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STABLE ITERATION SCHEMES FOR LOCAL STRONGLY PSEUDOCONTRACTIONS AND NONLINEAR EQUATIONS INVOLVING LOCAL STRONGLY ACCRETIVE OPERATORS

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Abstract. Let T be a local strongly pseudocontractive and uniformly continuous operator from an arbitrary Banach space X into itself. Under certain conditions, we establish that the Noor iteration scheme with errors both converges strongly to a unique fixed point of T and is almost T-stable. The related results deal with the convergence and almost stability of the Noor iteration scheme with errors of solutions of nonlinear equations of the local strongly accretive type.

1. Introduction

Let X be an arbitrary Banach space, X^* be its duel space and $\langle x, f\rangle$ be the generalized duality pairing between $x \in X$ and $f \in X^*$. The mapping $J\colon X \to 2^{X^*}$ defined by

$$J(x) = \{f \in X^* : \mathrm{Re}\langle x, f\rangle = \|x\| \cdot \|f\|, \|f\| = \|x\|\}, \quad \forall\, x \in X,$$

is called the normalized duality mapping. An operator T with domain $D(T)$ and range $R(T)$ in X is said to be local strongly pseudocontractive if for each $x \in D(T)$ there exists $t_x > 1$ such that for all $y \in D(T)$ and $r > 0$

$$\|x - y\| \le \|(1 + r)(x - y) - rt_x(Tx - Ty)\|. \tag{1.1}$$

An operator T is called local strongly accretive if given $x \in D(T)$ there exists $k_x \in (0, 1)$ such that for each $y \in D(T)$ there is $j(x - y) \in J(x - y)$ satisfying

$$\mathrm{Re}\langle Tx - Ty, j(x - y)\rangle \ge k_x\|x - y\|^2. \tag{1.2}$$

In particular, the operator T is called strongly pseudocontractive (respectively, strongly accretive) if $t_x \equiv t$ (respectively, $k_x \equiv k$) is independent of $x \in D(T)$. In the sequel, we denote by $I, F(T)$ and $S(T)$ the identity mapping on X, the set of all fixed points of T, and the set of all solutions of the equation $Tx = f$, respectively.

Clearly, each strongly pseudocontractive operator is local strongly pseudocontractive and each strongly accretive operator is local strongly accretive. It is known

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that T is local strongly pseudocontractive if and only if $I-T$ is local strongly accretive and $k_x = 1 - \frac{1}{t_x}$, where t_x and k_x are the constants appearing in (1.1) and (1.2), respectively.

Suppose that T is an operator on X. Assume that $x_0 \in X$ and $x_{n+1} = f(T, x_n)$ defines an iteration scheme which produces a sequence $\{x_n\}_{n=0}^{\infty} \subset X$. Suppose, furthermore, that $\{x_n\}_{n=0}^{\infty}$ converges strongly to $q \in F(T) \neq \emptyset$. Let $\{y_n\}_{n=0}^{\infty}$ be any sequence in X and put $\varepsilon_n = \|y_{n+1} - f(T, y_n)\|$.

Definition 1.1 ([14]). (i) The iteration scheme $\{x_n\}_{n=0}^{\infty}$ defined by $x_{n+1} = f(T, x_n)$ is said to be T- stable if $\lim_{n\to\infty} \varepsilon_n = 0$ implies that $\lim_{n\to\infty} y_n = q$;

(ii) The iteration scheme $\{x_n\}_{n=0}^{\infty}$ defined by $x_{n+1} = f(T, x_n)$ is said to be almost T- stable if $\sum_{n=0}^{\infty} \varepsilon_n < \infty$ implies that $\lim_{n\to\infty} y_n = q$.

Note that $\{y_n\}_{n=0}^{\infty}$ is bounded provided that the iteration scheme $\{x_n\}_{n=0}^{\infty}$ defined by $x_{n+1} = f(T, x_n)$ is either T-stable or almost T-stable. Therefore we revise Definition 1.1 as follows:

Definition 1.2. (i) The iteration scheme $\{x_n\}_{n=0}^{\infty}$ defined by $x_{n+1} = f(T, x_n)$ is said to be T- stable if $\{y_n\}_{n=0}^{\infty}$ is bounded and $\lim_{n\to\infty} \varepsilon_n = 0$ implies that $\lim_{n\to\infty} y_n = q$;

(ii) The iteration scheme $\{x_n\}_{n=0}^{\infty}$ defined by $x_{n+1} = f(T, x_n)$ is said to be almost T- stable if $\{y_n\}_{n=0}^{\infty}$ is bounded and $\sum_{n=0}^{\infty} \varepsilon_n < \infty$ implies that $\lim_{n\to\infty} y_n = q$.

Definition 1.3 ([49, 50, 56]). Let K be a nonempty convex subset of an arbitrary Banach space X and $T: K \to K$ be an operator.

(i) For any given $x_n \in K$ the sequence $\{x_n\}_{n=0}^{\infty}$ defined by

$$\begin{aligned} x_{n+1} &= a_n x_n + b_n T y_n + c_n u_n, \\ y_n &= a'_n x_n + b'_n T z_n + c'_n v_n, \\ z_n &= a''_n x_n + b''_n T x_n + c''_n w_n, \quad \forall\, n \geq 0, \end{aligned}$$

is called the Noor iteration sequence with errors, where $\{u_n\}_{n=0}^{\infty}$, $\{v_n\}_{n=0}^{\infty}$, and $\{w_n\}_{n=0}^{\infty}$ are arbitrary bounded sequences in K and $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$, $\{b'_n\}_{n=0}^{\infty}$, $\{c'_n\}_{n=0}^{\infty}$, $\{a''_n\}_{n=0}^{\infty}$, $\{b''_n\}_{n=0}^{\infty}$ and $\{c''_n\}_{n=0}^{\infty}$, are real sequences in $[0, 1]$ such that $a_n + b_n + c_n = a'_n + b'_n + c'_n = a''_n + b''_n + c''_n$ for all $n \geq 0$;

