphs of these two may look identical, the procedures are different. For example, consider y1 = int(x), y1 = − int(x) and y1 = int(−x) from the MATH NUM menu. Graphing these should help you explain *how* these two transformations differ.)

5.2 The transformation game (Trigonometric functions)

	Function	Horizontal	Vertical
3	$y = \sin x$	Stretch by 3	Stretch by 3
2	$y = \cos x$	Stretch by 2	Stretch by 2
1	$y = \tan x$	Left $\pi/2$	Up 1 unit
0	Your Choice	Flip over y-axis	Flip over x-axis
-1	$y = \tan x$	Right $\pi/2$	Down 1 unit
-2	$y = \cos x$	Shrink by 2	Shrink by 2
-3	$y = \sin x$	Shrink by 3	Shrink by 3

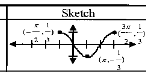
Procedures

0. "Roll dice" by using int(rand(7)−3) on the calculator. Opponents both roll dice. The high scorer goes first; this person is called the "grapher."
1. The grapher rolls the die again to determine which of the functions will be used in the current round, and both contestants record this on the sheet in the first column.
2. The grapher then rolls the die again to determine the horizontal transformation; both contestants record this on the sheet in the second column.
3. The grapher then rolls the die to determine the vertical transformation; both contestants record this on the sheet in the third column.
4. Both contestants must independently determine the equation and the sketch of the transformed function, indicating important points. This must be done WITHOUT the aid of the graphing calculator.
5. Once both contestants have completed the transformation, the answer should be checked on the graphing calculator.
6. If the grapher is correct, the grapher wins. If not, the opponent's answer is checked. If the opponent is also incorrect, no points are awarded. If there is a winner, the die is rolled again and the ABSOLUTE value of the roll is awarded to the winner and should be recorded in the last column. (A contestant who rolls a zero may roll again.)
7. Contestants now exchange roles and continue, starting from step 1. Play alternates until time is called.

Name ____________________ Points _______

Opponent __________________ Points _______

(Record each roll and required information as shown in the example below.)

f(x)	H	V	Transformation	Sketch	Points
(2) y=cosx	(1) Left $\frac{\pi}{2}$	(-3) Shrink by 3	$y = \frac{1}{3}\cos(x + \frac{\pi}{2})$		(-1) 1

⋮

5.3 The transformation game (Floor, ceiling, and absolute value functions)

	Function	Horizontal	Vertical
3	$y = \lfloor x \rfloor$	Right 3	Up 3
2	$y = \lvert x \rvert$	Stretch by 2	Stretch by 2
1	$y = \lceil x \rceil$	Stretch by 2	Stretch by 2
0	Your Choice	Flip over y-axis	Flip over x-axis
-1	$y = \lceil x \rceil$	Shrink by 3	Shrink by 3
-2	$y = \lvert x \rvert$	Shrink by 2	Shrink by 2
-3	$y = \sin x$	Left 3	Down 3

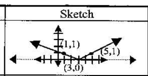
Procedures

0. "Roll dice" by using int(rand(7)−3) on the calculator. Opponents both roll dice. The high scorer goes first; this person is called the "grapher."
1. The grapher rolls the die again to determine which of the functions will be used in the current round, and both contestants record this on the sheet in the first column.
2. The grapher then rolls the die again to determine the horizontal transformation; both contestants record this on the sheet in the second column.
3. The grapher then rolls the die to determine the vertical transformation; both contestants record this on the sheet in the third column.
4. Both contestants must independently determine the equation and the sketch of the transformed function, indicating important points. This must be done WITHOUT the aid of the graphing calculator.
5. Once both contestants have completed the transformation, the answer should be checked on the graphing calculator.
6. If the grapher is correct, the grapher wins. If not, the opponent's answer is checked. If the opponent is also incorrect, no points are awarded. If there is a winner, the die is rolled again and the ABSOLUTE value of the roll is awarded to the winner and should be recorded in the last column. (A contestant who rolls a zero may roll again.)
7. Contestants now exchange roles and continue, starting from step 1. Play alternates until time is called.

