RF03: A PROGRESSION OF EARLY NUMBER CONCEPTS

Kathleen Hart

The purpose of the research forum is to describe the research evidence available concerning a Progression in Early Number Concepts. Children around the world are taught some Arithmetic as soon as they start school. The content matter may be dictated by the school or teacher, a book or very often by the curriculum laid down by the government. The word 'curriculum' may describe the range of experience to which the child is introduced when first attending school. For the purpose of this forum we limit discussion to the Number Syllabus for grades 1 to 4. The speakers involved have some evidence of what appears hard and what easy for young children in different parts of the world. The aim is to consider what gives success for the majority of children not what is possible for a talented few.

Participants are urged to bring a copy of the syllabus [grades 1 to 4] from their own country and evidence from their own research with young children or national surveys carried out on the child population. The intention is not to compare performance among countries but to judge the progression of difficulty of concepts through pupils' success or failure. It is likely that there is a great deal in common.

The allocation of time for the forum is three hours and we want to end with some suggestions of what we know and the identification of areas about which we have little or no information. The following activities are planned:

1. In the first session to study and discuss what is required by the published syllabuses of various countries. In many countries these lists of topics form the base on which the efficiency of schools and teachers are judged. Inspectors and evaluators use the syllabus to judge what is happening in schools. How are these lists drawn up?

The syllabuses we have may have a lot in common. They may make assumptions on the relative difficulty of ideas. Do any of them alert the teacher to a great leap in intellectual demand? Is there an assumption that the great majority of pupils will succeed. Is success measured in terms of mastery of most/all of the content or is a pass mark assigned which admits to success in only 30-40 % of the topics?

2. Talks by invited researchers who have investigated the learning of Number with young children, the steps of increasing difficulty and the pitfalls.

3. Participants are encouraged to add their own evidence.

4. We have planned a debate on the idea of 'achievability' [does everything in the syllabus have to be achievable by the pupils?] with a proposer and opponent, speakers from the floor and a vote. There is however only half an hour available for this activity.

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The questions asked in this research forum are:

- From the accumulated evidence can we suggest a progression of Early Number Concepts that seem to achievable by children in even the most basic of learning circumstances?
- Can we identify from the available evidence parts of Arithmetic which cause problems ?
- Can we provide some help for the teachers concerning these 'bottlenecks'?
- Can we formulate some research questions which could add more evidence?

USING GROWTH POINTS TO DESCRIBE PATHWAYS FOR YOUNG CHILDREN'S NUMBER LEARNING

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One important outcome of the Early Numeracy Research Project was the development of a framework of growth points to describe young children's number learning. This paper provides a brief overview of the development and use of these growth points.

INTRODUCTION

The Early Numeracy Research Project ([ENRP], Clarke, 1999) was a three-year project initiated in 1999 by the then Victorian Department of Education, Employment and Training (DEET). The aim was to enhance the mathematical learning of young children (5-year-olds to 8-year-olds) through increasing the professional knowledge of their teachers. The project was conducted in 35 matched samples of trial and reference schools that were representative of the broader population across the state. It could be expected, therefore, that any underlying dimension of achievement, like most human characteristics, would approximate a normal distribution (Rowley, Horne et al., 2001). This was an underlying assumption of the data analysis undertaken throughout the ENRP.

GROWTH POINTS FOR DESCRIBING MATHEMATICAL LEARNING

A basic premise of the ENRP was that knowledge about children's mathematical understanding and development is needed for teachers to plan effective learning experiences for their students. To increase teacher's knowledge of children's mathematical development, the ENRP research team developed a framework of growth points to:

- describe the development of children's mathematical knowledge and understanding in the first three years of school, through highlighting important ideas in early mathematics understanding in a form and language that was useful for teachers;
- reflect the findings of relevant Australian and international research in mathematics education, building on the work of successful projects such as Count Me in Too (Bobis & Gould, 1999);
- reflect the structure of mathematics;
- form the basis of mathematics curriculum planning and teaching; and
- identify those students who may benefit from additional assistance or intervention.

