FORMING TEACHERS AS RESONANCE MEDIATORS

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Assumptions about knowledge construction, knowledge transmission and the nature of mathematics always underlie any teaching practice even if often unconsciously. In the paper we explain our theoretical assumptions about these cognitive and epistemological issues and derive from them a "model" of teacher. Finally we discuss why and how participation in a modelization process can constitute a suitable strategy for disciplinary and professional training of future teachers conforming to the model.

INTRODUCTION

In this paper we intend to bring into discussion a handful of theoretical remarks that we see as meaningful and relevant for the actual development of research on the crucial problem of teachers training in mathematics areas[1]. Our focus will be in particular on the search for a reasonably satisfactory model of cognitive dynamics, adjusted to non-specialistic knowledge levels: in fact, such a model plays a key role in both teachers formation and the teaching process. In planning, handling and evaluating teachers formation paths we recognize, very schematically, at least four basic "model-ingredients"[2]: i) a realistic, even rough, model of "natural" cognitive dynamics ii) a global, epistemologically founded view of mathematics as an internally structured scientific discipline; iii) a modulated view of the variety of interferences of mathematical thinking with other cultural fields (mainly scientific and technological ones), and with everyday culture(s); iv) a pragmatically successful, even rough model of cultural transmission in knowledge areas, in particular scientific ones. Such ingredients, obviously crucial to teaching profession, are obviously correlated to each other: in particular it is clear the basic framing role assumed by i) with respect to other aspects.

A distinctive feature of our cognitive modelling is the core relevance of basic resonance dynamics[3] assumed to work at the root of all the modulations (from

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[1] The actuality of this debate is witnessed for example by the argument of the forthcoming ICMI study 15 (ICMI, 2004).

[2] But see also for example (Malara, 2003), where other influential variables like *emotion* and *belief* are put in evidence.

[3] We borrow the word *resonance* as a Physics metaphor: and in this sense resonance is actually much more than consonance. In his/her proposals, each teacher in fact will reasonably take into account the consonance of mathematical tools (equations, graphs, examples, an so on), employed in a particular context, in such a way that their notes are not played as *dissonant* with each other, and in reference to reality and thought structures. But when a complex interaction is driven by resonance dynamics, this implies that different

perception to abstract thinking) and interferences characterizing the knowledge of an individual. In our view, in fact, such dynamics always play the central role on the frontier of the progressive adjustment/fit between different, ever-present dimensions: the actual potentialities of individually developing cognitive structures, the framing patterns supplied by implicitly as well as explicitly codified cultures, the constraints of physical (at large) reality. The aim of the present work is therefore a twofold one: a) to discuss about the productivity of looking at the teacher's main role as one of resonance inducing mediation, on both planes of understanding and of motivation to understanding; b) to point out that critical awareness and a responsible assumption of such a role can actually be developed and supported by suitable teachers formation strategies, in turn resonance-exploiting vs individual learning experiences as well as resonance-emphasizing vs cooperative professional formation.

The next sections are organized according to the following path. After presenting some general theoretical references, we illustrate our "model" of the understanding process. From this we derive the necessity of characterizing the role of teachers as resonance mediators. Then we discuss some aspects of teachers formation strategy finalized to this purpose. Finally an example of teaching practices/events will be presented, in order to illustrate, as a conclusion, the potential impact and possible outcomes of our assumptions.

OUTLINE OF A GENERAL THEORETICAL FRAMEWORK

Before stressing in the next section some relevant "resonance features" characterizing our cognitive modelling, we feel it necessary to very briefly state its location/rooting within the complex landscape of the cognitive theories and interpretations available nowadays. In particular we would like to draw attention to the fact that many critical aspects of cognition have been variously noticed and variously entangled along times within different (often reciprocally contrasting) cognitive theories and/or epistemological positions. Actually, our basic research finding is that most of such aspects appear relevant in interpreting experimental teaching/learning evidences: and this directly implies their reciprocal complementarity. For example:

It is now quite common to refer to Vygotskij's views about the crucial role assumed by natural language in mediating natural culture, and viceversa, since the earliest ages. However such a mediation is only in a minor part an automatic, passive one: careful observation of cognitive transactions shows that strong resonance/ dissonance effects always take place on the fuzzy background of "implicit acculturation", and that an early, careful, active adult mediation plays a key role in fostering resonances and preventing dissonances ("misconceptions" appear at most as the result of missing/wrong/misleading mediations between developing cognition and culture).

