MATHEMATICAL KNOWLEDGE OF PRE-SERVICE PRIMARY TEACHERS

Beth Southwell Marina Penglase
University of Western Sydney

Seventy eight primary pre-service teachers participated in a survey of arithmetical content knowledge at the conclusion of an elective mathematical content course designed for those with a poor background in mathematics. Not only was the aim of this first stage of a research project to ascertain current knowledge but also to adjust current courses to better suit the students in teacher preparation courses. Analysis of the results of this survey indicate weaknesses in understanding in particular areas including place value, operations with common fractions, multiplication of decimal fractions, percentages and measurement. These areas are related to the curriculum the pre-service teachers will be expected to teach on their graduation.

INTRODUCTION

Several recent reports and movements have emphasized the need to enhance the mathematical content knowledge of students. The *NCTM Principles and Standards for School Mathematics* (2000) states that 'Teachers need different kinds of mathematical knowledge – knowledge about the whole domain; deep, flexible knowledge about curriculum goals and about the important ideas that are central to their grade level …' (p. 17). *The AAMT Standards for Excellence in Teaching Mathematics in Australian Schools* is one of the most recent documents to say that 'excellent teachers of mathematics have a sound, coherent knowledge of the mathematics appropriate to the student level they teach' (2002). If teachers are not confident in their mathematical knowledge, they may find it difficult to ensure that their students gain confidence and competence.

Then, too, the reports arising from the Third International Study of Mathematics and Science (Lokan, Ford & Greenwood, 1996, 1997) indicate that there were deficiencies as well as strengths in student achievement and understanding in mathematical content knowledge. Several researchers (Morris, 2001; Chick, 2002; Amarto & Watson, 2003) have found that pre-service teachers do not always possess the conceptual understanding of the mathematics content they will be expected to teach.

In 2002, the Board of Studies, NSW, released the new *Mathematics K-6 Syllabus* that became mandatory in NSW primary schools from 2004. This Syllabus has many differences from the 1989 *Mathematics K-6 Syllabus* that has been in use in these schools. Included among these differences are many involving different concepts or new approaches to mathematics. Primary teacher education students currently undertaking their teacher preparation courses will be expected to implement the new syllabus when they graduate. Many of them did not complete mathematics courses in

2005. In Chick, H. L. & Vincent, J. L. (Eds.). *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 4, pp. 209-216. Melbourne: PME.

their senior high school years, often because they had not been successful in their junior years. As a consequence of all of this, it is possible that primary teacher education students start their teacher preparation courses without key mathematical knowledge and with some negative attitudes towards the teaching and learning of the subject.

Over several decades there has been a change in the way mathematics has been taught and in the curriculum that has been followed. Constructivism heralded in a different emphasis on the process of teaching and learning. Unfortunately, many teachers saw it as a way of ignoring their own lack in mathematical content knowledge and concentrated on what they perceived to be the process required in a constructivist based classroom. Von Glasersfeld (1994), however, reminded educators of the possibility of enhancing mathematics achievement and understanding through a constructivist approach. In relation to the learning and teaching of arithmetic he stated:

> … if we really want to teach arithmetic, we have to pay a great deal of attention to the mental operations of our students. Teaching has to be concerned with understanding rather than performance … (von Glasersfeld, 1994, p. 7)

It is important to note that the outcome of learning implied in this statement is understanding and conceptual development.

The aim of this stage of the project is to ascertain the mathematical knowledge of primary teacher education students in a NSW university teacher preparation course at a particular stage in their courses. This will enable the researchers to tailor courses to help fill any gaps that may be found. It will also provide an ongoing measure against which students and university staff can judge the students' learning and the courses being undertaken.

METHODOLOGY

Sample. The 78 participants at a NSW university included both undergraduates in a four year program from second, third or fourth year of their course and students in a one year graduate entry program. They were doing this subject either because they lacked a sound mathematical background or because they had a particular interest in mathematics.

Procedure. The survey was administered during class time at the conclusion of a special elective mathematics content course. A survey methodology was considered most appropriate for this study. McMillan (2004, p. 195) describes surveys as popular because of their 'versatility, efficiency and generalizability'. Their versatility lies in their ability to 'address a wide range of problems or questions, especially when the purpose is to describe the attitudes, perspectives and beliefs of the respondents'. Their limitation, according to Mertler and Charles (2005), is that they do not allow the researcher to probe further as would be possible in an interview. In this current study, the 20 questions used in the survey were designed to ascertain whether the participants had the necessary mathematical knowledge on topics they were expected to teach and any further probing was considered possible if necessary after the initial responses were analysed. The project was approved by the University Ethics Committee.