As the impetus for the ENRP was a desire to improve young children's mathematics learning, in order to document any improvement, it was necessary to develop quantitative measures of children's growth. It was considered that a framework of key growth points in numeracy learning could fulfill this requirement. Further, the framework of growth points enabled the identification and description of any improvements in children's mathematical knowledge and understanding, where it existed, by tracking children's progress through the growth points. Trial school students' growth could then be compared to that of students in the reference schools.

In developing the framework of growth points, the project team studied available research on key "stages" or "levels" in young children's mathematics learning (Bobis, 1996; Boulton-Lewis, 1996; Fuson, 1992b; Mulligan & Mitchelmore, 1996; Pearn & Merrifield, 1998; Wright, 1998) as well as frameworks developed by other authors and groups to describe learning. A major influence on the project design was the New South Wales Department of Education initiative Count Me In Too (Bobis & Gould, 1999; New South Wales Department of Education and Training, 1998) that developed a learning framework in number (Wright, 1998) that was based on prior research and, in particular, on the stages in the construction of the number sequence (Steffe et al., 1988; Steffe et al., 1983). The *Count Me In Too Project* used an interview designed to measure children's learning against the framework of stages. It was decided to use a similar approach for the ENRP, but to expand the content of the interview to include domains in measurement and space, and to extend the range of tasks so that is was possible to measure the mathematical growth of all children in the first three years of school.

Following the review of available research, the ENRP team developed a framework of growth points for Number (incorporating the domains of Counting, Place value, Addition and Subtraction Strategies, and Multiplication and Division Strategies), Measurement (incorporating the domains of Length, Mass and Time), and Space (incorporating the domains of Properties of Shape, and Visualisation and Orientation). Within each mathematical domain, growth points were stated with brief descriptors in each case. There are typically five or six growth points in each domain (see Appendix 1, at the end of the Forum papers), and each growth-point was assigned a numeral so that the growth points reached by each child could be entered into a database and analysed. For example, the six growth points for the Counting domain are:

1. *Rote counting*
   Rote counts the number sequence to at least 20, but is unable to reliably count a collection of that size.
2. *Counting collections*
   Confidently counts a collection of around 20 objects.
3. *Counting by 1s (forward/backward, including variable starting points; before/after)*
   Counts forwards and backwards from various starting points between 1 and 100; knows numbers before and after a given number.
4. *Counting from 0 by 2s, 5s, and 10s*
   Can count from 0 by 2s, 5s, and 10s to a given target.
5. *Counting from x (where x>0) by 2s, 5s, and 10s*
   Can count from x by 2s, 5s, and 10s to a given target beginning at variable starting points.
6. *Extending and Applying*
   Can count from a non-zero starting point by any single digit number, and can apply counting skills in practical tasks.

Each growth point represents substantial expansion in mathematical knowledge, and it is acknowledged that much learning takes place between them. In discussions with teachers, the research team described growth points as key "stepping stones" along paths to mathematical understanding. They provide a kind of conceptual landscape upon which mathematical learning occurs (Rowley, Gervasoni et al., 2001). As with any journey, it is not claimed that every student passes all growth points along the way. Indeed, (Wright, 1998) cautioned that "it is insufficient to think that all children's early arithmetical knowledge develops along a common developmental path" (p. 702). Also, the growth points should not be regarded as necessarily discrete. As with Wright's (1998) framework, the extent of the overlap is likely to vary widely across young children. However, the order of the growth points provides a guide to the possible trajectory (Cobb & McClain, 1999) of children's learning. In a similar way to that described by Owens & Gould (1999) in the *Count Me In Too* project: "the order is more or less the order in which strategies are likely to emerge and be used by children" (p. 4).

