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TWO SAMPLE NONPARAMETRIC PROCEDURES BASED ON SAMPLE COVERAGES FOR UNCENSORED DATA

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Abstract. Suppose that $X_1, \dots, X_n$ and $Y_1, \dots, Y_m$ be random samples from cumulative distributions $F(x)$ and $G(y)$ respectively. Let $B_i = (X_{i-1}, X_i]$ be a random interval constructed from the first sample. Let $\hat{U}_i$ be the proportion of Y_i's that lies in B_i $(i = 1, \dots, n+1)$. $\hat{U}_i$ are called the sample coverages. A class of two-sample tests on $\hat{U}_i$ is proposed for Interquartile, Chi-square, and Modified Wilcoxon.

1. Introduction

Suppose we have two independent random samples $X_1, \dots, X_n$ and $Y_1, \dots, Y_m$ with respectively cumulative distribution functions $F(x)$ and $G(y)$. Moreover, no knowledge is assumed concerning the distribution functions F and G, except that they are continuous. The problem considered here is that of testing the null hypothesis $F = G$ against the alternative $F \neq G$.

The procedures developed in this paper offer a nonparametric test of the null hypothesis that, $F = G$.

In section 2, the descriptions and asymptotic distributions of the tests are given. Also, an example to illustrate the use of these tests is presented.

2. Description of Proposed Tests

Let $X_1, \dots, X_n$ be a random sample from cumulative distribution function $F(x)$ and $X_{(1)} < X_{(2)} < \dots < X_{(n)}$ be the order statistics corresponding to the sample of $X_1, \dots, X_n$.

Let $V_i = (F(x_{(i)}) - F(x_{(i-1)})) = (U_{(i)} - U_{(i-1)})$ for $i = 1, \dots, n+1$. Where $X_{(0)} = -\infty$ and $X_{(n+1)} = +\infty$. V_i's are called coverages of the random interval $B_i = \left(X_{(i-1)}, X_{(i)}\right], i = 1, \dots, n+1$.

Let $Y_1, \dots, Y_m$ be second random sample, independent from the X_i's from a cumulative distribution function $G(y)$.

Also let $G_m(y)$ be the sample distribution of $Y_1, \dots, Y_m$.

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Assuming $F = G$, define $\hat{V}_j = \int_{B_j} dG_m(y)$. Let I_s be a subset of $\{1, \ldots, n\}$ with s distinct elements and define $U_s = \sum_{i \in I_s} V_i$ and $\hat{U}_s = \sum_{j \in I_s} \hat{V}_j$, then $\hat{U}_s$ is sample analog to the U_s and will be called the sample coverage. The proposed tests are based on the sample coverages.

2.1 The Interquartile test

Let $s_1/n = 0.25$ and $s_2/n = 0.75$. Then $\hat{U}_{s_1} - \hat{U}_{s_2} = \hat{U}_{(s_1 - s_2)}$ is equal to the proportion of Y_i's which lie in $(X_{(s_1)}, X_{(s_2)}]$. Let $s/n = s_2/n - s_1/n \simeq 0.5$. Now, if $F = G$, then $\hat{U}_s$ should be near $s/(n+1)$ because $E(\hat{U}_s) = s/(n+1)$ (see Fligner and Wolf 1976). On the other hand if $F \neq G$, then $\hat{U}_s - s/(n+1)$ will be either too large or too small. This suggests that α – level for testing will be of the $(m\hat{U}_{s_<} \hat{r}_1$ or $m\hat{U}_{s_>} \hat{r}_2)$ where the interval contains and satisfies the condition that

$$\sum_{r=r_1}^{r_2} P[\hat{U}_s = r/m] \geq 1 - \alpha.$$

Of course, ideally we want $r_2 - r_1$ to be as small as possible. The probability mass function of $\hat{U}_s$ can be found in Fligner and Wolf (1976). We have tabulated the distribution of $\hat{U}_s$ for $5 \leq n$, $m \leq 60$, which is used to tabulate the critical values.

Table 1 contains some of these values.

When $F = G$, upon suitable standardization, $\hat{U}_s$ is asymptotically normally distributed (Yang 1984). If the normal approximation is used, the critical region would be of the form $\{|Z| > Z_{\alpha/2}\}$ where $Z = \left[\hat{U}_s - E(\hat{U}_s)\right] \Big/ \left[\mathrm{Var}(\hat{U}_s)\right]^{1/2}$ and $Z_{\alpha/2}$ is the $(1 - \alpha/2)100\%$ quantile of the standard normal distribution. We found that the normal approximation to the exact distribution of $\hat{U}_s$ is quite accurate for sample sizes $m, n \geq 15$.

