DEVELOPMENT OF MATHEMATICAL NORMS IN AN EIGHTH-GRADE JAPANESE CLASSROOM

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Mathematical norms are important cultural knowledge of mathematical activities. This paper reports an analysis of mathematical norms in ten consecutive lessons taught by an eighth-grade Japanese teacher. The lessons were located in the unit of simultaneous linear equations. The videotapes of the lessons, their transcripts, and the interview data were analyzed qualitatively. Several major mathematical norms were found in the lessons. The teacher's deliberate strategies to develop them were identified: using students' work, making a comparison, and being considerate of those students who did not follow a norm. Complexities of research on mathematical norm are discussed.

INTRODUCTION

This paper reports an analysis of the ten consecutive lessons taught by one of the eighth-grade Japanese teachers who participated in the *Learner's Perspective Study* (*LPS*), an international research project coordinated by David Clarke (see Clarke, 2004). The analysis focuses on mathematical norms introduced by the teacher.

As Clarke (2004) points out, one of the goals of *LPS* is to complement TIMSS 1999 Video Study. In Hiebert et al. (2003), the analysis of mathematics teaching focused on mathematical knowledge, procedures, and reasoning involved in the problems presented in the lessons. Teaching of mathematical norms was beyond their analysis. Though the mathematical norms are often not explicitly taught by teachers nor written in textbooks, they are crucial when the learning process of mathematics is conceived as mathematical activities.

Mathematical norms are knowledge "about" doing mathematics; therefore, they belong to the domain of metaknowledge in mathematics. It is hypothesized that beginning teachers are often occupied with covering curriculum content, paying their attention to mathematical knowledge and skills: Competent teachers as selected in *LPS* by design would invest more time and effort in teaching of metaknowledge. The major questions guided this analysis of Japanese data are, What mathematical norms would surface in the lessons? How would the teacher introduce, negotiate or establish those norms during the lessons? In the future those questions will be investigated in *LPS*'s lesson data of other countries, too.

THEORETICAL FRAMEWORK

Scientists search for patterns, regularities, rules, or laws in the real world, and try to build causal theories, so as to be able to explain the phenomena in which they are interested; Understanding of causal relationships is useful for prediction and control.

Social sciences, likewise, study those patterns, norms, regularities, rules, or laws appearing in human activities (cultures), so that they can explain and understand human activities. Positivist sociologies are known to have assumed a "normative" conception of human action: It has three main components, "actors," "rules," and "situations," and presumes that "actors know and follow rules in social situations" (Mehan & Wood, 1975, p. 74). This conception closely parallels that of natural phenomenon: "Physical objects follow natural laws in the physical world."

Ethnomethodologists had also studied people's "rule" use in social situations, but they made strong attacks to the positivist's normative conception. They claimed that actors, rules, and situations were mutually shaped in practice, in their terminology, "reflexively" related to each other (Mehan & Wood, 1975, pp. 75-76).

Cobb and his colleagues (Yackel & Cobb, 1996; McClain & Cobb, 2001) introduced the notion of "norms" of classroom process as a device to interpret classroom processes and clarify how children's beliefs and values develop. They identified several classroom social norms working in their project classroom, such as "Students were obliged to explain and justify their reasoning."

They also pointed out that there were norms specific to mathematics in the classroom, which they called "sociomathematical" norms. By using the prefix "socio-" they seem to be trying to stress that norms of mathematical activities depend on the community (Yackel & Cobb, 1996, p. 461). They contend that the mathematical activity has norms as constituent, and that norms are reflexively related to beliefs and values of mathematical activities.

In this paper I will use a simpler word "mathematical" norm to refer to a norm in the mathematical activity, rather than "sociomathemtical" norm. This is because I consider that mathematics is intrinsically sociocultural activity as current philosophies of mathematics and sociocultural theories inform. The prefix "socio-" of the sociomathematical norm is redundant as far as we accept this understanding.