Analysis. Data were analysed using descriptive statistics only for the first part of this research on the mathematical competence of pre-service teachers. Surveys of attitudes and beliefs will be considered at a later stage. Because the first and the last items had 3 and 2 parts respectively, these were treated in the analysis as separate items, thus making 23 items.

RESULTS AND DISCUSSION

At this stage of the research, there are two main areas that need to be reported. They are the item analysis for the 23 items and the relative difficulty of areas of arithmetic as indicated by responses.

Item Analysis. Table I presents the former of these for items of greatest interest indicated by the difficulties observed.

Table 1. Number and Percentage of Correct Responses and Most Common Incorrect Responses

Item	No. correct	Percentage correct	Most common incorrect responses
3	0	0	11 participants did not answer, one saying he/she could not understand the item. Other responses ranged from 0 to 38 with 7 giving alternatives such as '9 or 18'.
7	1	1.3	19 responses as 46 092 340. 23 did not attempt. 6 gave correct response except for units digit, 2 listed each power but did not join them with +. Three wrote expansions but not expressed as powers of 10.
11	34	43.6	9 did not attempt.
13	48	61.5	19 wrote fractions largest to smallest. See comments below.
14	54	69.2	15 subtracted numerators and denominators respectively. 6 did not attempt. One used addition.
15	45	57.7	8 did not attempt. 12 added numerators and denominators respectively. One added denominators only.
17	45	57.7	13 gave 17/100 as their answer. 3 wrote both 0.17 and 17/100 and 4 gave 17/100 0.17.
19	21	26.9	30 gave correct digits but with decimal point placed the first 2 digits of their answer. 10 did not attempt or crossed out work.
20	58	74.4	11 had each of the algorithms within the brackets correct but gave the wrong answer. 8 had 1 algorithm incorrect but all other working correct. One removed the last bracket, writing the last sign as a minus.
21	48	61.5	6 did not attempt. 10 gave 16 as answer. 14 were completely incorrect though 5 at least started to work a division algorithm.
22	59	75.6	10 gave 623 as response. 4 gave 6023.
23	62	79.5	5 did not attempt. 3 gave 598.7, 3 gave 598700 as response.

Comments on Specific Items

Item 3. No of pairs of numbers that sum to 19. The poor response to this item raises the question of language and its relationship to mathematics. The item was not expressed as clearly as most participants needed. The inclusion of the word ‘whole’, for instance, may have made the item more specific though this particular difficulty had not arisen in previously given similar questions. It could be that this cohort of students had been introduced to inclusive sets of numbers in such a way as to not question the item further and hence the number of non-attempts and the cry “I don’t understand this”. Unfortunately this view is not borne out by the types of responses given as they all refer to whole numbers anyway. It would appear, then, that these participants have been confused with the other wording in the item or are fairly rigid in their arithmetical thinking. ‘Different numbers’ in the item statement could be interpreted as meaning that 0 and 19 are the same numbers regardless of the order. In that case the correct answer would not be the same as it would be if they are considered different. This could explain the number of participants who gave alternative responses, e.g. 9 or 18.

Item 7. Expand 4 609 234 using powers of 10. As this item requires understanding fundamental to the decimal place value system and since numeration was a topic in the course participants followed, it is surprising that this item was so poorly completed. One can understand the omission of the index for the unit figure but again, perhaps the language used was not as familiar as expected. One participant actually wrote that he/she did not understand the word ‘expand’ and this possibly was the case for many others. The response 46 092 340 seems to indicate that participants did not know what was meant by ‘expand’ and thought that by multiplying by ten they were using a power of ten.

Item 11. Calculate 47 x 25 using a different method (N.B. previous question was Calculate 47 x 25) A ‘different method’ caused several problems for participants. Many used the commutative property and did not acknowledge that multiplying 47 by 25 used the same method as multiplying 25 by 47. Only 3 students made use of the distributive law and added each partial product. Several made use of the distributive property but only in part. Only two participants took advantage of the fact that the multiplier was 25 and is therefore one quarter of 100. The picture painted by the responses to this item indicates a fairly inflexible idea of multiplication with the emphasis on the standard algorithm. The NSW Syllabus K-6 Mathematics (1989, 2002) recommends that teachers encourage students to use their own natural methods to complete computations and to explore different ways in which this can be done. In the particular course this group of students has completed is the opportunity for many approaches to algorithms. Only one student used the lattice method and one attempted but was not able to complete the duplation or doubling method. Number sense and flexibility of arithmetical understanding are worthwhile aims at any level and pre-service teachers have the responsibility and opportunity to acquaint themselves with methods of an historical nature as well as being able to accept unorthodox methods created by their students.

