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A MATHEMATICAL EXAMINATION OF SQUEEZING AND STRETCHING OF SPHERICAL VESICLES

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Abstract. The geometry of a few simple models closely related to the Cole's and Yoneda's experiments are studied in some detail. Their principal differences are through the boundary conditions at the points where the membranes detach themselves from the compressing planes. Mathematically this affect the equilibrium configurations as in all these cases the mean curvature suffers a discontinuity while this is not so for the tangent vector. A new parametrization of the Delaunay's nodoids is derived.

1. Introduction

The mechanism maintaining the distinctive biconcave shape of the red blood cell has been a subject of considerable curiosity since its discovery more than two century ago. Actually, the shape of red blood cells and other closed biological membranes (vesicles) is closely related to the formation of lipid bilayer vesicles in aqueous medium. The interface, i.e., the lipid membrane is treated as a thin elastic shell and possesses four modes of deformation – dilatation, bending, shearing and torsion. From the geometrical viewpoint the bending and torsion are related to variations of the two principal curvatures of the interface.

Quantitatively, the elastic properties of the open lipid membranes and vesicles are described by the free energy which itself depends on the principal curvatures of the surfaces that represent them. By taking the first order variation of the free energy one derives a highly nonlinear equation which describes the possible equilibrium shapes [12]. The difficulties which one encounters in any serious attempt to solve this equation suggests that all kind of information providing more deep understanding of the observed highly nonlinear behaviour of vesicles is without any doubt of a definite value.

The curvature dependence of the interfacial tension has been investigated for the fist time by Young and Laplace in their study of the Euler-Lagrange variational