

A NOTE ON POISSON LIE ALGEBROIDS*

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Abstract. In this paper we study some properties of a Lie algebroid and its prolongation over the vector bundle projection of the dual bundle. We generalize some results on Poisson manifolds to the level of a Lie algebroid. The notions of canonical Poisson bivector and horizontal lift are studied and their compatibility conditions are pointed out.

1. Introduction

The Lie algebroid [10] is a generalization of both concepts of Lie algebra and integrable distribution, being a vector bundle (E, π, M) with a Lie bracket on his space of sections with properties very similar to those of a tangent bundle. The Poisson manifolds are the smooth manifolds equipped with a Poisson bracket on their ring of functions. I have to remark that the cotangent bundle of a Poisson manifold has the natural structure of a Lie algebroid [13]. In the last years diverse aspects of these subjects have been studied in a lot of papers (see for instance [13], [14], [12], [1] and [7]). In the present paper we study some geometrical structures on the prolongation of a Lie algebroid to its dual bundle and investigate some aspects of the Lie algebroid geometry endowed with a Poisson structure. In this way we generalize some results on Poisson manifolds.

The paper is organized as follows. In the Section 2 we recall the Cartan calculus and the Schouten-Nijenhuis bracket at the level of a Lie algebroid and present the Poisson structures on the Lie algebroid. The Section 3 deals with the prolongation of a Lie algebroid [5], [8] to its dual bundle and continue the investigation starting in [6]. We study the properties of the canonical Poisson bivector and introduce the notion of horizontal lift. Finally, the compatibility conditions of these bivectors

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