Let μ_F and μ_G be respectively the means of F and G, and σ_F and σ_G be respectively the standard deviations of F and G.

Then if $\sigma_F = \sigma_G$, $\hat{U}_s$ will tend to be small regardless of whether $\mu_F > \mu_G$ or $\mu_F < \mu_G$. Therefore, we can not use $\hat{U}_s$ to form one-sided tests for testing $\mu_F = \mu_G$. However, when $\mu_F = \mu_G$, if $\sigma_F > \sigma_G$ $(\sigma_F < \sigma_G)$, $\hat{U}_s$ tends to be large (small). Hence, for testing $\sigma_F = \sigma_G$, we can form one sided tests.

2.2 Chi-square test

The second proposed test is analogous to the Chi-square goodness of fit.

Let $I_{s_1}, I_{s_2}, \ldots, I_{s_k}$ be mutually disjoint with $s_i/n \to p_i$ $(0 < p_i < 1)$ as $n \to \infty$. Furthermore, suppose that $m/n \to \lambda$, $(0 \leq \lambda < \infty)$ as $m, n \to \infty$ then, Yang

(1984) showed the asymptotic distribution of

$$\left\{m^{1/2}(\hat{U}_{s_1} - \frac{s_1}{n+1}), \dots, m^{1/2}(\hat{U}_{s_k} - \frac{s_k}{n+1})\right\}$$

is k-variate normal distribution with zero mean vector and covariance matrix with element σ_{ij}, where

$$\sigma_{i,j} = \begin{cases} p_i(1-p_i)(1+\lambda), & \text{for } i = j \\ p_j p_i(1+\lambda), & \text{for } i \neq j. \end{cases}$$

Using the above result, we can easily prove the following theorem which is the basis of our second test.

Theorem. *Suppose* $I_{s_1}, I_{s_2}, \dots, I_{s_k}$ *be such that* $A_i = \cup_{j \in I_{s_i}} B_j$ $(i = 1, \dots, k)$ *form a partition of the real line and that* $s_i/n \to p_i$ $(0 < p_i < 1)$ *as* $n \to \infty$, *where* $\sum_{i=1}^{k} p_i = 1$. *Then if* $F = G$ *and* $m, n \to \infty$, $m/n \to \lambda$, $(0 \leq \lambda < \infty)$, *the statistic*

$$Y = C \, \frac{\sum\limits_{i=1}^{k} (\hat{U}_{s_i} - \hat{p}_i)^2}{\hat{p}_i^2},$$

(where $C = m(n+2)/(m+n+1)$*), converges in distribution to a Chi-square distribution with* $k-1$ *degree of freedom.*

Proof. We know from Yang(1984) that the vector

$$V = m^{1/2}\left[(\hat{U}_{s_1} - \hat{p}_1), \dots, (\hat{U}_{s_k} - \hat{p}_k)\right]^{\top}$$

has asymptotically normal distribution $N(0, \Sigma)$ where $\Sigma_{ii} = p_i(1-p_i)(1+\lambda)$ and $\Sigma_{ij} = -p_i p_j(1+\lambda)$ for $i \neq j$. We follow the approach of [**Anděl 1985**] and introduce the following notations:
put $\psi = (p_1^{1/2}, \dots, p_k^{1/2})^{\top}$ and $Q = I - \psi\psi'$.

We have

$$\psi'\psi = \sum_{i=1}^{k} p_i = 1.$$

Hence

$$Q^2 = I - 2\psi\psi' + \psi\psi'\,\psi\psi' = I - \psi\psi' = Q.$$

The matrix Q is idempotent , hence

$$\operatorname{rank}(Q) = \operatorname{tr} Q = \operatorname{tr} I - \operatorname{tr} \psi\psi' = k - 1.$$

It easy to check that [see **Anděl 1985**] $\sum = D.Q.D$, where $D = \operatorname{diag}\{[p_1(1+\lambda)]^{1/2}, \dots, [p_k(1+\lambda)^{1/2}]\}$.

Hence rank $\sum = \operatorname{rank} Q = k-1$.

