ON A BASIC PROBLEM FOR A SECOND ORDER DIFFERENTIAL EQUATION WITH A DISCONTINUOUS COEFFICIENT AND A SPECTRAL PARAMETER IN THE BOUNDARY CONDITIONS

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Abstract. In the present paper we investigate the completeness, minimality and basic properties of the eigenfunctions of one discontinuous Sturm–Liouville problem with a spectral parameter in boundary conditions and transmission conditions.

1. Introduction

We consider the discontinuous boundary value problem with a spectral parameter in the boundary conditions for a second order ordinary differential equation:

$$l(u) \equiv -p(x)u'' + q(x)u = \lambda u, \quad x \in [a,c) \cup (c,b] \tag{1}$$

$$\alpha_{11}u(a) - \alpha_{12}u'(a) = \lambda\left(\alpha_{21}u(a) - \alpha_{22}u'(a)\right) \tag{2}$$

$$\beta_{11}u(b) - \beta_{12}u'(b) = \lambda\left(\beta_{21}u(b) - \beta_{22}u'(b)\right) \tag{3}$$

$$u(c+0) - u(c-0) = 0 \tag{4}$$

$$u'(c+0) - u'(c-0) = -\lambda\delta_1 u(c) \tag{5}$$

where $p(x) = \frac{1}{p_1^2}$ for $x \in [a,c)$ and $p(x) = \frac{1}{p_2^2}$ for $x \in (c,b]$, $q(x)$ is a real-valued continuous function on the intervals $[a,c)$ and $(c,b]$ and has a finite limits $q(c \pm 0) = \lim_{x \to c \pm 0} q(x)$; p_i, α_{ij}, β_{ij} $(i,j = 1,2)$ are real constants. We assume also that $\delta_1 > 0$ and

$$\rho_1 = \begin{vmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{vmatrix}, \qquad \rho_2 = \begin{vmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{vmatrix}.$$

In the present work we investigate the completeness, the minimality and the basic properties of the system of eigenfunctions of the discontinuous boundary value problem (1)–(5). Note that the spectral properties of the boundary-value problem