(ii) If $b''_n = c''_n = 0$ for all $n \geq 0$ or $b'_n = c'_n = b''_n = c''_n = 0$ for all $n \geq 0$, then the sequence $\{x_n\}_{n=0}^{\infty}$ defined by (i) is called the Ishikawa or Mann iteration sequence with errors, respectively;

(iii) If $c_n = c'_n = c''_n = 0$ for all $n \geq 0$, or $c_n = c'_n = c''_n = b''_n = 0$ for all $n \geq 0$, or $c_n = c'_n = c''_n = b'_n = b''_n = 0$ for all $n \geq 0$, then the sequence $\{x_n\}_{n=0}^{\infty}$ defined by (i) is called the Noor, or Ishikawa or Mann iteration sequences, respectively.

It is clear that the Noor, Ishikawa and Mann iteration sequences are all special cases of the Noor, Ishikawa and Mann iteration sequences with errors, respectively.

Chidume [3] studied the Mann iteration sequence in $L_p(or\ l_p)$ and proved that the sequence converges strongly to the unique fixed point of T in case T is a Lipschitz strongly pseudocontractive mapping from a bounded closed convex subset K of $L_p(or\ l_p)$ into itself. Tan and Xu [54] extended the result of Chidume to both p-uniformly smooth Banach space and the Ishikawa iteration method, and they established that the Mann and Ishikawa iteration methods converge strongly to the unique solution of the equation $Tx = f$ in case T is a Lipschitzian and strongly accretive operator from a p-uniformly smooth Banach space into itself. Recently, some researchers have generalized the results in [3] and [54] either to smooth Banach spaces, uniformly smooth Banach spaces, Banach spaces or to the Mann iteration method, the Mann iteration method with errors, the Ishikawa iteration method, the Ishikawa iteration method with errors, the Noor iteration method, the Noor iteration method with errors, or to strongly accretive operators, local strongly accretive operators, strongly pseudocontractive operators, local strongly pseudocontractive operators, ϕ-strongly accretive operators and ϕ-hemicontractive operators (cf. [1, 2, 4–13, 18, 20–47, 50, 51, 55, 57]).

A few stability results for certain classes of nonlinear mappings have been established by several authors (see, e.g. [14–16, 18, 20, 24, 25, 28, 30, 31, 35, 39–41, 44, 46, 47, 51, 52]). Rhoades [53] proved that the Mann and Ishikawa iteration methods may exhibit different behaviors for different classes of nonlinear mappings. Harder and Hicks [15] revealed that the importance of investigating the stability of various iteration procedures for various classes of nonlinear mappings. Harder [14] obtained applications of stability results to first order differential equations. Osilike [51, 52] studied the stability of certain Mann and Ishikawa iteration sequences for fixed points of Lipschitz strong pseudocontractions and solutions of nonlinear accretive operator equations in real q-uniformly smooth Banach spaces.

It is our purpose in this paper to establish the convergence and almost stability of the Noor iteration scheme with errors for local strongly pseudocontractive operators in arbitrary Banach spaces. The related results deal with the convergence and almost stability of the Noor iteration scheme with errors of solutions of nonlinear equations of the local strongly accretive type. Our convergence results are generalizations and improvements of the results in [1–7, 11, 14, 20, 55].

2. Preliminaries

Lemma 2.1 ([18]). *Let X be an arbitrary Banach space and $x, y \in X$. Then $\|x\| \leq \|x+ry\|$ for every $r > 0$ if and only if there is $f \in J(x)$ such that $Re\langle y, f\rangle \geq 0$.*

Lemma 2.2 ([22]). *Suppose that $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$ and $\{\omega_n\}_{n=0}^{\infty}$ are nonnegative sequences such that*

$$\alpha_{n+1} \leq (1-\omega_n)\alpha_n + \beta_n\omega_n, \quad \forall\ n \geq 0,$$

with $\{\omega_n\}_{n=0}^{\infty} \subset [0,1]$, $\sum_{n=0}^{\infty} \omega_n = \infty$ and $\lim_{n\to\infty} \beta_n = 0$. Then $\lim_{n\to\infty} \alpha_n = 0$.

3. Main results

Our main results are as follows.

Theorem 3.1. *Let X be an arbitrary Banach space and $T\colon X \to X$ be local strongly pseudocontractive and uniformly continuous. Let $F(T) \neq \emptyset$ and $R(T)$ be bounded. Suppose that $\{u_n\}_{n=0}^{\infty}$, $\{v_n\}_{n=0}^{\infty}$ and $\{w_n\}_{n=0}^{\infty}$ are arbitrary bounded sequences in X and $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$, $\{b'_n\}_{n=0}^{\infty}$, $\{c'_n\}_{n=0}^{\infty}$, $\{a''_n\}_{n=0}^{\infty}$, $\{b''_n\}_{n=0}^{\infty}$, $\{c''_n\}_{n=0}^{\infty}$ and $\{r_n\}_{n=0}^{\infty}$ are any sequences in $[0,1]$ satisfying*

$$a_n + b_n + c_n = a'_n + b'_n + c'_n = a''_n + b''_n + c''_n = 1, \quad \forall\ n \geq 0; \tag{3.1}$$

$$c_n(1-r_n) = r_n b_n, \quad \forall\ n \geq 0; \tag{3.2}$$

$$\lim_{n\to\infty} r_n = \lim_{n\to\infty} b'_n = \lim_{n\to\infty} c'_n = 0; \tag{3.3}$$

$$\sum_{n=0}^{\infty}(b_n + c_n) = \infty. \tag{3.4}$$

Suppose that $\{x_n\}_{n=0}^{\infty}$ *is the sequence generated from an arbitrary* $x_0 \in X$ *by*

$$(3.5)\qquad \begin{aligned} x_{n+1} &= a_n x_n + b_n T y_n + c_n u_n, \\ y_n &= a'_n x_n + b'_n T z_n + c'_n v_n, \\ z_n &= a''_n x_n + b''_n T x_n + c''_n w_n, \qquad \forall\ n \geq 0. \end{aligned}$$

