PERSONAL EXPERIENCES AND BELIEFS IN EARLY PROBABILISTIC REASONING: IMPLICATIONS FOR RESEARCH

Sashi Sharma

The University of Waikato

Data reported in this paper is part of a larger study which explored form five (14 to 16 year olds) students' ideas in probability and statistics. This paper presents and discusses the ways in which students made sense of task involving independence construct obtained from the individual interviews. The findings revealed that many of the students used strategies based on beliefs, prior experiences (everyday and school) and intuitive strategies such as representativeness. While more students showed competence on coin tossing question they were less competent on the birth order question. This could be due to contextual or linguistic problems. The paper concludes by suggesting some implications for further research.

INTRODUCTION

Over the past years, there has been a movement in many countries to include probability and statistics at every level in the mathematics curricula. In western countries such as Australia (Australian Education Council, 1991), New Zealand (Ministry of Education, 1992) and the United States (Shaughnessy & Zawojewski, 1999) these developments are reflected in official documents and in materials produced for teachers. In line with these moves, Fiji has also produced a new mathematics prescription at the primary level that gives more emphasis to statistics at this level (Fijian Ministry of Education, 1994). Clearly the emphasis is on producing intelligent citizens who can reason with statistical ideas and make sense of statistical information.

Despite its decade-long presence in mathematics curricular, statistics is an area still in its infancy. Research shows that many students find probability difficult to learn and understand in both formal and everyday contexts (Barnes, 1998; Fischbein and Schnarch, 1997; Fischbein, Nello, & Marino, 1991; Shaughnessy & Zawojewski (1999). We need to better understand how learning and understanding may be influenced by ideas and intuitions developed in early years.

Concerns about the importance of statistics in everyday life and a lack of research in this area determined the focus of my study. Overall, the study was designed to investigate the ideas form five students have about statistics and probability. This paper presents and discusses data obtained from the probability task involving independence construct. Prior to discussing the details of my own research, I will briefly mention the theoretical framework and some related literature.

2005. In Chick, H. L. & Vincent, J. L. (Eds.). *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 4, pp. 177-184. Melbourne: PME.

THEORETICAL FRAMEWORK

Much recent research suggests that socio-cultural theories combined with elements of constructivist theory provide a useful model of how students learn mathematics. Constructivist theory in its various forms, is based on a generally agreed principle that learners actively construct ways of knowing as they strive to reconcile present experiences with already existing knowledge (von Glasersfeld, 1993). Students are no longer viewed as passive absorbers of mathematical knowledge conveyed by adults; rather they are considered to construct their own meanings actively by reformulating the new information or restructuring their prior knowledge (Cobb, 1994). This active construction process may result in alternative conceptions as well as the students learning the concepts intended by the teacher.

Another notion of constructivism derives its origins from the work of socio-cultural theorists such as Vygotsky (1978) and Lave (1991) who suggest that learning should be thought of more as the product of a social process and less as an individual activity. There is strong emphasis on social interactions, language, experience, catering for cultural diversity and contexts for learning in the learning process rather than cognitive ability only. Mevarech and Kramarsky (1997) claim that the extensive exposure of our students to statistics outside schools may create a unique situation where students enter the mathematics class with considerable amount of knowledge. This research was therefore designed to identify students' ideas, and to examine how they construct them.

PREVIOUS RESEARCH ON PROBABILITY

A number of research studies from different theoretical perspectives seem to show that students tend to have intuitions which impede their learning of probability concepts. Some prevalent ways of thinking which inhibit the learning of probability include the following:

- Representativeness: According to this strategy students make decisions about the likelihood of an event based upon how similar the event is to the population from which it is drawn or how similar the event is to the process by which the outcome is generated (Tversky & Kahneman, 1974). For instance, a long string of heads does not appear to be representative of the random process of flipping a coin, and so those who are employing representativeness would expect tails to be more likely on subsequent tosses until things evened out. Of course, the belief violates independence construct which is a fundamental property of true random sampling.
- Equiprobability bias: Students who use this bias tend to assume that random events are equiprobable by nature. Hence, the chances of getting different outcomes, for instance, three fives or one five on three rolls of a die are viewed as equally likely events (Lecoutre, 1992).
- Outcome orientation: Falk and Konold (1992) point out that the fundamental difference between formal and informal views of probability concerns the

perceived objective in reasoning about uncertainty. Formal probability is mostly concerned with deriving measures of uncertainty, or answering the question How often will event A occur in the long run? On the other hand what most people want is to predict what will occur in a single instance to answer the question Will A occur or not? Thus, the goal in dealing with uncertainty is to predict the outcome of a single next trial rather than to estimate what is likely to occur at the series of events. Konold (1989) refers to this perspective as the outcome approach.