Item 17. Convert 17% to a decimal fraction. Participants indicated some degree of uncertainty in their responses to this item. Perhaps there exists some confusion as to the difference between a decimal fraction and a common fraction since so many (13) gave their responses in the common fraction form. This confusion may have arisen from the common (sometimes incorrect) practice of referring to 'fractions' for common fractions and 'decimals' for decimal fractions. This practice overlooks the fact that all our numbers are decimal numbers since they are based on ten in the same way as binary numbers are based on two. Decimal fractions are fractions or rational numbers expressed with a decimal point. The difficulty may also have arisen because the method of changing a percentage to a decimal fraction was unfamiliar to the participants. Several knew that 17% meant 17/100 but then became uncertain as to the place value represented by the fraction. Those who put both 0.17 and 17/100 or its reverse may have been 'hedging their bets' hoping that one of their responses would be counted correct. Taplin (1998) reported that on medium difficulty items approximately half of the participants in her study with an incorrect answer to a question asking them to find 12% of $68 did not know that 12% meant 12/100.

Item 21. Solve: 1023 students, 63 per bus. How many buses needed? The responses to this item indicated that for some participants there was no need to consider the context of the problem. The answer 16.23 was arithmetically correct but was not a sensible answer to the question. Two participants did realise that they would need a whole number of buses but opted for 16 instead of 17 qualifying their responses by saying some students would stand or be left behind. As the item said that the students were to 'fit in' the buses, not necessarily be seated, the responses of 16 were not considered correct. The question of context in word problems - indeed problems of any kind – is one that needs to be considered as a vital aspect of problem solving. Contreras and Martinez-Cruz (2001) report that only 28% of their sample of pre-service teachers were able to give a realistic response to a problem using the same context as this one and only 6 % were able to give an explanation of their answer.

Items 14, 15. Find 5/8 - 2/5. Find 4/5 + 2/3. The error recorded by a number of participants is a recognised common one and indicates not only a faulty algorithm for common fractions but also a lack of flexibility of arithmetical thinking, as a simple check of the reasonableness of the answer would alert the participant to the error. Rational numbers, particularly in the common fraction form, have been recognised for some time as an area of great difficulty for all students. Skemp (1986) claimed that this is partially because students have to apply a process of *accommodation* when they meet rational numbers and this is different to the *assimilation* process that has been possible with all the previous work they have experienced in mathematics. Taplin (1995) found that pre-service primary teachers found difficulty in fractions concepts including operations. One interesting aspect of these items is that a few students were able to complete one of these two items correctly but followed the common error for the other. This seems to indicate an unstable concept of operations with common fractions.

Item 19. Calculate 14.83 x 0.06. Taplin (1995) also identified difficulties in the multiplication of decimal fractions. This same difficulty arose in this study because of the participants' placement of the decimal point.

Items 22, 23. Convert 6.23km to m. Convert 5.987L to ml. These two items seem to indicate a general weakness in measurement which could be linked to a place value deficiency or to a lack of understanding of the metric system of measurement. As the metric system is usually considered an application of the place value system, and is used in that way, it is disturbing that the numbers correct in these items are as low as they are, unless the fact that they are the last items caused participants to think less about them than other items. Morris (2001) also reports a similar deficiency in converting metric measures between units.

Item 13. Put 5/6, 2/3, 4/5 in order. Although responses in the wrong order were accepted as correct with Item 5, they were not in Item 13, mainly because of the possibility that participants do not understand that the magnitude of common fractions is different to that of whole numbers. This point is supported by Leinhardt and Smith (1985, p. 269).

Analysis of Content Areas. The second area of reporting for this stage of this project is in relation to the particular topic that participants found difficult. For this purpose, the 23 items have been linked in groups and each group considered separately. Table 2 shows the items in groups and the relevant statistics related to each group.

Table 2. Number and Percentages of Correct Responses for each Category of Items.

Category	No. of items in each category	No. correct responses	Percentage correct responses
Basic concepts, numeration	3 (5,6,7)	155	65.8 (*98.7)
Basic facts	3 (1,2,3)	154	66.2 (*98.7)
Four operations	5 (8-12)	389	83.1
Order of common fractions	1 (13)	48	61.3
Operations with common fractions	2 (14,15)	99	63.5
Decimal fractions	1 (19)	21	26.9
Percentages	3 (16,17,18)	156	66.7
Measurement	2 (22,23)	121	76.6
Order of operations	1 (20)	58	74.4
Word problems	1 (21)	48	61.5

* Percentage of non-responses to items 3 and 7 are not included.