Let $\{f_n\}_{n=0}^{\infty}$ *be any bounded sequence in* X *and define* $\{\varepsilon_n\}_{n=0}^{\infty}$ *by*

$$(3.6)\qquad \begin{aligned} \varepsilon_n &= \|f_{n+1} - a_n f_n - b_n T s_n - c_n u_n\|, \\ s_n &= a'_n f_n + b'_n T t_n + c'_n v_n, \\ t_n &= a''_n f_n + b''_n T f_n + c''_n w_n, \quad \forall\ n \geq 0. \end{aligned}$$

Then there exist real sequences $\{h_n\}_{n=0}^{\infty}$, $\{g_n\}_{n=0}^{\infty}$ *and constant* $M > 0$ *such that*

(i) $\{x_n\}_{n=0}^{\infty}$ *converges strongly to the unique fixed point* q *of* T *and*

$$\|x_{n+1} - q\| \leq \Big(1 - (b_n + c_n)k_q\Big)\|x_n - q\| + \frac{1}{k_q}(b_n + c_n)h_n + \frac{M}{k_q}c_n,\ \forall\ n \geq 0,$$

where $k_q = 1 - \frac{1}{t_q}$;

(ii)

$$\|f_{n+1} - q\| \leq \Big(1 - (b_n + c_n)k_q\Big)\|f_n - q\| + \frac{1}{k_q}(b_n + c_n)g_n + \frac{M}{k_q}c_n + \varepsilon_n,\ \forall\ n \geq 0;$$

(iii) $\sum_{n=0}^{\infty} \varepsilon_n < \infty$ *implies that* $\lim_{n\to\infty} f_n = q$, *so that* $\{x_n\}_{n=0}^{\infty}$ *is almost* T*-stable;*

(iv) $\lim_{n\to\infty} f_n = q$ *implies that* $\lim_{n\to\infty} \varepsilon_n = 0$;

(v) $\lim_{n\to\infty} h_n = \lim_{n\to\infty} g_n = 0$.

Proof. Set $p_n = a_n f_n + b_n T s_n + c_n u_n$, $d_n = b_n + c_n$, $d'_n = b'_n + c'_n$, $\forall\ n \geq 0$. Since T is local strongly pseudocontractive and $F(T) \neq \emptyset$, it follows that $F(T)$ is a singleton and $S = I - T$ is local strongly accretive. Let $F(T) = \{q\}$. Put

$$\begin{aligned} M = &\sup\{\|Tx - q\| : x \in X\} + \sup\{\|f_n - q\| : n \geq 0\} \\ &+ \sup\{\|u_n - q\| : n \geq 0\} + \sup\{\|v_n - q\| : n \geq 0\} + \|x_0 - q\|. \end{aligned}$$

It is easy to show that

$$(3.7)\qquad \sup\{\|x_n - q\|, \|y_n - q\|, \|z_n - q\| :\ n \geq 0\} \leq M;$$

$$(3.8)\qquad \sup\{\|p_n - q\|, \|s_n - q\|, \|t_n - q\|\ n \geq 0\} \leq M.$$

It follows from (1.2) that there exists $k_q = 1 - \frac{1}{t_q}$ such that

$$\operatorname{Re}\langle Sx - Sq, j(x - q)\rangle \geq k_q\|x - q\|^2, \quad \forall\ x \in X,$$

which implies that

$$\operatorname{Re}\langle (S - k_q I)x - (S - k_q I)q, j(x - q)\rangle \geq 0, \quad \forall\ x \in X.$$

Thus Lemma 2.1 ensures that

$$(3.9)\qquad \|x - q\| \leq \|x - q + r\Big((S - k_q I)x - (S - k_q I)q\Big)\|, \quad \forall\ x \in X,\ \forall\ r > 0.$$

Using (3.1) and (3.5), we obtain that for all $n \geq 0$,

$$(3.10)\qquad \begin{aligned} (1 - d_n)x_n = &x_{n+1} - d_n T y_n - c_n(u_n - T y_n) \\ = &\Big(1 - (1 - k_q)d_n\Big)x_{n+1} + d_n(S - k_q I)x_{n+1} \\ &+ d_n(T x_{n+1} - T y_n) - c_n(u_n - T y_n), \end{aligned}$$

and

$$(1-d_n)q = \Big(1-(1-k_q)d_n\Big)q + d_n(S-k_qI)q. \tag{3.11}$$

It follows from (3.9)–(3.11) that for any $n \geq 0$,

$$\begin{aligned}(1-d_n)\|x_n - q\| \geq &\Big(1-(1-k_q)d_n\Big)\|x_{n+1} - q \\ &+ \frac{d_n}{1-d_n(1-k_q)}[(S-k_qI)x_{n+1} - (S-k_qI)q]\| \\ &- d_n\|Tx_{n+1} - Ty_n\| - c_n\|u_n - Ty_n\|.\end{aligned} \tag{3.12}$$