For the framework of Yackel & Cobb (1996), the notion of sociomathematical norm appears to hold a central position of classroom mathematical activity:

> These sociomathematical norms are intrinsic aspects of the classroom's mathematical microculture. Nevertheless, although they are specific to mathematics, they cut across areas of mathematical content by dealing with mathematical qualities of solutions, such as their similarities and differences, sophistication, and efficiency. Additionally, they encompass ways of judging what counts as an acceptable mathematical explanation. (Yackel & Cobb, 1996, p. 474)

However, this strong emphasis on norms has a danger of leading to the positivist's normative conception. The symbolic interactionism and ethnomethodology do not put norms on the central place in explaining social conduct:

> Rather than the major criterion people employ to regulate their own and other's conduct, social norms are one of several forms of knowledge that people employ in their everyday conduct. ... It should not be thought, however, that norms are constantly implicated in

acts, nor that people behave by finding the appropriate norms that govern each and every social situation. (Hewitt, 1994, p.160)

We should avoid the tendency to explain classroom processes with too much emphasis on abstract norms. Rather, as Waschescio (1998, p. 235) also pointed out, norms should be understood as cultural "tools," which may or may not enhance mathematical activities.

RESEARCH PROCESSES

Unlike TIMSS 1999 Video Study, in *LPS* project, eighth-grade teachers were not randomly selected. Only those who were considered "competent" by local educators were selected. In addition, for each teacher, ten consecutive lessons were videotaped by three cameras (teacher camera, student camera, whole class camera). Students were interviewed by the stimulated-recall method using videotapes of the lessons.

This paper analyzes one Japanese teacher's ten consecutive lessons that were located in the unit of simultaneous linear equations, covering the linear combination method ("addition or subtraction method"), the substitution method, and part of application problems. The videotapes of the lessons, their transcripts, and the interview data were analyzed qualitatively. To let mathematical norms emerge from the data, any piece of the data that appeared to indicate beliefs on how to work on mathematics was coded, and the normative aspects behind those beliefs were repeatedly analyzed.

The eighth-grade students had experienced huge amount of mathematical activities since entering schools. They must have been equipped with many mathematical norms, some of which would have been working when they participated in this research. This report analyzed only the ones that the teacher emphasized during the lessons because I was interested in how the teacher introduced or developed mathematical norms in the classroom.

MATHEMATICAL NORMS IN THE CLASSROOM

Norm 1: Efficiency

The value of pursuing efficient ways of solving problems is generally shared among mathematicians. Many theories, theorems, and formulae in mathematics have been produced to improve efficiency. In this class also, the teacher encouraged the students to pursue efficient ways of solving simultaneous equations.

In the first lesson (L1), the class discussed a simultaneous equations: $5x + 2y = 9$…(1)□$-5x + 3y = 1$ …(2). The teacher asked a student KORI write his solution on the board. He subtracted (2) from (1), obtaining $10x - y = 8$. Solving it for x, he put it into (1), obtaining the value of y. Finally, he put the value of y into (1), and got the value of x. After KORI explained his solution to the class, the teacher asked to the class: "OK, any question? Can you understand? Well, do you have any thoughts as work out this question? Any impressions of this explanation?" (L1 10'54")[This notation indicates that this talk occurred in 10 min. and 54 sec. from the start of L1].

A student SUZU responded to it: "I think there is much simpler one." SUZU wrote his solution on the board: He added (1) and (2), and got an equation without variable x. And he solved it for y, and got the value of y. He then put it into one of the given equations, and got the value of x.

The students were then seeing two different solutions on the board. The teacher explained the reason why he asked KORI to write his solution on the board. The teacher intentionally chose KORI because he had observed at the previous lesson that KORI had solved the problem differently from the other students: "Almost, actually almost students have this opinion that I saw the class that we did yesterday. And in fact, the way which KORI did was different so that I wanted them to write on the blackboard" (15'27").

The teacher thought that by comparing solutions with different degrees of difficulties, students would be able to appreciate an easier one well: "I think you can know which point was difficult as you compare the difficult way and the easier one" (15'48"). Finally, the teacher concluded that SUZU's solution was easier and better than KORI:

> Now, actually that way is much better than this way, when we compare the calculations so far. As a result, it is better to notice that this way, which SUZU wrote, is better, you know? (16'24")

He asked the students where they thought KORI's solution was more complicated than SUZU. This question tried to elaborate inefficiency of KORI's solution.

