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# ON MULTICOMPONENT DERIVATIVE NONLINEAR SCHRÖDINGER EQUATION RELATED TO SYMMETRIC SPACES

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**Abstract.** We study derivative nonlinear Schrödinger equations related to symmetric spaces of the type **A.III**. We discuss the spectral properties of the corresponding Lax operator and develop the direct scattering problem connected to it. By applying an appropriately chosen dressing factor we derive soliton solutions to the nonlinear equation. We find the integrals of motion by using the method of diagonalization of Lax pair.

## 1. Introduction

A classical example of a nonlinear evolution equation integrable by means of the inverse scattering transform is provided by derivative nonlinear Schrödinger equation (DNSE)

$$\mathrm{i}q_t + q_{xx} + \mathrm{i}(|q|^2 q)_x = 0 \tag{1}$$

where function $q\colon \mathbb{R}^2 \to \mathbb{C}$ is infinitely smooth. DNSE occurs in plasma physics to describe the propagation of nonlinear Alfvén waves with circular polarization [16, 17]. Equation (1) is equivalent to compatibility condition $[L, A] = 0$ for $L$ and $A$ chosen in the form [13]

$$\begin{aligned} L(\lambda) &:= \mathrm{i}\partial_x + \lambda Q(x,t) - \lambda^2\sigma_3 \\ A(\lambda) &:= \mathrm{i}\partial_t + \sum_{k=1}^{3} A_k(x,t)\lambda^k - 2\lambda^4\sigma_3 \end{aligned} \tag{2}$$

where $\lambda \in \mathbb{C}$ is a spectral parameter and

$$Q(x,t) = \begin{pmatrix} 0 & q(x,t) \\ q^*(x,t) & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$