In view of (3.2),(3.7) and (3.12), we have for all $n \geq 0$,

$$\begin{aligned}\|x_{n+1} - q\| \leq &\frac{1-d_n}{1-(1-k_q)d_n}\|x_n - q\| \\ &+ \frac{d_n}{1-(1-k_q)d_n}\|Tx_{n+1} - Ty_n\| \\ &+ \frac{c_n}{1-(1-k_q)d_n}\|u_n - Ty_n\| \\ \leq &(1-k_qd_n)\|x_n - q\| + \frac{1}{k_q}d_nh_n + \frac{1}{k_q}Mc_n,\end{aligned} \tag{3.13}$$

where $h_n = \|Tx_{n+1} - Ty_n\|$. Note that

$$\begin{aligned}\|x_{n+1} - z_n\| \leq &b_n\|x_n - Ty_n\| + c_n\|u_n - x_n\| \\ &+ b'_n\|x_n - Tz_n\| + c'_n\|v_n - x_n\| \\ \leq &2M(d_n + d'_n) \to 0\end{aligned}$$

as $n \to \infty$. Thus uniformly continuity of T means that

$$h_n = \|Tx_{n+1} - Ty_n\| \to 0 \text{ as } n \to \infty. \tag{3.14}$$

Put $\alpha_n = \|x_n - q\|$, $\omega_n = k_qd_n$, $\beta_n = (h_n + Mr_n)k_q^{-2}$, $\forall\ n \geq 0$. Then (3.13), (3.14), (3.1)–(3.4) and Lemma 2.2 imply that $\lim_{n\to\infty} \alpha_n = 0$. That is, $x_n \to q$ as $n \to \infty$.

Observe that for all $n \geq 0$,

$$\begin{aligned}(1-d_n)f_n = &p_n - d_nTs_n - c_n(u_n - Ts_n) \\ = &\Big(1-(1-k_q)d_n\Big)p_n + d_n(S-k_qI)p_n \\ &+ d_n(Tp_n - Ts_n) - c_n(u_n - Ts_n).\end{aligned} \tag{3.15}$$

By virtue of (3.15), (3.11), (3.9) and (3.2), we get that

$$\begin{aligned}(1-d_n)\|f_n - q\| \geq &\Big(1-(1-k_q)d_n\Big)\|p_n - q \\ &+ \frac{d_n}{1-(1-k_q)d_n}[(S-k_qI)p_n - (S-k_qI)q]\| \\ &- d_n\|Tp_n - Ts_n\| - c_n\|u_n - Ts_n\| \\ \geq &\Big(1-(1-k_q)d_n\Big)\|p_n - q\| \\ &- d_n\|Tp_n - Ts_n\| - c_n\|u_n - Ts_n\|,\end{aligned}$$

which means that

$$\begin{aligned}\|p_n - q\| \le & \frac{1-d_n}{1-(1-k_q)d_n}\|f_n - q\| \\ & + \frac{d_n}{1-(1-k_q)d_n}\|Tp_n - Ts_n\| \\ & + \frac{c_n}{1-(1-k_q)d_n}\|u_n - Ts_n\| \\ \le & (1-k_q d_n)\|y_n - q\| + \frac{1}{k_q}d_n g_n + \frac{M}{k_q}c_n\end{aligned} \tag{3.16}$$

for any $n \ge 0$, where $g_n = \|Tp_n - Ts_n\|$. Since

$$\begin{aligned}\|p_n - s_n\| \le & b_n\|f_n - Ts_n\| + c_n\|f_n - u_n\| \\ & + b'_n\|f_n - Tt_n\| + c'_n\|f_n - v_n\| \\ \le & 2M(d_n + d'_n) \to 0\end{aligned}$$

as $n \to \infty$, and T is uniformly continuous, so that $g_n = \|Tp_n - Ts_n\| \to 0$ as $n \to \infty$. Thus (3.16) implies that

$$\begin{aligned}\|f_{n+1} - q\| \le & \|p_n - q\| + \|f_{n+1} - p_n\| \\ \le & (1-k_q d_n)\|f_n - q\| + \frac{1}{k_q}d_n g_n + \frac{M}{k_q}c_n + \varepsilon_n\end{aligned}$$

for all $n \ge 0$.

Suppose that $\sum_{n=0}^{\infty}\varepsilon_n < \infty$. Set $\alpha_n = \|f_n - q\|$, $\omega_n = k_q d_n$, $\beta_n = (g_n + Mr_n)k_q^{-2} + m_n k_q^{-1}$, $\gamma_n = \varepsilon_n$, $\forall\ n \ge 0$. Using Lemma 2.2, (3.3) and (3.4), we conclude immediately that $\alpha_n \to 0$ as $n \to \infty$. Therefore $f_n \to q$ as $n \to \infty$. That is, $\{x_n\}_{n=0}^{\infty}$ is almost T-stable.

Suppose that $\lim_{n\to\infty} f_n = q$. Then

$$\begin{aligned}\varepsilon_n \le & \|f_{n+1} - q\| + \|p_n - q\| \\ \le & \|f_{n+1} - q\| + (1-k_q d_n)\|f_n - q\| + \frac{1}{k_q}d_n g_n + \frac{M}{k_q}c_n \to 0\end{aligned}$$

as $n \to \infty$. That is, $\varepsilon_n \to 0$ as $n \to \infty$. This completes the proof. □

Using the methods of proof in Theorem 3.1, we obtain the following results.

Theorem 3.2. *Let X, T, $R(T)$, $F(T)$, $\{u_n\}_{n=0}^{\infty}$, $\{v_n\}_{n=0}^{\infty}$, $\{w_n\}_{n=0}^{\infty}$, $\{x_n\}_{n=0}^{\infty}$, $\{y_n\}_{n=0}^{\infty}$, $\{z_n\}_{n=0}^{\infty}$, $\{s_n\}_{n=0}^{\infty}$, $\{t_n\}_{n=0}^{\infty}$, $\{f_n\}_{n=0}^{\infty}$, and $\{\varepsilon_n\}_{n=0}^{\infty}$ be as in Theorem 3.1. Suppose that $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$, $\{b'_n\}_{n=0}^{\infty}$, $\{c'_n\}_{n=0}^{\infty}$, $\{a''_n\}_{n=0}^{\infty}$, $\{b''_n\}_{n=0}^{\infty}$ and $\{c''_n\}_{n=0}^{\infty}$ are any sequences in $[0,1]$ satisfying (3.1) and*

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty} b'_n = \lim_{n\to\infty} c'_n = 0; \tag{3.17}$$

$$\sum_{n=0}^{\infty} c_n < \infty; \tag{3.18}$$

$$\sum_{n=0}^{\infty} b_n = \infty. \tag{3.19}$$

Then the conclusions of Theorem 3.1 hold.