Up to this point, the teacher seems to be putting more value on efficient solutions. The students seem to be encouraged pursuing as efficient solutions as they can. KORI's solution seems to be devalued. This does not mean that inefficient solutions are useless, however. First, the teacher soon pointed out that KORI's method gave the same result as SUZU. Second, he suggested that KORI's method contained an important idea: "There are some important ideas in this [KORI's] process, I think" (19'21"), which I discuss next.

Norm 2: Even inefficient attempts could contain important ideas

Efficiency is not the only value in pursuing mathematics. New ideas for developing new ways of solving problems are equally important in mathematics. Those could be discovered through numerous inefficient, or failed attempts as the history of mathematics shows. In this class, the teacher once gave an opportunity for the whole class to appreciate an important idea found in an "inefficient" solution.

In L1 the teacher pursued "KORI's idea," and went into the idea of the substitution method, which was formally introduced at L7. This pursuit continued well into the next lesson L2. Thus, he seems to believe that *even inefficient attempts could contain important ideas.*

In addition to this normative action, the teacher paid respect and care to both solutions. Devaluing one's idea may hurt his or her feeling. When KORI received negative opinions to his solution, the teacher encouraged KORI: “It’s OK. Don’t be depressed as it didn’t go well. It is better to get some comments, right? Don’t worry”(L1 13’42”). By pursuing KORI’s idea with the whole class, the teacher showed further care to the student whose idea had been devalued.

Norm 3: In mathematics you cannot write what you have not shown to be true yet

Mathematics is traditionally written in the deductive way: It must begin with axioms, definitions, or already proved theorems, and proceed logically. Therefore, you cannot write what you have not shown to be true yet. This norm is emphasized especially in the teaching of proof in geometry in Japan.

In L3, the teacher reviewed the solution of a simultaneous equation: $3x + 2y = 23$, $5x + 2y = 29$. As homework, he had asked the students to do checking of the solution. First, he asked UCHI to put up his work on the board (Figure 1). As a “different way,” he then asked KIZU to put up his work on the board (Figure 2).

By putting $x = 3$, $y = 7$ into $3x + 2y = 23$ and $5x + 2y = 29$

3X3 □ 2X7 = 23   5X3 □ 2X7 = 29
9 □ 14 = 23   15 + 14 = 29
23 = 23   29 = 29

Figure 1: UCHI’s writing on the board.

By putting $x = 3$, $y = 7$ into $3x + 2y = 23$ By putting $x = 3$, $y = 7$ into $5x + 2y = 29$

3x3 + 2x7   5x3 + 2x7
= 9 + 14 = 23   = 15 + 14 = 29

Figure 2: KIZU’s writing on the board.

The teacher posed the class a question what differences they noticed between them. The students discussed the question with nearby students. After that, UCHI and KIZU explained their work in front. The teacher mentioned that most of the students did the same way as UCHI did. Reviewing the checking of the solution of linear equations studied at previous year, the teacher pointed out UCHI’s writing used an unconfirmed fact:

> This is just substituting *x* as three, and *y* as seven into the equation, right? It’s just substitution, right? It’s just substitution but this is already an equality, so the right side and the left side have to be equivalent, doesn’t it? But you can’t confirm that yet, can you? Right? Which means, if you write it this way, actually,[Writes on the blackboard]you’ve already shown that the right side and the left side are equivalent at this point. But you haven’t confirmed that yet (L3 36’28”).

Here the teacher was trying to let the students be aware of a mathematical norm that if you write an equation in your solution, it means that you have already shown the equality, or that in a mathematical explanation you cannot write what you have not shown to be true yet.

Based on this norm, the teacher accepted KIZU's way of checking, and devalued UCHI's way. Again, the teacher did not forget reminding the students of the fact that most of the students did the UCHI's way: You were not the only one who did wrong.

Norm 4: Accuracy is more valued than speed

In mathematics, establishing truths is one of the most important goals. In the history of mathematics, numerous mathematicians have strived to establish the truths of "conjectures." Therefore, the accuracy of the solution is often more valued than the efficiency, though in the application of mathematics to the real world, efficiency is sometimes more valued.