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elements (in cognition: an idea, a mental construct, an image, an action, etc.) are simultaneously activated whenever one is evoked, producing by reciprocal interference a mutual reinforcement effect. Resonance dynamics are obviously at work also along the paths of cultural evolution at large – but this is an even more complex subject.

Apart from the "stages" machinery, some insights by Piaget appear to be crucial to outline features of cognitive dynamics. Assimilation, accommodation, (temporary) equilibration… lively define the main modes of any resonance process, adjusting to each other partially mismatching external and internal terms, and recognizing as such the resulting reciprocal fit. Something similar, though in a "divergent" modality, correlates the dynamics of the "physical abstraction" to the ones of the "reflecting abstraction", ending up with separate though entangled scientific and formal models. Cognitive activity is, exactly, an activity – structured according to possible, effective, meaningful … internal actions, monitored by internal "convergence sense" and counterpointed by external action and discourse.

There is evidently no sense at all to counter Vygotskij's views to Piaget's: both their ways-to-look-at cognition account in fact for crucial aspects of what actually happens. The point is to correlate such views within a comprehensive dynamical model: and the resonance dynamics frame actually lends itself to account for many crucial correlation aspects. Something very similar can be said about most of the presently debated views about mathematically relevant cognitive structures: from the "embodied cognition" ones (Lakoff & Núñez, 2000), naturally referring to some neurocognitive studies (Changeux, 2000), (Dehaene, 1997), to the "language referred" (Sfard, 2000) or more generally "semiotic" ones (Radford, 2000), to the "information processing" ones … and so on. It is evident from our research experience, and it can be shown by careful analysis of learning experience, that all these ingredients appear to be crucial in some respects within resonantly converging dynamics of meaningful learning.

LEARNING THROUGH UNDERSTANDING

Our basic theoretical assumption about "learning through understanding" has been developed and refined, within the above wide theoretical framework, managing multi-year, multi-classroom action-research projects mainly devoted to a coherent reorganization of both school teaching and formation activity for pre-elementary and elementary in-service teachers in science-mathematics area[4]. We briefly synthesize as follows the main points of our model of cognitive dynamics and knowledge transmission[5]:

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[4] We are mainly referring to the long-term Italian project *Capire si può* (*It is possible to understand*) partially documented in http://www5.indire.it:8080/set/capire_per_modelli/capire.htm

[5] Correspondent views of mathematics as discipline and as educational task are implicit in our model, too. The prevailing view of mathematics today, pervading most curricula, and in particular university curricula for future teachers, stressing its **apriori separateness** from other scientific areas, conflicts with natural cognitive processes, and appears to be at the origin of many students' difficulties. On the contrary, if mathematics is conceived as an **aposteriori abstraction** coming for example through a modelization process, its "cognitive" resonance stimulates students motivated interest toward its structural development and allows them to reach quite high levels of formalization.

a) Understanding, as different from learning, and motivation, as different from acceptance, are strictly correlated, for pupils as well as for teachers: based on feelings and feedback of competence in dealing with increasingly complex situations.

b) Learning through understanding is the result of a process of resonance between individual cognition, social culture and reality structures, along cognitive paths efficiently addressed and controlled in their meaning-driven dynamics. It requires, at any level, also resonance between various "dimensions" of *natural thinking* (Guidoni, 1985): perception, language, action, representation, planning, interpretation, etc.

c) Learning through understanding requires long-term, longitudinal along years, and wide-range, transversal across disciplines, processes, mediated and supported by a simultaneous development of language, with increasing awareness of usages and functions of all its components (syntax, semantics, pragmatics, semiotics).

Two kinds of activities appear as critical, self-developing keys for both teachers formation and pupils learning: modelization processes from everyday experience contexts and word problems.