Remark 3.1. The following examples reveal that Theorem 3.1 and Theorem 3.2 are independent.

Example 3.1. Let R denote the reals with the usual norm and define $T\colon R \to R$ by $Tx = \frac{1}{4}(\sin x)^2$ for all $x \in R$. Clearly, $F(T) = \{0\}$ and $R(T) = [0, \frac{1}{4}]$. Observe that

$$|Tx - Ty| \le \frac{1}{4}|\sin x - \sin y| \cdot |\sin x + \sin y| \le \frac{1}{2}|x - y| \quad \forall\, x, y \in R. \tag{3.20}$$

Hence T is uniformly continuous on R. For each $x \in R$, choose $t_x = 2$. Then (3.20) ensures that

$$\begin{aligned} |(1+r)(x-y) - rt_x(Tx - Ty)| \ge& (1+r)|x-y| - rt_x|Tx - Ty| \\ =& |x-y| + r\Big(|x-y| - t_x|Tx - Ty|\Big) \\ \ge& |x-y| \end{aligned}$$

for any $y \in R$. That is, T is local strongly pseudocontractive. Put

$$\begin{aligned} &a_n = 1 - (n+1)^{-\frac{1}{4}},\ b_n = (n+1)^{-\frac{1}{4}} - (n+1)^{-\frac{1}{2}},\ c_n = (n+1)^{-\frac{1}{2}}, \\ &r_n = (n+1)^{-\frac{1}{4}},\ a_n' = 1 - \frac{1}{n+1},\ b_n' = c_n' = \frac{1}{2(n+1)}, \\ &a_n'' = 1 - 3(7n+3)^{-1},\ b_n'' = (7n+3)^{-1},\ c_n'' = 2(7n+3)^{-1},\ \forall n \ge 0. \end{aligned}$$

Thus Theorem 3.2 is not applicable since $\sum_{n=0}^{\infty} c_n = \infty$. It is easy to verify that the conditions of Theorem 3.1 are fulfilled.

Example 3.2. Let R, T, $\{a_n'\}_{n=0}^{\infty}$, $\{b_n'\}_{n=0}^{\infty}$, $\{c_n'\}_{n=0}^{\infty}$, $\{a_n''\}_{n=0}^{\infty}$, $\{b_n''\}_{n=0}^{\infty}$, $\{c_n''\}_{n=0}^{\infty}$ be as in Example 3.1. Set

$$a_n = 1 - b_n - c_n,\ b_{2n} = (2+2n)^{-2},\ b_{2n+1} = (3+2n)^{-1},\ c_n = (2+n)^{-2},\ \forall n \ge 0.$$

Then all the conditions of Theorem 3.2 are satisfied. But Theorem 3.1 is not applicable since

$$\lim_{n\to\infty} \gamma_{2n} = \lim_{n\to\infty} \frac{c_{2n}}{b_{2n} + c_{2n}} = \frac{1}{2} \ne 0.$$

Remark 3.2. Theorem 3.1 and Theorem 3.2 show that, under certain conditions, the Noor iteration scheme considered in Theorem 3.1 and Theorem 3.2, respectively, is almost T-stable. The example below proves that the iteration scheme is not T-stable.

Example 3.3. Let R denote the reals with the usual norm and define $T\colon R \to R$ by $Tx = \frac{1}{3}\sin x$ for all $x \in R$. Then $F(T) = \{0\}$, $R(T) = \Big[-\frac{1}{3}, \frac{1}{3}\Big]$, T is uniformly continuous on R and

$$|x - y| \le |(1+r)(x-y) - t_x r(Tx - Ty)|, \quad \forall\, x, y \in R,$$

where $t_x = 3$ for all $x \in R$. Suppose that $\{u_n\}_{n=0}^{\infty}$, $\{v_n\}_{n=0}^{\infty}$, $\{w_n\}_{n=0}^{\infty}$, $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a_n'\}_{n=0}^{\infty}$, $\{b_n'\}_{n=0}^{\infty}$, $\{c_n'\}_{n=0}^{\infty}$, $\{a_n''\}_{n=0}^{\infty}$, $\{b_n''\}_{n=0}^{\infty}$ and $\{c_n''\}_{n=0}^{\infty}$ satisfy the conditions of Theorem 3.2. It follows from Theorem 3.2 that the Noor iteration scheme $\{x_n\}_{n=0}^{\infty}$ with errors defined by (3.4) both converges strongly to the unique fixed point 0 of T and is almost T-stable. We next prove that it is not T-stable. Choose $f_n = \frac{n}{2n+1}$, $\forall\, n \ge 0$. Since

$$\lim_{n\to\infty} a_n = 1, \quad \lim_{n\to\infty} b_n = \lim_{n\to\infty} c_n = 0$$

and $\lim_{n\to\infty} f_n = \frac{1}{2}$, it follows that

$$\begin{aligned}\varepsilon_n =&|f_{n+1} - a_n f_n - b_n T s_n - c_n u_n|\\ \leq&|f_{n+1} - a_n f_n| + b_n|Ts_n| + c_n|u_n| \to 0\end{aligned}$$

as $n \to \infty$. That is, $\lim_{n\to\infty} \varepsilon_n = 0$. However, $\lim_{n\to\infty} f_n = \frac{1}{2} \neq 0 = T0$. Thus $\{x_n\}_{n=0}^{\infty}$ is not T-stable.