- Beliefs: Research shows that a number of children think that their results depend on a force, beyond their control, which determines the eventual outcome of an event. Sometimes this force is God or some other force such as wind, other times wishing or pleasing (Amir and Williams, 1994;; Truran, 1994).
- Human Control: Research designed to explore children's ability to generalise the behaviour of random generators such as dice and spinners show that a number of children think that their results depend on how one throws or handles these different devices (Shaughnessy & Zawojewski, 1999; Truran, 1994).

Whether one explains the reasoning in probabilistic thinking by using naive strategies such as representativeness and equiprobability or by deterministic belief systems such outcomes can be controlled, the fact remains that students seem very susceptible to using these types of judgements and in some sense all of these general claims seem to be valid. Different problems address different pieces of this knowledge.

OVERVIEW OF THE STUDY

Sample. The study took place in a co-educational private secondary school in Fiji. The class consisted of 29 students aged 14 to 16 years. According to the teacher, none of the students in the sample had previously received any *in-depth* instruction in statistics. Fourteen students were chosen from the class, the criteria for selection included gender and achievement.

Task. The baby (Item 1A) and the coin questions (Item 1B) were used to explore students' understanding of the independence concept and responses demanded both numerical and qualitative descriptions.

Item 1A: The baby problem

The Singh family is expecting the birth of their fifth child. The first four children were girls. What is the probability that the fifth child will be a boy? Please explain your answer.

Item 1B: Coin problem

(i) If I toss this coin 20 times, what do you expect will occur?

(ii) Suppose that the first four tosses have been heads. That's four heads and no tails so far. What do you now expect from the next 16 tosses? Why do you think so?

Interviews. Each student was interviewed individually by myself in a room away from the rest of the class. The interviews were tape recorded for analysis. Each interview lasted about 40 to 50 minutes.

RESULTS

This section describes the patterns of thinking identified in response to the two questions. Extracts from typical individual interviews are used for illustrative purposes. Throughout the discussion, I is used for the interviewer and Sn for the *nth* student.

A few students in my study believed in the independence of events, that is, that each successive trial is independent of the previous trials. For example, Student 12 was able to use the independence concept for both questions. With respect to the baby problem, she explained that since the fifth child could be a boy or a girl, the chance of getting a boy or a girl was 50%. For item 1B, she explained that since getting heads or tails were equally likely, she would expect about 8 heads and 8 tails.

Prior beliefs and experiences played an important role in the thinking of many students. On the baby problem, four students related to their religious beliefs and experiences. The students thought that one can not make any predictions because the sex of the baby depends on God. The religious aspect is revealed in the response of Student 17 who explained:

> We can not say that Mrs Singh is going to give birth to a boy or a girl because whatever God gives, you have to accept it.

It must be noted that the birth order problem is equivalent to the coin question (Item 1B). However, when a different context is introduced, students are comfortable thinking deterministically. For instance, Student 2 was considered statistical on Item 1B but she used the religious perspective on Item 1A. Student 3 tended to draw upon experiences gained from other subjects. She explained that the outcome was managed by the parents and tried to relate her previous knowledge of biology in responding to Item 1A.

Two students in the present study thought that the results depend on individual control (Item 1B). The students said that people can control the outcome by throwing it in a certain direction or throwing it fast. This is reflected in the following interview:

S20: Eh ... it will have 5 heads and 15 tails.

I: Why do you think that there will be 5 heads and 15 tails?

S20: Eh ... because when you throw each time it comes head or tail

I: But you said more tails. Why do you think you will get more tails?

S20: I will throw the coin in one direction so I will get HHH, when I change in another direction I will get all tails.It depends on how fast you throw and how fast the coin swings.

Some students based their reasoning on inappropriate rules and intuitions such as representativenes. Two students applied the n(E)/n(s) rule inappropriately. For example, with respect to Item 1A, student 25 said,

> Chance will be one upon five. Four girls and one to be born; don't know whether it will be a boy or a girl. Like in dice there is one side and the total is six but one is the chance eh.

Although the students had learnt finding probabilities using sample space, they applied this rule inappropriately. The data revealed that while two students used the representativeness strategy for the baby problem, six used it for the coin problem. Students using the representativeness strategy on Item 1B thought that there would be a balancing out so they would expect more tails. Even repeated probing did not produce any probabilistic thinking.

