Seventh International Conference on
Geometry, Integrability and Quantization
June 2–10, 2005, Varna, Bulgaria
Ivaïlo M. Mladenov and Manuel de León, Editors
SOFTEX, Sofia 2006, pp 176–186

MEMBRANE APPROACH TO BALLOONS AND SOME RELATED SURFACES

MARIANA HADZHILAZOVA and IVAÏLO M. MLADENOV

Institute of Biophysics, Bulgarian Academy of Sciences
Acad. G. Bonchev Str., Bl. 21, 1113 Sofia, Bulgaria

Abstract. The well known Laplace–Young equation asserts that the pressure difference across the film or a membrane in a equilibrium is proportional to the mean curvature with a proportionality constant the surface tension of the interface. Here we present two variants of this equation leading to the surfaces of Delaunay and the mylar balloon and in this way provide their nonvariational characterization.

1. Introduction

The equations describing axisymmetric membranes are used not only in Biology but also in many other areas including the design and development of scientific balloons. The balloons are used by many space agencies to carry out researches in the upper stratosphere. The equilibrium equations of such surfaces are expressed as some boundary value problems. In order to study all external and internal forces acting on to the surface we are lead to many different cases of equilibrium conditions. Some of them can be solved exactly to determine the shape. The physical meaning of these parameters plays a very important role. The most crucial quantities as the membrane weight density, circumferential and meridional stresses, the differential pressure could variate. Guided by mechanical ideas we will derive two classes of shapes having quite interesting geometrical properties.

2. Axisymmetric Membranes

As usual we will think of the axisymmetric surface $\mathcal{S}$ by specifying its meridional section, i.e., a curve $u \longrightarrow (r(u), z(u))$ in the XOZ plane, assuming that u is the so called natural parameter provided by the corresponding arc length. We will denote the total arc length by L. The surface $\mathcal{S}$ can be presented in the ordinary