So that the stability of the growth point scale could be determined, test-retest correlations over one school year and for a 12 month period were calculated. The correlations for March to November ranged from 0.48 to 0.71 in the trial group and from 0.43 to 0.68 in the reference group (Rowley, Horne et al., 2001). With the addition of the summer break, twelve-month test-retest correlations dropped slightly, as would be expected. Over such a long period of time, when children are developing at a great rate, this represents a high level of stability, in that the relative order amongst the children is preserved quite well, although, as the data showed, considerable growth took place (Rowley, Horne et al., 2001).

The framework of growth points formed the structure for the creation of the assessment items used in the ENRP Assessment Interview. Both the interview and the framework of growth points were refined throughout the first two years of the project in response to data collected from more than 20,000 assessment interviews with children participating in the project. The assessment interviews provided teachers with insights about children's mathematical knowledge that otherwise may not have been forthcoming. Further, teachers were able to use this information to plan instruction that would provide students with the best possible opportunities to extend their mathematical understanding. These themes were also present in responses to a survey asking trial school teachers to explain how their teaching had changed as a result of their involvement in the ENRP (Clarke et al., 2002).

The longitudinal nature of the ENRP and the detailed information collected about individual children's mathematical knowledge meant that the data could be analysed to identify particular issues related to mathematical learning. For example, the complexity of the teaching process was highlighted by the spread of growth points within any particular grade level. For Grade 2 children in 2000, the spread in the Counting domain was from Growth Point 1 to Growth Point 6. It is clear that in providing effective learning experiences for children, teachers needed to cater for a wide range of abilities. This is important knowledge for teachers, and implies that the curriculum in which the children engage needs to be broad enough to cater for the differences. This type of professional knowledge also makes it possible for teachers to transform the curriculum and the mathematics instruction they provide. However, while the aim is for all teachers to be so empowered, the reality is that it is difficult for teachers to cater for all children's learning needs in the classroom. This is why alternative learning opportunities are beneficial for some children.

CONCLUSION

The ENRP framework of growth-points, the professional knowledge gained through the ENRP assessment interview and the professional development program, and the analysis of ENRP data about children's mathematical learning provided teachers with many insights about effective mathematics assessment, learning and teaching. This culminated in teachers being more confident that they were meeting the instructional needs of children, and more assured about the curriculum decisions they made.

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NUMBER ATTAINMENT IN SRI LANKAN PRIMARY SCHOOLS

Kathleen Hart

From 1998 to 2003 the Primary Mathematics Project *was operative in Sri Lanka. Part of the project was a longitudinal survey with a number of cohorts of children. Here only the progress in Number is quoted and only one cohort is considered. Other data are available. For the purpose of the forum the data are used to identify what in the syllabus for Number appears to be available to all the pupils and what concepts cause difficulty.*

Sri Lanka is an island off the southern tip of India having an area of some 66 000 square kilometres. The population is composed of Sinhala,Tamils, and Muslims. About 74% are Sinhala who are predominantly Buddhist , about 18% are Tamil and are predominantly Hindu, the 7% who are Muslim speak mainly Tamil. A civil war has continued for 20 years, waged mainly in the north but with sporadic bombings in the cities and resulting in many refugees in the east of the country.

The country has very nearly universal primary education. There is a school within walking distance of each village and the pupils are provided with school uniforms and learning materials by the government. The literacy rate on the island is one of the highest in Asia [87% in 1986] but repeated surveys have shown that mathematics attainment is low. The *Primary Mathematics Project*, funded by DfID of Great Britain and the Sri Lankan government , from which these data are produced, worked in schools all over the island but had limited access to the north because of the war. Part of the project was the National Basic Mathematics Survey [NBMS] designed to provide information on which reforms could be based. Here we report only those aspects of NBMS which concern mathematics attainment. In 1998, a total of 7400 children in grades 3, 5 and 7 were tested with written papers and a smaller sample from grades 1 and 2 were interviewed. The papers were designed to match the curriculum and to cater for what was emphasised in the school textbooks. A group of 30 teachers studied them , tried the questions in schools and revised items. The papers were produced in Tamil and Sinhala. These teachers became the evaluation team and carried out the testing in the nationwide school sample. The emphasis was on the child completing as much of the test as possible so members of the evaluation team were told to read items to pupils who appeared to have trouble reading them and to allow about an hour for completion. The report of the survey appeared in 1999 (Hart & Yahampath, 1999).