One student drew upon the equiprobability bias on the coin problem. The student reasoned that if one tosses a coin 20 times, one expects to get 10 heads and 10 tails because one does not know which side will fall. Hence equal chance should be given to both events. In three cases, students could not explain their responses. For instance, Student 9 said that there will be equal number of heads and tails but could not explain her reasoning.

DISCUSSION

This section first discusses the results in a broader context. Then limitations of the study are discussed and suggestions made for directions for further research.

Probability: A broader Context

The results show students think that outcomes on random generators such as coins (Item 1B) can be controlled by individuals. The general belief is that results depend on how one throws or handles these different devices. The finding concurs with the results of studies by Amir and Williams (1994), Shaughnessy and Zawojewski (1999) and Truran (1994). It must be noted that the students using the control strategy in this study were boys. One explanation for this could be that boys are more likely to play sports and chance games that involve flipping coins and rolling dice to start these games.

Although this study provides evidence that reliance upon control assumption can result in biased, non-statistical responses, in some cases this strategy may provide useful information for other purposes. For example, student 20's knowledge of physics may have been reasonable. The responses raise further questions. Is there a weakness in the wording of this question in that it is completely open-ended and does not focus the students to draw on other relevant knowledge? Perhaps, including cues such as "fair" in the item would have aided in the interpretation of this question. Are the students aware of the differences in probabilistic reasoning compared with reasoning in other contexts? Although in probability theory we work with an idealised die or coin, deterministic physical laws govern what happens during these trials. It does not make sense to say that the coin has a probability of one-half to be heads because the outcome can be completely determined by the manner in which it is thrown. Additionally, a good Bayesian statistician might not give 50/50 heads/tails as the likely outcome after a run of heads with a particular coin. Such a person might start looking at prior experience to inform a particular situation.

With respect to students' beliefs, experiences and learning, it is evident that other researchers have encountered similar factors. Amir and Williams (1994) note that children's reasoning appeared to be related to their religious, superstitious and causal beliefs. In some respects, the findings of the present investigation go beyond those discussed above. The findings demonstrate how students' other school experiences also influence their construction of statistical ideas. At times the in-school experiences appear to have had a negative effect on the students. An example of negative effect that arose from other school experiences was the student who was deeply convinced that the father decides the sex of the baby. Gal (1998) suggests that such responses constitute what students know about the world, they cannot be judged as inappropriate until a students' assumptions about the context of the data are fully explored. For instance, the students confronted with the problem concerning birth order (Item 1A) may not know which model is appropriate. The statistical model implies that both events are equally probable but the student does not know whether biologically there is some tendency for families to have offspring of a particular gender or the end result of boys to girls should be equivalent. We know now that giving birth to boys and girls is **not** random but affected by things like times of conception and genetic dispositions of the parents. Although the outcomes are independent across births, there are rare occasions of identical or fraternal twins and triplets. In short it is not possible to determine the nature of the error unambiguously on the basis of the students' response.

In the study described here, background knowledge, that is often invoked to support a student's mathematical understanding, is getting in the way of efficient problem solving. Given how statistics is often taught through examples drawn from "real life" teachers need to exercise care in ensuring that this intended support apparatus is not counterproductive. This is particularly important in light of current curricula calls for pervasive use of contexts (Meyer, Dekker, & Querelle, 2001; Ministry of Education, 1992) and research showing the effects of contexts on student' ability to solve open ended tasks (Cooper & Dunne, 1997; Sullivan, Zevenbergen, & Mousley, 2002). Conversely, in spite of the importance of relating classroom mathematics to the real world, the results of my research indicate that students frequently fail to connect the mathematics they learn at school with situations in which it is needed. For instance, Student 2 used statistical principles on Item 1B whereas on Item 1 she refereed to her religious convictions. The findings support claims made by Lave (1991) that learning for students is situation specific and that connecting students' everyday contexts to academic mathematics is not easy.

Limitations

It must be acknowledged that the open-ended nature of the tasks and the lack of guidance given to students regarding what was required of them certainly influenced how students explained their understanding. The students may not have been particularly interested in these types of questions as they are not used to having to describe their reasoning in the classroom. Some students in this sample clearly had difficulty explaining explicitly about their thinking. Another reason could be that such questions do not appear in external examinations. Although the study provides some valuable insights into the kind of thinking that high school students use, the conclusions cannot claim generality because of a small sample. Additionally, the study was qualitative in emphasis and the results rely heavily on my skills to collect information from students. Some directions for future research are implied by the limitations of this study.

