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SOME EXAMPLES RELATED TO THE DELIGNE–SIMPSON PROBLEM*

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Abstract. We consider the variety of $(p{+}1)$-tuples of matrices M_j from given conjugacy classes $C_j \subset GL(n,\mathbb{C})$ such that $M_1 \cdots M_{p+1} = I$. This variety is connected with the *Deligne–Simpson problem: give necessary and sufficient conditions on the choice of the conjugacy classes $C_j \subset GL(n,\mathbb{C})$ so that there exist irreducible $(p{+}1)$-tuples of matrices $M_j \in C_j$ whose product equals I.* The matrices M_j are interpreted as monodromy operators of regular linear systems on Riemann's sphere. We consider among others cases when the dimension of the variety is higher than the expected one due to the presence of $(p+1)$-tuples with non-trivial centralizers.

1. Introduction

1.1. The Deligne–Simpson Problem

In the present paper we consider some examples related to the **Deligne–Simpson Problem** (DSP) which is formulated like this:

Give necessary and sufficient conditions upon the choice of the $p{+}1$ conjugacy classes $c_j \subset gl(n,\mathbb{C})$, resp. $C_j \subset GL(n,\mathbb{C})$, so that there exist irreducible $(p+1)$-tuples of matrices $A_j \in c_j$, $A_1 + \cdots + A_{p+1} = 0$, resp. of matrices $M_j \in C_j$, $M_1 \cdots M_{p+1} = I$.

By definition, the **weak DSP** is the DSP in which the requirement of irreducibility is replaced by the weaker requirement the centralizer of the $(p+1)$-tuple of matrices to be trivial.

The matrices A_j, resp. M_j, are interpreted as *matrices-residua* of *Fuchsian* systems on Riemann's sphere (i. e. linear systems of ordinary differential equa-

*To the memory of my mother.