In 1999 a longitudinal study was started, taking three regions of the country and following a sample from schools of the four types found in the state education system, both Tamil and Sinhala speaking and with both boys and girls. Over two hundred children from each of grades 3 and 5, at this time, were tested in consecutive years until 2002. The pupils who were first and second graders in 1999 were tested each year until they were in grade five. The data from these youngest cohorts are reported here. In 1999 we took five children from the first grade and five from the second in each of six schools, in three towns. Tasks which matched contents of the class syllabus and which employed manipulatives and symbols were used. Each child was interviewed by a teacher from another school who had been trained on the tasks. An audio tape of the interview and notes from two observers provided the data.

COHORT ONE

The 87 first grade pupils interviewed in 1999 had only been in school for five months. The syllabus indicated what was considered suitable at this stage and so the tasks were chosen to reflect this. Sorting tasks, the use of vocabulary for 'front', middle' and 'behind' were included but here we will concentrate on Number. A form of the classic Piagetian conservation task was used with questions such as 'Are there the same number?' referring to two piles of objects and then a displacement of one set was made to see if the child changed his/her opinion. Under half the sample responded correctly [47, 42, 40 per cent.] Another task was the recognition of symbols for 1,2,and 3. A card with the symbol was shown and the child asked 'Give me that number of toys'.' Read the card for me'.

Ninety five percent could read '1' and 78% could give the correct amount of toys. For the number '2' this was reduced to 85% and 55% and for '3' the results were 70% and 56%. Given a card with '3' written on it but only two toys with it, 55% could rectify the situation. When asked to count beads [16], 50% could do it correctly, with a further 20% completing part of the count.

We did not interview this group of pupils for another 17 months, towards the end of their second year in school but another group of first graders were interviewed towards the end of their first year in 2000. They were from the same schools. The Piagetian conservation task was more successful, 64, 50 and 58 per cent but it is clear that this task cannot be assumed to be within the grasp of the great majority of the children. However matching groups of objects to the symbols'1', '2', '3', was achieved by 100, 93 and 97% of the children and 90% could correct the number of toys to give '3' In this group 90% could accurately give seven objects, matching the symbol.. The range of objects which could be counted was also extended, so that 88% could count up to 16. However when ,as the syllabus suggested, the children were asked to add 3 and 4 [written on cards] only 67% could do it. Forty five percent counted on their fingers to add these two numbers.

When we tested cohort one towards the end of their second year in school, they were again interviewed on tasks which reflected the class syllabus. By now over 90% of the group [the sample was reduced to 79 from 87] could read number symbols of 1 to 9, say which number was smaller and identify that the cards for 5, 7 and 9 were missing from a sequence of cards. Given a set of dominoes they could total the number of dots on two touching sections, that is provided with objects to count they could provide a total over ten.

In 2000 the interviewers added some questions on subtraction, since it was at the end of the second grade. 'Eight birds were in the tree and three flew away, how many were left?'. Eighty percent had this correct and 95 % when the question was repeated with '8 flew away'.

All the questions given to grades 1 and 2 reported so far were given orally. The syllabus does contain some written computations so the following were given to the pupils, written on paper. The percentage success is shown below in Table 1.

5 +3 ——	7 +8 ——	4 +4 ——	2+4=……	6+6=……	
86%	62%	91%	80%	68%	Success rate
5 -2 ——	9 -2 ——	7 -7 ——	8-4……	3-3……	
72%	61%	58%	58%	54%	Success rate

Table 1. Written Computations Year Two .[2000]

The questions are now too difficult for nearly half the pupils so the syllabus seems to be ahead of the children.