Implications for Further Research

One direction for further research could be to replicate the present study and include a larger sample of students from different ethnic backgrounds. Secondly, this small scale investigation into identifying and describing students' reasoning from constructivism has opened up possibilities to do further research at a macro-level on students' thinking and to develop explicit categories for responses. Such research would validate the framework of response levels described in literature (Watson & Callingham, 2003) and raise more awareness of the levels of thinking that need to be considered when planning instruction and developing students' statistical thinking. The place of statistics has changed in the revised mathematics prescription. Statistics appears for the first time at all grade levels (Fijian Ministry of Education, Women, Culture, Science and Technology, 1994). Like the secondary school students, primary school students are likely to resort to non-statistical or deterministic explanations. Research efforts at this level are crucial in order to inform teachers, teacher educators and curriculum writers.

References

Amir, G. & Williams, J. (1994). The influence of children's culture on their probabilistic thinking, In J. da Ponte & J. Matos (Eds.), *Proceedings of the Eighteenth International Conference for the Psychology of Mathematics Education, 29 July-3 August, 1994*, Vol. II (pp. 24-31). Lisbon, Portugal, The Program Committee of the 18th PME Conference.

Australian Education Council. (1991). *A National Statement on Mathematics for Australian Schools*. Canberra, Curriculum Corporation.

Barnes, M. (1998). Dealing with misconceptions about probability. *Australian Mathematics Teacher,* 54(10), 17-20.

Cobb, P. (1994). Where is the mind? Constructivist and sociocultural perspectives on mathematical development' *Educational Researcher,* 23(7), 13-20.

Cooper, B. & Dunne, M. (1999). *Assessing Children's Mathematical Ability*. Buckingham: Open University Press.

Falk, R. & Konold, C. (1992). The psychology of learning probability. In F. Sheldon and G. Sheldon (Eds.), *Statistics for the Twenty-First Century* (pp. 151-164). Washington, Mathematical Association of America.

Fijian Ministry of Education, Women, Culture, Science and Technology. (1994). *Primary Mathematics Prescriptions.* Suva: Curriculum Development Unit.

Fischbein, E. & Schnarch, D. (1997). The evolution with age of probabilistic, intuitively based misconceptions. *Journal for Research in Mathematics Education,* 28, 96-105.

Konold, C. (1989). An outbreak of belief in independence? In C. Maher, G. Goldin & B. Davis (Eds.), *The Proceedings of the eleventh Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education,* Vol II (pp. 203-209). Rutgers, Rutgers University Press.

Lave, J. (1991). Cognition in practice; Mind, mathematics and culture in everyday life. New York: Cambridge University Press.

Lecoutre, M. (1992). Cognitive models and problem spaces in purely random situations. *Educational Studies in Mathematics*, 23, 557-568.

Mevarech, Z. R. & Kramarsky, B. (1997). From verbal descriptions to graphic representations: Stability and change in students' alternative conceptions. *Educational Studies in Mathematics,* 32, 229-263.

Meyer, M. R, Dekker, T. & Querelle, N. (2001). Context in mathematics curricula, *Mathematics Teaching in the Middle School,* 6(9), 522-527.

Ministry of Education. (1992). *Mathematics in the New Zealand Curriculum.* Wellington: Ministry of Education.

Shaughnessy, J.M. & Zawojewski, J. S. (1999). Secondary students' performance on data and chance in the 1996 NAEP, *The Mathematics Teacher,* 92(8), 713-718.

Sullivan, P., Zevenbergen, R., & Mousley, J. (2002). Contexts in Mathematics teaching: snakes or ladders? In B. Barton, K. C. Irwin, M. Pfannncuch and M. Thomas (Eds.), *Mathematics Education in the South Pacific.* Proceedings of the 25th annual conference of the Mathematics Education Research Group of Australasia (pp 649-656). Auckland.

Truran, K. M. (1994). Children's understandings of random generators in C. Beesey & D. Rasmussen (Eds.), *Mathematics Without Limits, Proceedings of the 31st Annual Conference of the Mathematical Association of Victoria* (pp. 356-362). Melbourne, Mathematics Association of Victoria.

Tversky, A. & Kahneman, D. (1974). Judgement under uncertainty: Heuristics and biases. *Science,* 185, 1124-1131.

Von Glasersfeld, E. (1993). Questions and answers about radical constructivism. In K. Tobin (Ed.), *The practice of constructivism in science education* (pp. 24-38). Washington, American Association for the Advancement of Science.

Vygotsky, L. (1978). Mind in society: The development of higher psychological processes. Cambridge: Harvard University Press.

Watson, J. M. & Callingham R. (2003). Statistical literacy: A complex hierarchical construct. *Statistics Education Research Journal,* 2(2), 3-46.