As noticed in (Verschaffel, 2002), in mathematics education literature word problems have been used in many ways and with several different goals, dating back from the classical proposals by Pólya (Pólya, 1962). The same applies to modelization, where, moreover, the meanings themselves assigned to the word "modelization" considerably vary. Therefore, it is necessary to begin with explaining what "modelization" means for us (contrasting for example Verschaffel's definition), (although the example in the last section will better clarify our own point of view): we interpret it as a very complex, neither deterministic nor a one-way process where the formal structures are seen as one of the different correlated ways into which the cognitive reconstruction of external world structures take form. In other words what really counts is not a standard hierarchy of multi-representations (actions, words, graphs, and so on) whose top is identifiable by the algebraic formulation of a physical law: due to the subtended cognitive dynamics, what is most effective is a *continuous* (quasi subliminal, in expert situations) *shifting from one cognitive dimension to another in a mutual progressive enhancement.*

As for word problems, for reasons of space we will limit ourselves to the restrictions which make them appropriate for our goal (for further details see Guidoni *et al*., 2003; Tortora, 2001): they ought to be problems which could be tackled in parallel through the use of symbolic tools and direct action, led in such a way as to illustrate that the solution process converges if sustained by a multiplicity of representations (in particular graphic: see Guidoni *et al*., 2005) brought along by the individual cognitive dynamics themselves, and thus, reciprocally enlightening. The final task is extracting their structures which requires a double "variation of the subject": one is the varying of the numerical data in the same context; and one is facing "isomorphic" situations, recognizing them as such (for an analogous detailed analysis see Mason, 2001).

Therefore, it is possible to understand, and the attention devoted to the mechanics of understanding takes priority over the mechanics involved in non- understanding. However, on the condition that adult mediation be sufficiently flexible and incisive in order to generate the conditions which set off and fuel the comprehension process: assigning a crucial role to teaching mediation as *resonance inducing*.

TEACHER'S ROLE AND ITS IMPLEMENTATION

"*Pick them up where they are, then find a path which guides them to the place you want them to reach*". According to this famous Wittgenstein's mot, a teacher must manage, among other things, specific skills. This implies defining the space of cognitive configurations (multi-dimensional) and, based on available resources, the designing of possible learning trajectory paths. Overall to adopt teaching strategies that are progressive, coherent, not imposing but supportive of potentialities. The teacher should create, on a local level, the many possible links between individual cognition, social culture and reality structure through the use of dynamics of abstraction and de-abstraction (modelling and de-modelling) with coherence, flexibility and competence.

In working with in-service teachers (as in above mentioned projects – see footnote 4), since teachers and students are simultaneously involved, the critical awareness and the assumption of the teacher role are supported by the immediate and long term interaction with the learners cognitive processes. For the teachers in training this important support is missing, making the development of the above mentioned skills ever more complex. However, five years of research on our part in the formation of elementary school teachers in training, based on conceptual paths and didactic strategies gradually validated, have convinced us that in each case the guided collective participation in modelization or problem solving processes makes up a privileged entrance into the world of the combined acquisition of knowledge and professionalism. Here, we limit ourselves to illustrating through a single example, underlining some aspects which systematically emerge in the modelization process, the way in which the theoretical hypothesis of resonance mediation in cultural transmission is effective when put into practice in the classroom. It is important to highlight that, in this as in other work contexts experimented in different environments, the underlying cognitive dynamics put into play are very similar between both working in-service teachers and pre-service teachers in training, and in substance correspond to what takes place in class; likewise the crucial role played by a meta-cognitive attitude is analogous both on an individual and group scale. We have also noticed that it is important in all situations to alternate auto-directed work of manipulation and interpretation either individually or in small groups (including substantial homework) with collective guided work of comparison and analysis of partial results, yet leaving to the individual the final systemizing of results and interpretation of the processes being adopted.

The choice of the context to be explored is always addressed by some conditions. It is in fact important to deeply commit to the task both conscious perception/action, and the construction of elementary logical relationships to support in integrated way the complexity of the experience. In other words, the context must lend itself to approaches characterized, since the beginning of the cognitive path, by direct manipulations guided by reflection on what is being observed: a context at the same time complex enough to demand a careful, previous individuation of interacting systems as of pertinent variables, and simple enough to allow for an exploration not too rigidly guided.

