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COMPRESSED PRODUCT OF BALLS AND LOWER BOUNDARY ESTIMATES ON BERGMAN KERNELS

AZNIV KASPARIAN

Department of Mathematics and Informatics, "St Kliment Ohridski" University
5 James Bourchier Blvd, 1126 Sofia, Bulgaria

Abstract. The image $B_{p^\sigma,q}$ of a product of balls $B_p \times B_q$ under a compression $c_\sigma(X,V) = (X, V(1 - {}^t\bar{X}X)^{\frac{\sigma}{2}})$ is called a compressed product of balls of exponent $\sigma \in \mathbb{R}$. The present note obtains the group $\mathrm{Aut}(B_{p^\sigma,q})$ of the holomorphic automorphisms and the $\mathrm{Aut}(B_{p^\sigma,q})$-orbit structure of $B_{p^\sigma,q}$ and its boundary $\partial B_{p^\sigma,q}$ for $\sigma > 1$. The Bergman completeness of $B_{p^\sigma,q}$ is verified by an explicit calculation of the Bergman kernel. As a consequence, local lower boundary estimates on the Bergman kernels of the bounded pseudoconvex domains are obtained, which are locally inscribed in $B_{p^\sigma,q}$ at a common boundary point.

For a strictly pseudoconvex domain $\mathcal{D} = \{z \in \mathbb{C}^n\,;\ \rho(z) < 0\}$ with a smooth boundary, Fefferman [6] and Boutet de Monvel–Sjöstrand [2] have expanded the diagonal values $k_{\mathcal{D}}(z) := k_{\mathcal{D}}(z,z)$ of the Bergman kernel in the form $k_{\mathcal{D}}(z) = \varphi_{\mathcal{D}}(\rho)\rho^{-n-1} + \psi_{\mathcal{D}}(\rho)\log(-\rho)$, where $\varphi_{\mathcal{D}}(\rho)$ and $\psi_{\mathcal{D}}(\rho)$ are power series in the defining function $\rho = \rho(z)$.

Only few results are known for the boundary behavior of the Bergman kernel of a weakly pseudoconvex domain. For arbitrary $m = (m_1, \ldots, m_n) \in \mathbb{N}^n$ let $\mathcal{E}_m := \{z \in \mathbb{C}^n\,;\ \rho_m(z) = \sum_{j=1}^n |z_j|^{2m_j} - 1 < 0\}$, $z^o \in \partial\mathcal{E}_m$,

$$P_m := \{j \in \mathbb{N}\,;\ z_j^o = 0 \text{ and } m_j > 1\}, \quad Q_m := \{j \in \mathbb{N}\,;\ z_j^o \neq 0 \text{ or } m_j = 1\}\,.$$

Kamimoto has established in [11] the existence of an open subset $U \subset \mathbb{C}^n$ with $z^o \in \partial U$ and a real analytic function $\Phi_m \colon U \to \{r \in \mathbb{R};\ r > 0\}$, such that $k_{\mathcal{E}_m}(z) = \Phi_m(z)\rho_m(z)^{-\sum_{j\in P_m} m_j^{-1} - \mathrm{card}\, Q_m - 1}$ on U. If $P_m = \emptyset$ then $\Phi_m(z)$ is bounded around z^o, while $\lim_{z\to z^o}\Phi_m(z) = \infty$ for $P_m \neq \emptyset$.