Name ____________________ Points _______

Opponent _________________ Points _______

(Record each roll and required information as shown in the example below.)

f(x)	H	V	Transformation	Sketch	Points
(2) y = |x|	(3) Right 3	(-2) Shrink by 2	$y = \frac{1}{2}\lvert x-3\rvert$		(3) 3

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5.4 The transformation game (Exponential and logarithmic functions)

	Function	Horizontal	Vertical
3	$y = 10^x$	Right 3	Up 3
2	$y = 2^x$	Stretch by 2	Stretch by 2
1	$y = x^e$	No Change	No Change
0	Your Choice	Roll Again	Roll Again
-1	$y = \ln(x)$	Flip over y-axis	Flip over x-axis
-2	$y = \log_2(x)$	Shrink by 2	Shrink by 2
-3	$y = \log x$	Left 3	Down 3

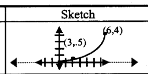
Procedures

0. "Roll dice" by using int(rand(7)−3) on the calculator. Opponents both roll dice. The high scorer goes first, this person is called the "grapher."
1. The grapher rolls the die again to determine which of the functions will be used in the current round, and both contestants record this on the sheet in the first column.
2. The grapher then rolls the die again to determine the horizontal transformation; both contestants record this on the sheet in the second column.
3. The grapher then rolls the die to determine the vertical transformation; both contestants record this on the sheet in the third column.
4. Both contestants must independently determine the equation and the sketch of the transformed function, indicating important points. This must be done WITHOUT the aid of the graphing calculator.
5. Once both contestants have completed the transformation, the answer should be checked on the graphing calculator.
6. If the grapher is correct, the grapher wins. If not, the opponent's answer is checked. If the opponent is also incorrect, no points are awarded. If there is a winner, the die is rolled again and the ABSOLUTE value of the roll is awarded to the winner and should be recorded in the last column. (A contestant who rolls a zero may roll again.)
7. Contestants now exchange roles and continue, starting from step 1. Play alternates until time is called.

Name ____________________ Points _______

Opponent _________________ Points _______

(Record each roll and required information as shown in the example below.)

f(x)	H	V	Transformation	Sketch	Points
(2) $y=2^x$	(3) Right 3	(-2) Shrink by 2	$y = \frac{1}{2}2^{(x-3)}$		(-2) 2

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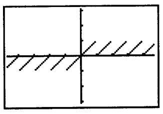
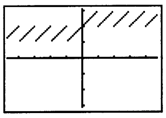
6. Solutions to the calculator lab experiment

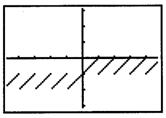
4. Technically, circles should be shown for open endpoints and the dots are connected, but students will probably not include these in their diagrams:



5. a)

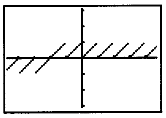


b)

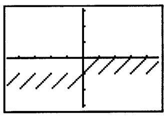


The graph shifts vertically b units.

6. a)

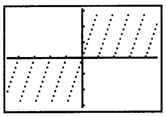


b)

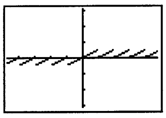


The graph shifts horizontally -a units.

7. a)

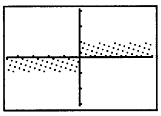


b)

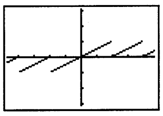


The graph stretches vertically by a factor of d.

8. a)

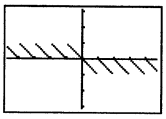


b)



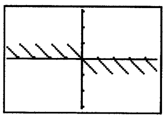
The graph shrinks horizontally by a factor of c.

9. a)



Flips the graph over the x-axis.

b)



Flips the graph over the y-axis.

Author
Camp, Dane R., Dr., 585 Summerdale Avenue, Glen Ellyn, IL 60137, USA. E-mail: DRCamp@aol.com

ATCM99

Applications of Technology in Mathematics Research and Teaching for the 21st Century

Guangzhou (P.R. China), December 17–21, 1999

ATCM99 will provide an interdisciplinary forum for researchers and teachers in education, mathematics and mathematical sciences, together with researchers and developers of computer technology, to present their results in using computer technology and exchange ideas and information in their latest development. The conference will cover a broad range of topics on the relevance of technology in mathematical research and teaching. These include but are not limited to:
– computer-aided teaching in mathematical sciences
– computer algebra (systems) in research and teaching
– multimedia and distance learning
– graphing calculators
– mathematical research and teaching using technology
– numerical analysis.

The conference will consist of plenary sessions by invited speakers and a special session (by G.T. Springer, Australia), parallel sessions by contributed papers, and tutorial sessions on software and hardware relevant to mathematical research and teaching. Books, software and hardware may also be on display.

Plenary speakers will be: Bruno BUCHBERGER (Austria), CHUAN Jen-chung (Taiwan), Ed CONNERS (USA), Keith O. GEDDES (Canada), LEE Peng Yee (Singapore), Katuhiko SHIMIZU (Japan), Bert WAITS (USA), WU Wen-tsün (China).

For further information please contact:
Prof. WU Wen-tsün
ATCM99
Guangzhou Normal University
Guangzhou 510400, P.R. China
Fax: +86 20 86234160 or +86 20 86234992
E-mail: atcm@runet.edu or atcm99@guangztc.edu.cn
http://www.cs.runet.edu/∼atcm/ATCM99/