THIRD GRADE. COHORT 1

Towards the end of the third grade the same cohort of children were asked questions pertaining to the syllabus. By now the expectation is that pupils are writing computations in their books and there is a third grade textbook. The tests, given in November, had some questions given orally and a test paper which had printed questions but which the evaluator could read to the child if needed [there were only five in a group]. The oral questions were about Number, Shape and Money and very similar to those asked in Year 2. For Number there was a further question about the number which comes before and after ' 7.' On this latter there was success at the 85% level and on the earlier questions success was at over 95%. The second year work tested here had been consolidated. When it came to the regular third grade questions on the test paper the mean score for the paper was 39%. Failure has arrived.

By the end of third grade the pupils are expected to deal with two and three digit numbers, do addition and subtraction algorithms including decomposition. cope with multiplication of two 2 digit numbers and even shade one half of a diagram. The only question which had a facility of over 85% was completing a sequence of numbers from the five times table. About half the pupils could correctly identify the number of hundreds, tens and ones given a three digit number. The two digit algorithms were adequately completed only if it was single digit work involved, that is no regrouping of tens. This is shown in Table 2 below.

75 -32 _____	81 -25 _____	39 +18 _____	305 +217 _____	
72%	41%	45%	36%	Success rate

Table 2 . Two Digit Algorithms. Grade 3

Cohort One was tested again in grades four and five. Other cohorts were followed and it was obvious that although performance was mixed, those who performed badly or even at a 'middle' level in grade 3 never achieved great success later. Grade 3 seems a very great hurdle. According to the teachers of these children 'place value' is a problem and certainly the algorithms quoted above become not just difficult but very difficult when decomposition is involved.

In the forum we will look at these and other data and try to sequence what is in the syllabus so that the difficulties become more obvious to a teacher. The aim is not to throw out what teachers, certainly in this sample, feel is the mathematics they want or intend to teach but to offer information which might provide a better chance of success. All participants are encouraged to bring data and also the Number Curriculum taught in the first four years of their primary schools.

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MATHEMATICS RECOVERY: FRAMEWORKS TO ASSIST STUDENTS' CONSTRUCTION OF ARITHMETICAL KNOWLEDGE.

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Mathematics Recovery was the outcome of a three-year research and development project at Southern Cross University in northern New South Wales, conducted in 1992-5. The project received major funding from the Australian Research Council and major contributions in the form of teacher time, from regional government and Catholic school systems. Over the 3-year period, the project involved working in 18 schools with 20 teachers and approximately 200 participating first-grade students (Wright, 2000).

MR can be regarded as consisting of two distinct but interrelated components. One component concerns an elaborated body of theory and practice for working with students, that is, teaching early number knowledge (Wright et al., 2000; & Wright et al., 2002). The second component concerns distinctive ways of working with teachers, that is, providing effective, long-term professional development in order to enable teachers to learn about working with students (Wright, 2000, pp.140-4).

The theoretical origins of MR are in the research program of Les Steffe, a professor in mathematics education, at the University of Georgia in the United States. In the 1970s and 1980s, Steffe's research focused almost exclusively on early number learning (e.g. Steffe & Cobb, 1988; Steffe, 1992). The goal of this research is to develop psychological models to explain and predict students' mathematical learning and development. Of particular interest in this approach, is the strategies – for which Steffe uses the Piagetian label of 'schemes', that the student uses in situations that are problematic for the student, and how these schemes develop and are re-organised over the course of an extended teaching cycle, as observed in teaching sessions mainly, but also in pre- and post- interview-based assessments.

Steffe's research and Mathematics Recovery have as their basic orientation, von Glasersfeld's theory of cognitive constructivism – an epistemological theory that has been developed and explicated over the last 30 or more years, (e.g. von Glasersfeld, 1978; 1995). Von Glasersfeld's theory is a theory about knowing – how humans come to know, rather than for example, an approach to teaching.