Put

$$W = D^{-1}V = m^{1/2}\left\{[\hat{U}_{s_1} - \hat{p}_1]/[p_1(1+\lambda)]^{1/2}, \dots, [\hat{U}_{s_k} - \hat{p}_k]/[p_k(1+\lambda)]^{1/2}\right\}^T$$

we have $W \sim N(0, Q)$. Therefore see [**Anděl 1985**] $Z = W'Q^{-1}W = \chi^2_{k-1}$, however $Z = m\sum_{i=1}^k (\hat{U}_{s_i} - \hat{p}_i)^2)/p_i(1+\lambda)$.

To prove that Y has also asymptotic distribution χ^2_{k-1}, it suffices to apply Cramer-Slucky theorem and the fact that the difference $Y - Z$ converges to 0 in probability.

For given m, n we have

$$\begin{aligned}
|Y - Z| &= \left| C\sum_{i=1}^k (\hat{U}_{s_i} - \hat{p}_i)/\hat{p}_i - \frac{m}{1+\lambda}\sum_{i=1}^k (\hat{U}_{s_i} - \hat{p}_i)^2/p_i \right| \\
&\le \frac{m}{1+\lambda}\sum_{i=1}^k (\hat{U}_{s_i} - \hat{p}_i)^2/p_i \cdot \left| \frac{(1+\lambda)C}{m}\frac{p_i}{\hat{p}_i} - 1 \right| \\
&\le Z \max_{i=1,\dots,k}\left\{ \left| \frac{(1+\lambda)C}{m}\frac{p_i}{\hat{p}_i} - 1 \right| \right\}.
\end{aligned}$$

For any given i the sequences

$$\frac{p_i}{\hat{p}_i} \quad \text{and} \quad \frac{(1+\lambda)C}{m} = \frac{(1+\lambda)(n+2)}{(m+n+1)} = \frac{(1+\lambda)(1+2/n)}{(1+m/n+1/n)}$$

converge to 1 according to our assumptions. This completes the proof of the theorem. □

Note that $\hat{p}_i$ is the estimate of the expected proportion of Y_i's lying in A_i and $\hat{U}_s$ is the observed proportion of Y_i's which occurred in A_i.

This suggests that we reject the null hypothesis H_0 that $F = G$ at level α $(0 < \alpha < 1)$ if $Q > q_{1-\alpha}$, where $q_{1-\alpha}$ is the $(1-\alpha)100\%$ quantile of the Chi-square distribution with $(k-1)$ degree of freedom. In practice the A_i are the intervals of the form $(X_{(r_{i-1})}, X_{(r_i)}]$, $s_i = r_i - r_{i-1}$.

2.3 Modified Wilcoxon test

Fligner and Wolf (1976) pointed out that under the assumption $F = G$, the Wilcoxon test statistic can be written as

$$W = \sum_{i=1}^n (n-i+1)\hat{V}_i + n(n+1)/2.$$

It is interesting to note that in the Wilcoxon test statistic the weight associated with each coverage $\hat{V}_i$ decreases as i increases. This may explain why the Wilcoxon test is very sensitive in detecting a shift in location, but is very very poor in detecting a change in scale .

This also suggests that the Wilcoxon test can be modified by putting more weight to sample coverages near the mid coverage $\hat{V}_s$, where $s = [n/2]$ so that it will also be sensitive in detecting scale change.

The modified Wilcoxon test considered in this paper is based on the statistic

$$MW = \left[\sum_{i=1}^{n}(n-i+1)\hat{V}_{s+w_i}\right] + n(n+1)/2$$

where

$$w_i = \begin{cases} i/2 & \text{for } i \text{ even,} \\ -(i-1)/2 & \text{for } i \text{ odd.} \end{cases}$$

This is a special case of the statistic of the form:

$$T = \sum_{i=1}^{n}(n-i+1)\hat{V}_{k_i} + n(n+1)/2.$$

Where $(k_1, \ldots, k_n)$ is a permutation of $(1, 2, \ldots, n)$.

By the exchangability property of the $\hat{V}_i$'s, MW and W have the same distribution when $F = G$, therefore, the critical values for MW for testing $F = G$ are readily available like the interquartile for testing $\sigma_F = \sigma_G$, but can not be used to form one-sided test for testing $\mu_F = \mu_G$.

2.4 Application

To illustrate the use of the proposed tests we apply them to example 16.5 given in Mendenhall [1983]. As experiment was conducted to compare the strength of the two types of kraft papers, one is a standard kraft paper of specified weight and the other is treated with a chemical substance.