Suppose that $\{u_n\}_{n=0}^{\infty}$, $\{v_n\}_{n=0}^{\infty}$, $\{w_n\}_{n=0}^{\infty}$, $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$ $\{b'_n\}_{n=0}^{\infty}$, $\{c'_n\}_{n=0}^{\infty}$, $\{a''_n\}_{n=0}^{\infty}$, $\{b''_n\}_{n=0}^{\infty}$, $\{c''_n\}_{n=0}^{\infty}$ and $\{r_n\}_{n=0}^{\infty}$ satisfy the conditions of Theorem 3.1. Similarly we can prove that the Noor iteration scheme $\{x_n\}_{n=0}^{\infty}$ with errors defined by (3.4) is almost $T-$stable, but not $T-$stable.

Theorem 3.3. *Let X be an arbitrary Banach space and $T\colon X \to X$ be local strongly accretive and uniformly continuous. Define $S\colon X \to X$ by $Sx = f + x - Tx$. Let $S(T) \neq \emptyset$ for some $f \in X$ and either $R(T)$ or $R(I-T)$ be bounded. Suppose that $\{u_n\}_{n=0}^{\infty}$, $\{v_n\}_{n=0}^{\infty}$ and $\{w_n\}_{n=0}^{\infty}$ are arbitrary bounded sequences in X and $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$, $\{b'_n\}_{n=0}^{\infty}$, $\{c'_n\}_{n=0}^{\infty}$ $\{a''_n\}_{n=0}^{\infty}$, $\{b''_n\}_{n=0}^{\infty}$, $\{c''_n\}_{n=0}^{\infty}$ and $\{r_n\}_{n=0}^{\infty}$ are any sequences in $[0,1]$ satisfying (3.1)–(3.4). For arbitrary $x_0 \in X$, the Ishikawa iteration scheme $\{x_n\}_{n=0}^{\infty}$ is defined by*

$$\begin{aligned}x_{n+1} =&a_n x_n + b_n S y_n + c_n u_n\\ y_n =&a'_n x_n + b'_n S z_n + c'_n v_n,\\ z_n =&a''_n x_n + b''_n S x_n + c''_n w_n, \quad \forall\ n \geq 0.\end{aligned} \tag{3.21}$$

Let $\{f_n\}_{n=0}^{\infty}$ be any bounded sequence in X and define $\{\varepsilon_n\}_{n=0}^{\infty}$ by

$$\begin{aligned}\varepsilon_n =&\|f_{n+1} - a_n f_n - b_n S s_n - c_n u_n\|,\\ s_n =&a'_n f_n + b'_n S t_n + c'_n v_n,\\ t_n =&a''_n f_n + b''_n S f_n + c''_n w_n, \quad \forall\ n \geq 0.\end{aligned} \tag{3.22}$$

Then there exist real sequences $\{h_n\}_{n=0}^{\infty}$, $\{g_n\}_{n=0}^{\infty}$ and constant $M > 0$ such that

(i) *$\{x_n\}_{n=0}^{\infty}$ converges strongly to the unique solution q of the equation $Tx = f$ and*

$$\|x_{n+1} - q\| \leq \Big(1 - (b_n + c_n)k_q\Big)\|x_n - q\| + \frac{1}{k_q}(b_n + c_n)h_n + \frac{M}{k_q}c_n, \quad \forall n \geq 0,$$

(ii) $\|f_{n+1} - q\| \leq \Big(1 - (b_n + c_n)k_q\Big)\|f_n - q\| + \frac{1}{k_q}(b_n + c_n)g_n + \frac{M}{k_q}c_n + \varepsilon_n, \quad \forall n \geq 0;$

(iii) *$\sum_{n=0}^{\infty} \varepsilon_n < \infty$ implies that $\lim_{n\to\infty} f_n = q$, so that $\{x_n\}_{n=0}^{\infty}$ is almost S-stable;*

(iv) $\lim_{n\to\infty} f_n = q$ *implies that* $\lim_{n\to\infty} \varepsilon_n = 0$;

(v) $\lim_{n\to\infty} h_n = \lim_{n\to\infty} g_n = 0$.

Proof. Since T is local strongly accretive and $S(T) \neq \emptyset$ for some $f \in X$, so that (1.2) implies that $S(T)$ is a singleton and S is local strongly pseudocontractive. Let $S(T) = \{q\}$. Then q is the unique fixed point of S. Now we prove that $R(S)$ is bounded. It is easy to see that $R(S)$ is bounded if $R(I-T)$ is bounded. Suppose that $R(T)$ is bounded. Observe that

$$\mathrm{Re}\langle Tx - Tq, j(x-q)\rangle \geq k_q\|x - q\|^2, \quad \forall\ x \in X,$$

which implies that

$$\|x-q\| \le \frac{1}{k_q}\|Tx-Tq\|, \quad \forall\, x \in X. \tag{3.23}$$

Using (3.23), we infer that for all $x \in X$,

$$\begin{aligned}\|Sx-Sq\| =& \|Tq-Tx+x-q\| \\ \le& \|Tx-Tq\|+\|x-q\| \le \Big(1+\frac{1}{k_q}\Big)\|Tx-Tq\|.\end{aligned}$$

That is, $R(S)$ is bounded. Thus the rest of the proof follows immediately as in the proof of Theorem 3.1, and is therefore omitted. This completes the proof. □

