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THE UNCERTAINTY WAY OF GENERALIZATION OF COHERENT STATES

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Abstract. The three ways of generalization of canonical coherent states are briefly reviewed and compared with the emphasis laid on the (minimum) uncertainty way. The characteristic uncertainty relations, which include the Schrödinger and Robertson inequalities, are extended to the case of several states. It is shown that the standard $SU(1,1)$ and $SU(2)$ coherent states are the unique states which minimize the second order characteristic inequality for the three generators. A set of states which minimize the Schrödinger inequality for the Hermitian components of the $su_q(1,1)$ ladder operator is also constructed. It is noted that the characteristic uncertainty relations can be written in the alternative complementary form.

1. Introduction

Coherent states (CS) introduced in 1963 in the pioneering works by Glauber and Klauder [1] pervade nearly all branches of quantum physics (see the reviews [1, 4]). This important overcomplete family of states $\{|\alpha\rangle\}$, $\alpha \in \mathbb{C}$, can be defined in three equivalent ways [3]:

D1) As the set of eigenstates of boson destruction operator (the ladder operator) $a:\ a|\alpha\rangle = \alpha|\alpha\rangle$,

D2) As the orbit of the ground state $|0\rangle$ ($a|0\rangle = 0$) under the action of the unitary displacement operators $D(\alpha) = \exp(\alpha a^\dagger - \alpha^* a)$ (which realize ray representation of the Heisenberg–Weyl group H_1) : $|\alpha\rangle = D(\alpha)|0\rangle$.

D3) As the set of states which minimize the Heisenberg uncertainty relation (UR) $(\Delta q)^2(\Delta p)^2 \geq 1/4$ for the Hermitian components q, p of a ($a = (q + \mathrm{i}p)/\sqrt{2}$) with equal uncertainties: $(\Delta q)^2(\Delta p)^2 = 1/4$, $\Delta q = \Delta p$. Note that one requires the minimization plus the equality of the two variances.