Standard (X_i)	1.21	1.43	1.35	1.51	1.39	1.17	1.48	1.42	1.29	1.40
Treated (Y_i)	1.49	1.37	1.67	1.50	1.31	1.29	1.52	1.37	1.44	1.53

I Quartile test:

Choose s_1 and s_2 so that $s_1/n = 0.25$ and $s_2/n = 0.75$. Therefore $s_1 = 2.5$ and $s_2 = 7.5$, $s = s_2 - s_1 = 5$. $\hat{U}_s$ is the proportion of Y_i's occurring in the interval $(X_{(2)}, X_{(7)}]$, where $X_{(2)} = 1.21$ $X_{(7)} = 1.42$. From table 1 with $n = m = 10$ and

$\alpha = 0.05$ we reject H_0 if $\hat{U}_s < 1/10$ or $\hat{U}_s > 8/10$ (or $10\hat{U}_s < 1$ or $10\hat{U}_s > 8$). Since $10\hat{U}_s = 4.5$, we fail to reject H_0 at the 5% level.

II Chi-square test:

For the Chi-square test, the real line is partitioned into three disjoint intervals $A_i = (-\infty, X_{(3)}], A_2 = (X_{(3)}, X_{(6)}]$ and $A_3 = (X_{(6)}, \infty)$. Hence $\hat{U}_{s_1} = 1/10 = 0.1, \hat{p}_1 = 3/11 \doteq 0.273, \hat{U}_{s_2} = 3/10 = 0.30, \hat{p}_2 = 3/11 \doteq 0.273$ and $\hat{U}_{s_3} = 6/10 = 0.6$ and $\hat{p}_3 = 4/11 \doteq 0.364$

$\therefore Y = 1.516$ and $q_{0.955} = 5.991$. We fail to reject H_0.

III Modified Wilcoxon test: We have

i	1	2	3	4	5	6	7	8	9	10
$\hat{V}_i$	0	0	1	1	2	0	0	0	1	2
$(n-i+1)$	2	4	6	8	10	9	7	5	3	1

Therefore $MW = 94$. From the Wilcoxon table $W_{0.025} = 79$, $W_{0.95} = 131$. So we fail to reject H_0 at $\alpha = 0.05$. Also the Wilcoxon test ($W = 86$) gives the same conclusion.

Table 1. The lower and upper 0.05 and 0.10 percentage for the distribution of $\hat{U}_s$.

	$\alpha = 0.5$				$\alpha = 0.10$			
Sample n	Size m	Lower r_1	Upper r_2	True α	s	Lower r_1	Upper r_2	True α
10	10	1	8	0.0449	5	2	8	0.0991
	11	2	11	0.0550	5	2	8	0.1184
	12	2	11	0.0462	5	2	9	0.0913
	13	2	11	0.0462	5	3	11	0.1034
	14	2	11	0.0561	5	3	11	0.1034
	15	2	12	0.0446	5	1	10	0.1106
	16	3	16	0.0513	5	2	11	0.1041
	17	2	13	0.0493	5	4	14	0.1048
	18	3	15	0.0469	5	4	14	0.1048
	19	1	14	0.0490	5	2	13	0.0959
	20	2	15	0.0476	5	3	14	0.0969
	21	4	19	0.0501	5	1	14	0.1015
	22	1	16	0.0515	5	3	15	0.1036
	23	2	17	0.0478	5	4	16	0.1085
	24	3	18	0.0449	5	6	24	0.1006
	25	5	25	0.0525	5	3	17	0.0965
11	10	1	8	0.0511	6	2	8	0.0851
	11	1	9	0.0379	6	1	8	0.0975
	12	2	10	0.0479	6	2	9	0.0946
	13	3	13	0.0551	6	3	10	0.1101
	14	1	11	0.0459	6	1	10	0.1008
	15	2	12	0.0454	6	2	11	0.0902
	16	3	13	0.0548	6	3	12	0.0917
	17	4	17	0.0524	6	5	17	0.1027
	18	2	14	0.0485	6	3	13	0.1032
	19	3	15	0.0501	6	4	14	0.1067
	20	3	16	0.0414	6	6	20	0.1058
	21	5	21	0.0502	6	3	15	0.0999
	22	3	17	0.506	6	4	16	0.0972
	23	4	18	0.0532	6	5	17	0.1016
	24	4	19	0.0452	6	3	17	0.0993
	25	2	19	0.0499	6	6	19	0.0969
12	10	2	10	0.0588	6	1	7	0.0944
	11	2	10	0.0479	6	2	8	0.0975
	12	1	9	0.0516	6	3	11	0.0988
	13	2	10	0.0601	6	2	9	0.1086
	14	2	11	0.0462	6	3	11	0.0866
	15	3	14	0.0502	6	3	11	0.0986