Surprisingly, the area requiring greatest attention is decimal fractions. Place value concepts seem to have caused most problems. This outcome could be the result of only having one item on decimal fractions, though the other related item on changing percentages to decimal fractions was not well done either. This seems to lend support to the premise that more time needs to be spent on work with decimal fractions. It could be that because the link is so obvious to teachers, they do not spend the necessary time to allow students to construct effective processes of understanding and using decimal fractions.

Language in mathematics is another area that needs more attention. Because mathematics is sometimes spoken about as the universal language, the assumption is made that no matter what language they speak every day, students will be able to understand all aspects of mathematics, including terminology and syntax of problems. This is not necessarily so as Bell and Ho Woo (1998) have found.

CONCLUSION

The study has caused certain possible future research topics to emerge. The concentration of research on specific topics in mathematics is necessary if pre-service teachers are to become properly equipped for their daunting task as teachers. This test needs to be extended to geometry and probability and further testing carried out.

This study reminds teachers and teacher educators in particular, that understanding the way in which learners construct their arithmetical knowledge is of prime importance in all mathematics courses. Much more can be done to analyse thought processes and develop approaches in the classroom that will assist students in their mathematical constructions. Leinhardt and Smith (1985), in a study with elementary teachers, concluded that the 'skills associated with lesson structure and subject matter knowledge are obviously intertwined (p. 247)'. This is a reminder that without sound mathematical knowledge many pedagogical processes are of little benefit. This current study also alerts teacher educators, particularly in New South Wales, to the need to assist pre-service teachers with specific topics as each requires. This can only be done in the time available in pre-service courses by a screening test to identify possible specific areas of weakness and the design of appropriate programs for them. It is anticipated that this study will lead to such a process.

References

AAMT (2002). *Standards for Excellence in teaching mathematics in Australian schools*. Adelaide: AAMT.

Amarto, S. A. & Watson, A. (2003). Improving student teachers' understanding of multiplication by two digit numbers. In N. A. Pateman, B.J. Dougherty, & J. Zilliox (Eds.). *Proceedings of the 27th Conference of the International Group for the Psychology of Mathematics Education and the 25th Conference of PME-NA*. Honolulu, HI: PME

Bell, G., & Ho Woo, J. (1998). Probing the link between language and mathematical conceptualization. *Mathematics Education Research Journal*, 10, 1, 51-74.

Chick, H. L. (2002). Pre-service primary teachers' assessment of and performance on multiplicative numeracy items. In A. D. Cockburn, & E. Nardi (Eds.). *Proceedings of the 26th Annual Conference of the International Group for the Psychology of Mathematics*, (Vol. 2, pp. 233-240). Norwich, UK: PME.

Contreras, J. N., & Martinez-Cruz, A. M. (2001). An investigation of pre-service elementary teachers' solution processes to problematic story problems involving division of fractions and their interpretation of solutions. In M. van den Heuvel-Panhuizen (Ed.). *Proceedings of the 25th Conferenece of the Internaltional group for the Psychology of Mathematics*, (Vol. 2, 289-296). Utrecht: PME.

Leinhardt, G., & Smith, D. A. (1985). Expertise in mathematics instruction: Subject matter knowledge. *Journal of Educational Psychology*, 77(3), 247-271.

Lokan, J, Ford, K., & Greenwood, L. (1996). *Maths and Science on the line*. Melbourne: ACER.

Lokan, J, Ford, K., & Greenwood, L. (1997). *Maths and Science on the line*. Melbourne: ACER.

McMillan, J. H. (2004). *Educational Research* (4th Ed.). Boston: Pearson.

Mertler, C. A., & Charles, C. M. (2005). *Introduction to Educational Research* (5th Ed.). Boston: Pearson.

Morris, H. (2001). Issues raised by testing trainee primary teachers' mathematical knowledge. *Mathematics Teacher Education and Development*, 3, 37-47.

National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics.* Reston, Va: NCTM.

Skemp, R. R. (1986). *The Psychology of Learning Mathematics* (2nd Ed.). Harmondsworth: Penguin.

Taplin, M. (1995). *Difficulties with the basic mathematics among pre-service teachers*. New Horizons in Education, 92, 3-19.

Taplin, M. (1998). Pre-service teachers' problem solving processes. *Mathematics Education Research Journal*, 10(3), 59-76.

von Glasersfeld, E. (1994). A radical constructivist view of basic mathematical concepts. In P. Ernest (Ed.) *Constructing mathematical knowledge*, (pp. 5-7). London: Falmer Press.