The teacher often emphasized to the students the importance of checking solutions by themselves. When writing a solution process, if one omits to write several intermediate steps, one could save time. But, then it may become harder to check the procedures. In L5, when the teacher was circling among the students, DOEN asked him if he could omit writing calculations in the solution process. The teacher advised him not to omit them:

> Oh, okay, maybe you should write down up to this expression. Because, when you want to check later, if you don't have this part, you suddenly come up with this expression. For example, for this question, negative seventeen y equals to negative fifty-one. So, it will be easier if you have a clue for what you have done by then, but what if you don't have it? I don't think you have to write down the whole process you took, so, maybe this part can come off, but you had better leave the calculations part if you think about the checking. When you try to check, you have another way from always substituting it, but following what you have done. I think that'll be easier for those situations. I recommend you to leave it for a while. In the future, it will be easy to do a sum in your head. [To Class]The thing is, *it's better to be able to do accurate calculations rather than quick calculations* [the italics are added by the author]. (L5 26'31")

PATTERNS IN NORM DEVELOPMENT

From the data discussed above, there seem to be at least three strategies the teacher used to develop mathematical norms.

Using students' work

The teacher explains a norm by using students' work as an exemplification of what it means to follow the norm.

Since any norm has generality, it could be communicated by using only general terms like "in a mathematical explanation you cannot write what you have not shown to be true yet." But the teacher in this study talked about norms almost always by using students' work. In addition, the teacher did not use any artificial example: He always used actual work of students.

Making a comparison

Sometimes the teacher lets the whole class to compare two of their work on the blackboard, and points out that one of them follows a norm properly, and the other do not (see Norms 1 and 3). Then, the teacher asks the students to follow the norm.

This seems to correspond to "neriage," which is an instructional strategy common in Japanese elementary schools. Japanese elementary teachers often ask children to present their own ideas or solutions on the blackboard. Then comparing their writing the children discuss what they notice of them. This process of comparative discussion is called "neriage." (kneading). Since "neriage" accompanies comparative discussion, this is not just "sharing ideas" (cf. McClain & Cobb, 2001, p. 247).

Being considerate of those students who did not follow a norm

The teacher often discussed that a student's work did not follow a norm. When doing it, he took careful measures to reduce psychological and social damage of the student.

DISCUSSION

The present paper identified three patterns in developing mathematical norms. The use of students' work seems very important. Since a norm is about how to work on mathematics, the use of mathematical work is natural for communicating a norm. Also, since students are familiar with their work, the use of students' work would facilitate students' understanding of the norm. Comparison of students' work would also be very helpful for students to produce clear understanding of the norm as well as their metacognition of their own work. Since pointing out students' violation of a norm may hurt their feeling, being considerate of those students who did not follow the norm seems a hallmark of "competent" teachers. In the data on Norms 1-3, the teacher made considerate moves explicitly. For the data on Norm 4, he did so implicitly by *not* pointing out any student's violation.

These three patterns would be found in common strategies of introducing norms. For example, Voigt (1995) discusses an "indirect" way of introducing a norm about "what counts as an elegant mathematical solution." The strategy highlights students' elegant solutions. Thus, it uses students' work, and has students compare their solutions implicitly. Also, avoiding explicit negative evaluation, the indirect way takes care of the feeling of those students who did not follow the norm.

Studying norms requires understanding of relationships between various norms. A classroom in Japanese schools constitutes a community where a teacher and students stay together, negotiate meanings, share common goals, and shape their identities. It forms a "community of practice." A community generates, maintains, modifies, or eliminates various kinds of patterns called norms, standards, obligations, rules, routines, and the like. Consider a mathematical norm that I identified above, "in mathematics you cannot write what you have not shown to be true yet." This is consistent with a general moral "You should not tell a lie to people." The mathematical norm seems to be backed or authorized by the social norm. That is why the norm appeals to educators and students. Also, consider the teacher's considerate treatment with unsatisfactory fulfilment of mathematical norm, which I identified. The teacher's treatment seems consistent with a *social* norm such as "Any attempt to explain his or her thinking should be respected" (cf. McClain & Cobb, 2001, p. 245).

Furthermore, norms may cause a dilemma. In fact, Norms 1 and 2 appear contradictory. Also, Norm 4 indicates that the efficiency is not always given the highest value. Which norm to use seems to depend on the context where participants are situated. As discussed at the theoretical framework, norms cannot prescribe participants' behaviour. Norms are no more than useful cultural knowledge.

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