	16	2	12	0.0518	6	4	13	0.1048
	17	3	14	0.0441	6	3	12	0.0976
	18	3	14	0.0508	6	1	12	0.0997
	19	2	14	0.0494	6	5	19	0.1001
	20	4	17	0.0490	6	5	16	0.1004
	21	4	17	0.0490	6	2	14	0.0997
	22	2	16	0.0492	6	5	16	0.1070
	23	3	17	0.0484	6	6	19	0.1019
	24	5	21	0.0479	6	3	16	0.1015
	25	2	18	0.0499	6	4	17	0.0978
13	10	1	8	0.0446	7	3	10	0.1111
	11	2	9	0.0548	7	2	8	0.1110
	12	2	10	0.0405	7	2	9	0.0836
	13	1	10	0.0510	7	3	10	0.0968
	14	2	11	0.0487	7	2	10	0.1005
	15	3	12	0.0580	7	3	11	0.0996
	16	3	13	0.0454	7	4	12	0.1125
	17	2	13	0.0503	7	2	12	0.0979
	18	3	14	0.0505	7	3	13	0.0900
	19	4	15	0.0588	7	4	14	0.0916
	20	5	20	0.0513	7	2	14	0.0976
	21	3	16	0.0498	7	4	15	0.0992
	22	4	17	0.0513	7	5	16	0.1021
	23	6	23	0.0563	7	3	16	0.1016
	24	5	19	0.0492	7	6	18	0.0952
	25	4	19	0.0495	7	5	18	0.0912
14	10	2	10	0.0507	7	1	7	0.089
	11	2	9	0.0548	7	2	8	0.0953
	12	1	9	0.0472	7	3	10	0.0968
	13	2	10	0.0524	7	2	9	0.1003
	14	3	14	0.0516	7	3	10	0.1117
	15	1	11	0.0482	7	4	13	0.1000
	16	2	12	0.0454	7	4	13	0.0902
	17	3	13	0.0533	7	4	13	0.0902
	18	1	13	0.0497	7	4	13	0.0997
	19	4	16	0.0479	7	5	15	0.1025
	20	4	16	0.0479	7	5	15	0.1025
	21	1	15	0.0510	7	3	14	0.1017
	22	5	22	0.0503	7	6	18	0.1009
	23	5	19	0.0492	7	1	15	0.1045
	24	5	19	0.0492	7	4	16	0.1042
	25	3	18	0.0484	7	7	22	0.0994
15	10	1	8	0.0399	8	3	10	0.1048
	11	2	9	0.0485	8	2	8	0.1018

	12	3	12	0.0578	8	3	9	0.1156
	13	1	10	0.0455	8	4	13	0.1058
	14	2	11	0.0424	8	2	10	0.0916
	15	3	12	0.0502	8	3	11	0.0892
	16	4	16	0.0502	8	4	12	0.1005
	17	3	13	0.0545	8	3	12	0.0997
	18	3	14	0.0430	8	4	13	0.0997
	19	4	15	0.0500	8	6	19	0.1049
	20	3	15	0.0521	8	3	14	0.0937
	21	4	16	0.0527	8	5	15	0.1081
	22	4	17	0.0430	8	7	22	0.1042
	23	2	17	0.0502	8	6	17	0.0984
	24	4	18	0.0497	8	5	17	0.0941
	25	5	19	0.0511	8	8	25	0.1034
16	10	2	9	0.0485	8	1	7	0.0858
	11	2	9	0.0485	8	3	11	0.0998
	12	1	9	0.0440	8	1	8	0.0941
	13	2	10	0.0468	8	2	9	0.0941
	14	3	12	0.0502	8	3	10	0.1019
	15	3	12	0.0502	8	4	12	0.1005
	16	2	12	0.0407	8	4	12	0.1005
	17	3	13	0.0463	8	5	17	0.1115
	18	4	15	0.0500	8	5	15	0.0979
	19	4	15	0.0500	8	4	13	0.1076
	20	4	15	0.0566	8	1	13	0.1066
	21	5	21	0.0508	8	6	18	0.0990
	22	5	18	0.0492	8	6	17	0.0984
	23	5	18	0.0492	8	1	15	0.1004
	24	2	17	0.0492	8	7	22	0.1000
	25	4	18	0.0501	8	7	19	0.1055
17	10	2	8	0.0622	9	3	10	0.0998
	11	2	9	0.0437	9	2	8	0.0947
	12	3	12	0.0535	9	3	9	0.1070
	13	2	10	0.0507	9	4	13	0.0999
	14	3	11	0.0590	9	3	10	0.1060
	15	3	12	0.0443	9	4	11	0.1178
	16	2	12	0.0499	9	1	11	0.0996
	17	3	13	0.0481	9	3	12	0.0913
	18	4	14	0.0552	9	4	13	0.0901
	19	5	19	0.0499	9	5	14	0.0998
	20	3	15	0.0459	9	4	14	0.0967
	21	4	16	0.0457	9	5	15	0.0972
	22	5	17	0.0519	9	3	15	0.1001
	23	4	17	0.0523	9	5	16	0.1013