Assessment in Mathematics Recovery involves a one-on-one interview, in which the student is presented with groups of tasks, where each group relates to a particular aspect of early number learning. The assessment has two broad purposes. First, it should provide a rich, detailed description of the student's current knowledge of early number. Second, the assessment should lead to determination of levels on the relevant tables in the framework of assessment and learning (Wright et al., 2000)

One of the key elements of the MR program is its framework for assessment and learning – usually referred to as the Learning Framework in Number. One important function of the framework is to enable summary profiling of students' current knowledge. The profiling is based on six aspects of number early number knowledge referred to as a model. Each model contains a progression of up to six levels indicating the development of students' knowledge on that particular aspect of early number learning. Taken together, the models can be regarded as laying out a multi-faceted progression of students' knowledge and learning in early number, and in this sense the models are analogous to a framework (Wright et al., 2002, e.g. p. 77).

The view in MR is that models consisting of progressions of levels of student knowledge constitute one important part of a learning framework. A comprehensive learning framework should also contain: (a) descriptions of assessment tasks that relate closely to the levels on each of the models, and thus enable determination of the student's level; (b) descriptions of other assessment tasks which might not relate directly to the models but nevertheless, have the potential to provide important information about early number knowledge; (c) comprehensive descriptions of the likely responses of students to the all assessment tasks; and (d) descriptions of other aspects of early number knowledge considered to be relevant to students' overall learning of early number. A framework as just described can rightly be regarded as a comprehensive framework for assessment and learning.

The Learning Framework in Number (LFIN) is regarded as a rich description of the students' early number knowledge. This includes, but is not limited to, the strategies that student uses to solve what adults might regard as simple number tasks (additive, subtractive). While it is important to document students early arithmetical strategies, it is not sufficient to describe students' knowledge merely in terms of the currently available strategies. As well, there are important aspects of students' knowledge not simply described in terms of strategies used to solve problems. These aspects include for example, facility with spoken and heard number words, and ability to identify (name) numerals.

The six aspects of the framework are described in terms of a progression of levels. These are: (a) strategies for counting and solving simple addition and subtraction tasks; (b) very early place value knowledge, that is, ability to reason in terms of tens and ones; (c) facility with forward number word sequences; (d) facility with backward number word sequences; (e) facility with numeral identification; and (f) early knowledge of multiplication and division. Other aspects of the framework relate to: (a) combining and partitioning small numbers without counting; (b) using five and ten as reference points in numerical reasoning; (c) use of finger patterns in numerical contexts; (d) relating number to spatial patterns; and (e) relating number to temporal sequences. While each aspect can be considered from a distinct perspective, it is also important to focus on the inter-relationships of the aspects.

MR assessment tells the teacher 'where the student is' and the learning framework indicates 'where to take the student', but teachers don't necessarily have the time to design and develop specific instructional procedures. In the period 1999-2000, Wright and colleagues developed an explicit framework for instruction. Thus the instructional settings and activities used in earlier versions of MR were incorporated into an instructional framework (usually referred to as the Instructional Framework for Early Number – IFEN). The instructional framework differs in form from the learning framework because its purpose is different. Nevertheless it is informed by and strongly linked to the learning framework (Wright et al., 2002). The framework sets out a progression of key teaching topics which are organized into three strands as follows:

- Counting — instruction to progressively develop use of counting by ones, to solve arithmetical tasks.
- Grouping — instruction to develop arithmetical strategies other than counting by ones.
- Number words and numerals — instruction to develop facility with FNWSs, BNWSs and a range of aspects related to numerals.