Theorem 3.4. *Let X, T, $R(T)$, $R(I-T)$, $S(T)$, f, $\{u_n\}_{n=0}^\infty$, $\{v_n\}_{n=0}^\infty$, $\{w_n\}_{n=0}^\infty$, $\{x_n\}_{n=0}^\infty$, $\{y_n\}_{n=0}^\infty$, $\{z_n\}_{n=0}^\infty$, $\{f_n\}_{n=0}^\infty$ and $\{\varepsilon_n\}_{n=0}^\infty$ be as in Theorem 3.3. Suppose that $\{a_n\}_{n=0}^\infty$, $\{b_n\}_{n=0}^\infty$, $\{c_n\}_{n=0}^\infty$, $\{a'_n\}_{n=0}^\infty$, $\{b'_n\}_{n=0}^\infty$, $\{c'_n\}_{n=0}^\infty$, $\{a''_n\}_{n=0}^\infty$, $\{b''_n\}_{n=0}^\infty$ and $\{c''_n\}_{n=0}^\infty$ are any sequences in $[0,1]$ satisfying (3.1) and (3.17)–(3.19). Then the conclusions of Theorem 3.3 hold.*

Remark 3.3. The convergence result in Theorem 3.4 extends Theorem 2 of Chidume [5], Theorems 5 and 6 of Chidume [6], Theorem 2 of Chidume [7] and Theorem 2 of Chidume and Osilike [10].

Using methods similar to those above, we can prove the following results.

Theorem 3.5. *Let X be an arbitrary Banach space, $f \in X$ and $T\colon X \to X$ be local strongly accretive and uniformly continuous. Define $G\colon X \to X$ by $Gx = f - Tx$. Let $R(T)$ be bounded and the equation $x+Tx = f$ has a solution $q \in X$. Suppose that $\{u_n\}_{n=0}^\infty$, $\{v_n\}_{n=0}^\infty$ and $\{w_n\}_{n=0}^\infty$ are arbitrary bounded sequences in X and $\{a_n\}_{n=0}^\infty$, $\{b_n\}_{n=0}^\infty$, $\{c_n\}_{n=0}^\infty$, $\{a'_n\}_{n=0}^\infty$, $\{b'_n\}_{n=0}^\infty$, $\{c'_n\}_{n=0}^\infty$, $\{a''_n\}_{n=0}^\infty$, $\{b''_n\}_{n=0}^\infty$, $\{c''_n\}_{n=0}^\infty$ and $\{r_n\}_{n=0}^\infty$ are any sequence in $[0,1]$ satisfying (3.1)–(3.4). For arbitrary $x_0 \in X$, the Noor iteration scheme $\{x_n\}_{n=0}^\infty$ with errors is defined by*

$$\begin{aligned} x_{n+1} =& a_n x_n + b_n G y_n + c_n u_n, \\ y_n =& a'_n x_n + b'_n G z_n + c'_n v_n, \\ z_n =& a''_n x_n + b''_n G x_n + c''_n w_n, \quad \forall\, n \ge 0. \end{aligned} \tag{3.24}$$

Let $\{f_n\}_{n=0}^\infty$ be any bounded sequence in X and define $\{\varepsilon_n\}_{n=0}^\infty$ by

$$\begin{aligned} \varepsilon_n =& \|f_{n+1} - a_n f_n - b_n G s_n - c_n u_n\|, \\ s_n =& a'_n f_n + b'_n G t_n + c'_n v_n, \\ t_n =& a''_n f_n + b''_n G f_n + c''_n w_n, \quad \forall\, n \ge 0. \end{aligned} \tag{3.25}$$

Then there exist real sequences $\{h_n\}_{n=0}^\infty$, $\{g_n\}_{n=0}^\infty$ and constant $M > 0$ such that

(i) *$\{x_n\}_{n=0}^\infty$ converges strongly to the unique solution q of the equation $x+Tx = f$ and*

$$\|x_{n+1}-q\| \le \Big(1-(b_n+c_n)k_q\Big)\|x_n-q\| + \frac{1}{k_q}(b_n+c_n)h_n + \frac{M}{k_q}c_n, \quad \forall\, n \ge 0,$$

(ii) $\|f_{n+1}-q\| \le \Big(1-(b_n+c_n)k_q\Big)\|f_n-q\| + \frac{1}{k_q}(b_n+c_n)g_n + \frac{M}{k_q}c_n + \varepsilon_n, \quad \forall\, n \ge 0,$

(iii) $\sum_{n=0}^\infty \varepsilon_n < \infty$ *implies that* $\lim_{n\to\infty} f_n = q$, *so that* $\{x_n\}_{n=0}^\infty$ *is almost* G*-stable,*

(iv) $\lim_{n\to\infty} f_n = q$ *implies that* $\lim_{n\to\infty} \varepsilon_n = 0$,

(v) $\lim_{n\to\infty} h_n = \lim_{n\to\infty} g_n = 0$.

Theorem 3.6. *Let X, T, $R(T)$, f, G, $\{u_n\}_{n=0}^\infty$, $\{v_n\}_{n=0}^\infty$, $\{w_n\}_{n=0}^\infty$, $\{x_n\}_{n=0}^\infty$, $\{y_n\}_{n=0}^\infty$, $\{z_n\}_{n=0}^\infty$, $\{f_n\}_{n=0}^\infty$ and $\{\varepsilon_n\}_{n=0}^\infty$ be as in Theorem 3.5. Suppose that $\{a_n\}_{n=0}^\infty$, $\{b_n\}_{n=0}^\infty$, $\{c_n\}_{n=0}^\infty$, $\{a'_n\}_{n=0}^\infty$, $\{b'_n\}_{n=0}^\infty$, $\{c'_n\}_{n=0}^\infty$, $\{a''_n\}_{n=0}^\infty$, $\{b''_n\}_{n=0}^\infty$, $\{c''_n\}_{n=0}^\infty$ are any sequences in $[0,1]$ satisfying (3.1) and (3.17)–(3.19). Then the conclusions of Theorem 3.3 hold.*

Remark 3.4. The convergence result in Theorem 3.6 generalizes Theorems 9 and 10 of Chidume [6] and Theorem 3.3 of Ding [13].