	24	5	18	0.0530	9	6	17	0.1028
	25	7	25	0.0558	9	4	17	0.1009
18	10	2	9	0.0437	9	2	7	0.1162
	11	2	9	0.0437	9	3	10	0.0943
	12	1	9	0.0414	9	1	8	0.1033
	13	3	13	0.0519	9	2	9	0.0892
	14	3	12	0.0443	9	4	13	0.0987
	15	2	11	0.0494	9	2	10	0.1015
	16	3	12	0.0538	9	3	11	0.0947
	17	4	15	0.0493	9	5	17	0.1017
	18	2	13	0.0465	9	5	14	0.0998
	19	3	14	0.0441	9	5	14	0.0998
	20	4	15	0.0498	9	1	13	0.1033
	21	3	15	0.0507	9	4	14	0.1025
	22	4	16	0.0505	9	5	15	0.1035
	23	1	16	0.0551	9	3	15	0.1008
	24	6	21	0.0504	9	7	19	0.0991
	25	6	20	0.0491	9	7	19	0.0934
19	10	2	8	0.0578	10	1	7	0.1002
	11	2	9	0.0400	10	2	8	0.0889
	12	3	12	0.0500	10	3	9	0.1000
	13	2	10	0.0464	10	1	9	0.0964
	14	3	11	0.0536	10	3	10	0.0988
	15	4	15	0.0547	10	4	11	0.1094
	16	2	12	0.0455	10	2	11	0.0972
	17	3	13	0.0431	10	4	12	0.1054
	18	4	14	0.0493	10	6	18	0.1141
	19	3	14	0.0519	10	3	13	0.0982
	20	4	15	0.0509	10	5	14	0.1101
	21	5	16	0.0573	10	7	21	0.1096
	22	2	16	0.0491	10	4	15	0.0993
	23	4	17	0.0465	10	5	16	0.0925
	24	5	18	0.0466	10	8	24	0.1059
	25	7	25	0.0506	10	5	17	0.1005
20	10	2	8	0.0578	10	2	7	0.1112
	11	1	8	0.0560	10	3	9	0.1000
	12	2	9	0.0550	10	3	9	0.1000
	13	3	12	0.0480	10	2	9	0.0852
	14	3	11	0.0536	10	4	12	0.0954
	15	2	11	0.0462	10	2	10	0.0980
	16	3	12	0.0491	10	3	11	0.0893
	17	4	14	0.0493	10	5	14	0.1008
	18	4	14	0.0493	10	3	12	0.0968
	19	4	14	0.0572	10	4	13	0.0928