AN EXAMPLE AND SOME CONCLUDING REMARKS

According to our experience, **springiness** can be a good example of a prototypical modelling context. As a matter of fact, the great pervasiveness of this family of phenomenologies within everyday experience is even marked by the corresponding structures of the natural language (in Italian, the adjective "elastic" is commonly used as a noun); while "what happens" to a stressed elastic object, together with the basic rules of such "happening", is first understood since childhood on the basis of direct bodily reference. As a consequence, a lot of pertinent thought-action-wording-representation aspects are actually available as crucial, interfering dimensions (not steps!) to support development and structuring of both phenomenological and formal competences, on the basis of an explicit restructuring of "what one already knows". In particular, it is important to remind some important aspects (not steps!) of the basic cognitive path: a rich, explicit, qualitative analysis of the behavioural patterns of different springing objects is always necessary: both to support more and more sophisticated modelling, and to correlate modelling itself to less and less evident springiness phenomena and features; once a general pattern of "forcing" is reasonably well controlled, and well represented by natural language, correlations between configuration variables, and between variables and systemic parameters, can be explored: first by "order relations and correlations" (the more… the less…; the more… the more…; the less… the less…; etc.); then by actual measuring what can be directly or indirectly measured, in properly arbitrary units. Always under the control of natural language, aspects of the explored situations can then be represented by qualitative line drawings, then by tables of number-pairs, gradually allowing also for (cautious) prediction of new facts.

As always, meaning really emerges from "*what is unchanged across change*" (Plato). To make cognitively explicit that here what does not change across systems and situations is a "form" of the relationship among pertinent variables and parameters, it is crucial that the relationship itself becomes explicitly and coherently multi-represented: by careful words; by symbols, standing for variables and parameters and their relationships; finally, by systematic use of the Cartesian plane. And the resonant interference of different representation features appears to allow for effectiveness and stability of the understanding.

Here is an excerpt from a "final" account of three pre-service teachers working with different rubber springs, differently configured in different stretching situations. From the initial "confusion" of random testing, a clear "rationalization" of observed patterns starts to emerge: not as a passive check within apriori imposed schemes and procedures, but as an active "reducing to stable, workable, schematic order" of the world observable variety.

"By comparing the stretching of single, serial and parallel rubber springs, we have observed, in spite of the inaccuracy of data, a piece of straight line in the central region of the broken lines. From this we have inferred that springs behave the same way in all three cases: a constant, direct proportionality shows up between the number of coins (or more in general the weight of the objects utilized) and the actual stretching. Let's represent what happens when $y=x/10$, where y corresponds to F (the force is the weight of the coins), x is $l-l_0$, and 1/10 corresponds to k… In conclusion, we cannot rely upon experimental data alone: the proportionality between the two variables can be seen when the error margin is small enough, and anyhow in the central part of measurements (when weight is neither too small nor too large)".

This way, one is giving sense at the same time to the linear function and to the physical law; to the intuition that the proportionality between force and stretch is but a part of a more complex behaviour, in both physical and formal terms; to the generalization of a "partially linear" behaviour as a powerful key of first interpretation; and so on. The (suggested) comparison/superposition of several graphs by overhead projection has then allowed to see, in the central part of the graphs, a real "linear cord" stemming from the merging of about twenty "broken lines": this simple contrivance incisively (resonantly) emphasizes both the abstraction process leading to the linear function, and all the "impediments" (in Galileo's words) which, from different origins, interpose themselves between the formal scheme, the reality of the spring and the practice of measurement.

"$F=k(l-l_0)$. If I trust the formula, I will look for a straight line at any cost, trying even to transform a broken line in a straight one. On one side we have mathematics, on the other side we have phenomena: how putting them together? The mathematical straight line is an abstraction. The spring of physicists cannot be identified with our elastic band, it looks like a straight line more than our graphical "cord" does. To understand, I've used both things: mathematics and phenomena. If I don't know the law, all springs are different, and I cannot identify the small region of their common behaviour. What is a spring? We succeeded in defining by words a real spring, the physicists' spring has its origin in the real ones, but is represented by a mathematical formula: $F=k(l-l_0)$".

From this example it is also emerging the meaning we attribute, within a reciprocally resonant convergency between phenomenology and formalism, to a cognitive process of "de-constructive de-modelling" seen as parallel and interfering to the one more commonly evoked of "constructive modelling". A continuous back-and-forth between what is "said" by facts and by their symbolization appears to be crucial. In particular we feel it important that contributions quite different in their physical origin, whose superposition determines phenomenological observations and measurements (intrinsic not-linearity of the deformation function, within its power expansion; deformations at the boundaries of the observed interval resulting from physical behaviour and measurement processes; measuring inaccuracies and errors; etc.) are not acritically merged and confused below the cover of an all-purpose "formalization", at this point very poorly significant in its not transparent stiffness. And it is evident that along a critical, guided sharing and comparison of the research results new phenomenological doubts can emerge, together with linguistic and formal discrepancies and further cultural needs: altogether defining the action space for further systematization levels.

A question is still open: how many of our students shall assume in their teachers life the proposed role, and how many shall turn back to old, reassuring models?

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