Each of the three strands spans a common set of five phases of instruction. Each key topic contains on average, six instructional procedures. Each instructional procedure includes explicit descriptions of the teachers' words and actions, as well as descriptions of the instructional setting (materials, instructional resources), and notes on purpose, teaching and students' responses. Finally, each instructional procedure typically is linked to a level in one or more of the models (aspects) of the learning framework. Thus the teacher is not only provided with exemplary instructional procedures suited to any particular student but is forearmed with detailed knowledge of ways the student is likely to respond to each instructional procedure.

Recent research (Wright, 1998; 2002), highlights the relative complexities of students' early number knowledge, and the usefulness of close observation and assessment in enabling detailed understanding of students' arithmetical knowledge and strategies. Critical to the efforts of teachers to address students' learning difficulties in mathematics are elaborated exemplars of theory-based practice directed at addressing mathematics learning difficulties.

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Appendix 1
ENRP Number Growth Points (Preparatory – Year 2)

Counting Growth Points 0. Not apparent. Not yet able to state the sequence of number names to 20. 1. Rote counting Rote counts the number sequence to at least 20, but is not yet able to reliably count a collection of that size. 2. Counting collections Confidently counts a collection of around 20 objects. 3. Counting by 1s (forward/backward, including variable starting points; before/after) Counts forwards and backwards from various starting points between 1 and 100; knows numbers before and after a given number. 4. Counting from 0 by 2s, 5s, and 10s Can count from 0 by 2s, 5s, and 10s to a given target. 5. Counting from x (where x >0) by 2s, 5s, and 10s Given a non-zero starting point, can count by 2s, 5s, and 10s to a given target. 6. Extending and applying counting skills Can count from a non-zero starting point by any single digit number, and can apply counting skills in practical tasks.	**Place Value Growth Points** 0. Not apparent Not yet able to read, write, interpret and order single digit numbers. 1. Reading, writing, interpreting, and ordering single digit numbers Can read, write, interpret and order single digit numbers. 2. Reading, writing, interpreting, and ordering two-digit numbers Can read, write, interpret and order two-digit numbers. 3. Reading, writing, interpreting, and ordering three-digit numbers Can read, write, interpret and order three-digit numbers. 4. Reading, writing, interpreting, and ordering numbers beyond 1000 Can read, write, interpret and order numbers beyond 1000. 5. Extending and applying place value knowledge Can extend and apply knowledge of place value in solving problems.
Strategies for Addition & Subtraction Growth Points 0. Not apparent Not yet able to combine and count two collections of objects. 1. Count all (two collections) Counts all to find the total of two collections. 2. Count on Counts on from one number to find the total of two collections. 3. Count back/count down to/count up from Given a subtraction situation, chooses appropriately from strategies including count back, count down to and count up from. 4. Basic strategies (doubles, commutativity, adding 10, tens facts, other known facts) Given an addition or subtraction problem, strategies such as doubles, commutativity, adding 10, tens facts, and other known facts are evident. 5. Derived strategies (near doubles, adding 9, build to next ten, fact families, intuitive strategies) Given an addition or subtraction problem, strategies such as near doubles, adding 9, build to next ten, fact families and intuitive strategies are evident. 6. Extending and applying addition and subtraction using basic, derived and intuitive strategies Given a range of tasks (including multi-digit numbers), can solve them mentally, using the appropriate strategies and a clear understanding of key concepts.	**Strategies for Multiplication & Division Growth Points** 0. Not apparent Not yet able to create and count the total of several small groups. 1. Counting group items as ones To find the total in a multiple group situation, refers to individual items only. 2. Modelling multiplication and division (all objects perceived) Models all objects to solve multiplicative and sharing situations. 3. Abstracting multiplication and division Solves multiplication and division problems where objects are not all modelled or perceived. 4. Basic, derived and intuitive strategies for multiplication Can solve a range of multiplication problems using strategies such as commutativity, skip counting and building up from known facts. 5. Basic, derived and intuitive strategies for division Can solve a range of division problems using strategies such as fact families and building up from known facts. 6. Extending and applying multiplication and division Can solve a range of multiplication and division problems (including multi-digit numbers) in practical contexts.