	20	5	17	0.0508	10	1	13	0.1004
	21	3	15	0.0469	10	6	16	0.0968
	22	4	16	0.0457	10	6	16	0.0968
	23	5	17	0.0512	10	7	19	0.1006
	24	4	17	0.0502	10	7	18	0.1012
	25	5	18	0.0506	10	7	18	0.1012
21	10	2	8	0.0542	11	1	7	0.0964
	11	2	9	0.0370	11	2	8	0.0842
	12	1	9	0.0489	11	3	9	0.0940
	13	2	10	0.0429	11	2	9	0.0998
	14	3	11	0.0493	11	3	10	0.0928
	15	4	15	0.0512	11	4	11	0.1025
	16	3	12	0.0519	11	3	11	0.1022
	17	4	13	0.0586	11	4	12	0.0984
	18	5	18	0.0538	11	5	13	0.1079
	19	3	14	0.0475	11	4	13	0.1039
	20	4	15	0.0460	11	5	14	0.1022
	21	5	16	0.0516	11	7	21	0.1041
	22	4	16	0.0523	11	5	15	0.1052
	23	5	17	0.0518	11	6	16	0.1049
	24	7	24	0.0566	11	8	24	0.1000
	25	4	18	0.0500	11	6	17	0.1062
22	10	2	8	0.0542	11	2	7	0.1071
	11	1	8	0.0542	11	3	8	0.0942
	12	2	9	0.0518	11	1	9	0.0990
	13	3	11	0.0493	11	4	9	0.1116
	14	3	11	0.0493	11	4	11	0.1025
	15	2	11	0.0436	11	4	11	0.1025
	16	4	15	0.0500	11	5	16	0.1106
	17	1	12	0.0514	11	5	14	0.0936
	18	3	13	0.0487	11	3	12	0.0928
	19	4	14	0.0524	11	6	19	0.1091
	20	1	14	0.0506	11	2	13	0.0990
	21	5	16	0.0516	11	4	14	0.0917
	22	6	22	0.0580	11	7	22	0.1076
	23	2	16	0.0502	11	4	15	0.0992
	24	4	17	0.0465	11	7	18	0.0925
	25	6	19	0.0463	11	1	16	0.1030
23	10	2	8	0.0512	12	1	7	0.0931
	11	3	11	0.0631	12	2	8	0.0803
	12	1	9	0.0464	12	3	9	0.0895
	13	2	10	0.0400	12	2	9	0.0956
	14	3	11	0.0458	12	3	10	0.0879
	15	1	11	0.0489	12	4	11	0.0967

	16	3	12	0.0481	12	3	11	0.0971
	17	4	13	0.0542	12	4	12	0.0926
	18	5	18	0.0505	12	5	13	0.1010
	19	4	14	0.0545	12	4	13	0.0980
	20	4	15	0.0420	12	5	14	0.0956
	21	6	21	0.0518	12	2	14	0.0999
	22	4	16	0.0480	12	5	15	0.0986
	23	5	17	0.0470	12	6	16	0.0977
	24	6	18	0.0521	12	4	16	0.0991
	25	5	18	0.0515	12	6	17	0.0990
24	10	2	8	0.0512	12	2	7	0.1036
	11	1	8	0.0526	12	3	9	0.0895
	12	2	9	0.0492	12	1	8	0.0973
	13	3	11	0.0458	12	4	13	0.1062
	14	1	10	0.0507	12	4	11	0.0967
	15	3	11	0.0576	12	4	11	0.0967
	16	4	14	0.0477	12	5	16	0.1046
	17	1	12	0.0496	12	5	13	0.1010
	18	3	13	0.0458	12	5	13	0.1010
	19	5	19	0.0500	12	6	19	0.1027
	20	2	14	0.0495	12	3	13	0.1003
	21	4	15	0.0496	12	6	15	0.1036
	22	6	22	0.0530	12	7	22	0.1008
	23	3	16	0.0496	12	4	15	0.0955
	24	6	18	0.0521	12	7	17	0.1052
	25	7	25	0.0554	12	1	16	0.1004
25	10	2	8	0.0487	13	1	7	0.0903
	11	3	11	0.0487	13	1	7	0.1215
	12	2	9	0.0537	13	3	9	0.0854
	13	3	10	0.0603	13	2	9	0.0920
	14	3	11	0.0429	13	3	10	0.0837
	15	2	11	0.0495	13	5	15	0.1071
	16	3	12	0.0450	13	3	11	0.0927
	17	4	13	0.0504	13	4	12	0.0876
	18	2	13	0.0489	13	6	18	0.1006
	19	4	14	0.0506	13	4	13	0.0930
	20	5	15	0.0561	13	5	14	0.0900
	21	2	15	0.0492	13	3	14	0.0975
	22	5	16	0.0550	13	5	15	0.0931
	23	5	17	0.0431	13	6	16	0.0915
	24	3	17	0.0502	13	5	16	0.1007
	25	5	18	0.0474	13	6	17	0.0930

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