Theorem 3.7. *Let K be a nonempty bounded closed convex subset of an arbitrary Banach space X and $T\colon K \to K$ be a uniformly continuous and local strongly pseudocontractive mapping. Let $F(T) \neq \emptyset$ and $\{u_n\}_{n=0}^\infty$, $\{v_n\}_{n=0}^\infty$ and $\{w_n\}_{n=0}^\infty$ be arbitrary sequences in K. Suppose that $\{a_n\}_{n=0}^\infty$, $\{b_n\}_{n=0}^\infty$, $\{c_n\}_{n=0}^\infty$, $\{a'_n\}_{n=0}^\infty$, $\{b'_n\}_{n=0}^\infty$, $\{c'_n\}_{n=0}^\infty$, $\{a''_n\}_{n=0}^\infty$, $\{b''_n\}_{n=0}^\infty$, $\{c''_n\}_{n=0}^\infty$ and $\{r_n\}_{n=0}^\infty$ are any sequences in $[0,1]$ satisfying (3.1)–(3.4). If $\{x_n\}_{n=0}^\infty$ is the sequence generated from an arbitrary $x_0 \in K$ by (3.4), then it converges strongly to the unique fixed point q of T.*

Theorem 3.8. *Let $X, K, T, F(T)$, $\{u_n\}_{n=0}^\infty$, $\{v_n\}_{n=0}^\infty$, $\{w_n\}_{n=0}^\infty$ and $\{x_n\}_{n=0}^\infty$ be as in Theorem 3.7 and $\{a_n\}_{n=0}^\infty$, $\{b_n\}_{n=0}^\infty$, $\{c_n\}_{n=0}^\infty$, $\{a'_n\}_{n=0}^\infty$, $\{b'_n\}_{n=0}^\infty$, $\{c'_n\}_{n=0}^\infty$, $\{a''_n\}_{n=0}^\infty$, $\{b''_n\}_{n=0}^\infty$ and $\{c''_n\}_{n=0}^\infty$ be any sequences in $[0,1]$ satisfying (3.1) and (3.17)–(3.19). Then $\{x_n\}_{n=0}^\infty$ converges strongly to the unique fixed point q of T.*

Remark 3.5. Theorem 3.8 extends, improves and unifies Theorem 3.4 of Chang [1], Theorems 3.4 and 4.2 of Chang et al. [2], the Theorem Chidume [3], Theorem 2 of Chidume [4], Theorem 4 of Chidume [6], Theorem 1 of Chidume [7], Theorem 1 of Chidume and Osilike [10], Theorem 1 of Liu [21] and Theorem 4.2 of Tan and Xu [54].

Question 3.1. Let X be an arbitrary Banach space and $T\colon X \to X$ be local strongly pseudocontractive and uniformly continuous. Let $F(T) \neq \emptyset$ and $R(T)$ be bounded. Suppose that $\{u_n\}_{n=0}^\infty$, $\{v_n\}_{n=0}^\infty$ and $\{w_n\}_{n=0}^\infty$ are arbitrary bounded sequences in X. What hypotheses on $\{a_n\}_{n=0}^\infty$, $\{b_n\}_{n=0}^\infty$, $\{c_n\}_{n=0}^\infty$, $\{a'_n\}_{n=0}^\infty$, $\{b'_n\}_{n=0}^\infty$, $\{c'_n\}_{n=0}^\infty$, $\{a''_n\}_{n=0}^\infty$, $\{b''_n\}_{n=0}^\infty$ and $\{c''_n\}_{n=0}^\infty \subset [0,1]$ are needed to guarantee the Noor iteration scheme with errors in (3.4) is T-stable?

Question 3.2. Let X be an arbitrary Banach space and $T\colon X \to X$ be local strongly accretive and uniformly continuous. Define $S\colon X \to X$ by $Sx = f + x - Tx$. Let $S(T) \neq \emptyset$ for some $f \in X$ and either $R(T)$ or $R(I - T)$ be bounded. Suppose that $\{u_n\}_{n=0}^\infty$, $\{v_n\}_{n=0}^\infty$ and $\{w_n\}_{n=0}^\infty$ are arbitrary bounded sequences in X. What hypotheses on $\{a_n\}_{n=0}^\infty$, $\{b_n\}_{n=0}^\infty$, $\{c_n\}_{n=0}^\infty$, $\{a'_n\}_{n=0}^\infty$, $\{b'_n\}_{n=0}^\infty$, $\{c'_n\}_{n=0}^\infty$, $\{a''_n\}_{n=0}^\infty$, $\{b''_n\}_{n=0}^\infty$ and $\{c''_n\}_{n=0}^\infty \subset [0,1]$ are needed to guarantee the Noor iteration scheme with errors in (3.21) is S-stable?

Question 3.3. Let X be an arbitrary Banach space, $f \in X$ and $T\colon X \to X$ be local strongly accretive and uniformly continuous. Define $G\colon X \to X$ by $Gx = f - Tx$. Let $R(T)$ be bounded and the equation $x + Tx = f$ has a solution $q \in X$. Suppose that $\{u_n\}_{n=0}^\infty$, $\{v_n\}_{n=0}^\infty$ and $\{w_n\}_{n=0}^\infty$ are arbitrary bounded sequences in X. What hypotheses on $\{a_n\}_{n=0}^\infty$, $\{b_n\}_{n=0}^\infty$, $\{c_n\}_{n=0}^\infty$, $\{a'_n\}_{n=0}^\infty$, $\{b'_n\}_{n=0}^\infty$, $\{c'_n\}_{n=0}^\infty$, $\{a''_n\}_{n=0}^\infty$, $\{b''_n\}_{n=0}^\infty$ and $\{c''_n\}_{n=0}^\infty \subset [0,1]$ are needed to guarantee the Noor iteration scheme with errors in (3.24) is G-stable?

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