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EXAMPLES OF PSEUDO-RIEMANNIAN G.O. MANIFOLDS

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Abstract. We modify the metrics on six-dimensional and seven-dimensional Riemannian g.o. manifolds constructed in previous published papers and we obtain pseudo-Riemannian g.o. manifolds. We describe geodesic graphs of corresponding g.o. spaces. We show that if these geodesic graphs are nonlinear, they are discontinuous on an nonempty set but they are continuous at the origin.

1. Introduction

Let M be a pseudo-Riemannian manifold. If there is a connected Lie group $G \subset I_0(M)$ which acts transitively on M as a group of isometries, then M is called a **homogeneous pseudo-Riemannian manifold**. Let $p \in M$ be a fixed point. If we denote by H the isotropy group at p, then M can be identified with the *homogeneous space* G/H. In general, there may exist more than one such group $G \subset I_0(M)$. For any fixed choice $M = G/H$, G acts effectively on G/H on the left. The pseudo-Riemannian metric g on M can be considered as a G-invariant metric on G/H. The pair $(G/H, g)$ is then called a **pseudo-Riemannian homogeneous space**.

If the metric g is a positive definite, then $(G/H, g)$ is always a *reductive homogeneous space*: We denote by $\mathfrak{g}$ and $\mathfrak{h}$ the Lie algebras of G and H respectively and consider the adjoint representation $\mathrm{Ad} : H \times \mathfrak{g} \to \mathfrak{g}$ of H on $\mathfrak{g}$. There exists a direct sum decomposition (*reductive decomposition*) of the form $\mathfrak{g} = \mathfrak{m} + \mathfrak{h}$ where $\mathfrak{m} \subset \mathfrak{g}$ is a vector subspace such that $\mathrm{Ad}(H)(\mathfrak{m}) \subset \mathfrak{m}$. If the metric g is indefinite, the reductive decomposition may not exist (see [6] for an example of nonreductive pseudo-Riemannian homogeneous space). For a fixed reductive decomposition $\mathfrak{g} = \mathfrak{m} + \mathfrak{h}$ there is a natural identification of $\mathfrak{m} \subset \mathfrak{g} = T_eG$ with the