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NOTE ON SEED GRAPHS WITH COMPONENTS OF GIVEN ORDER

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ABSTRACT. A **closed neighbourhood** $N_G[x]$ of a vertex x in a graph G is the subgraph of G induced by x and all neighbours of x. A **seed** of a vertex $x \in G$ is the subgraph of G induced by all vertices of $G \setminus N_G[x]$ and we denote it by $S_G(x)$. A graph F is a **seed graph** if there exists a graph G such that $S_G(x) \cong F$ for each $x \in G$. In this paper seed graphs with more than two components are studied. It is shown that if all components are of equal order, then they are all isomorphic to a complete graph. In the general case it is shown how the structure of any component F_i of a seed graph F depends on the structure of all components of smaller order.

1. Introduction

The notion of seed graphs arose in connection with a "complementary" (in a sense) approach to the study of local properties of graphs. A **closed neighbourhood** of a vertex $x \in G$, denoted $N_G[x]$, is the subgraph of G induced by x and all neighbours of x. We define a **seed** of a vertex x in a graph G as the subgraph of G induced by all vertices of $G \setminus N_G[x]$ and denote it by $S_G(x)$. A graph F is a **seed graph** if there exists a graph G such that $S_G(x) \cong F$ for each $x \in G$. The graph G is then called an **isomorphic survivor graph** with the seed F. We also say, as in the case of graphs with constant neighbourhoods, that such G is a realization of F. The "complementarity" of the notions of graphs with constant neighbourhoods and the isomorphic survivor graphs is obvious. We use the standard notation: A graph $\overline{G}$ is a complement of a graph $G = (V, E)$ if $V(\overline{G}) = V(G)$ and $E(\overline{G}) = \{uv | u, v \in V(G) \text{ and } uv \notin E(G)\}$. If G is an isomorphic survivor graph with a seed F, then $S_G(x) \cong F$ and hence $\overline{S_G(x)} \cong \overline{F}$ for every $x \in G$. But $\overline{S_G(x)}$ is the open neighbourhood of x in $\overline{G}$ and $\overline{S_G(x)} = N_{\overline{G}}(x)$. $\overline{G}$ is then a graph with constant neighbourhood $\overline{S_G(x)}$. The reason why seed graphs are studied instead of their complements as constant neighbourhoods (or isomorphic survivor graphs instead of graphs with complementary constant neighbourhoods) is that some properties are much easier to describe when seed graphs are considered. This occurs, for example, when a seed graph is a minimal connected or

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disconnected graph. The former case was studied by Gunther and Hartnell [**2**] and Hartnell and Kocay [**3**]. It is worth mentioning that while they proved in their papers, among other results, that the only cycles which are seed graphs are C_3 ([**2**]), C_4, C_5 and C_6 ([**3**]), the same result was obtained in terms of graphs with constant neighbourhoods by Zelinka [**5**]. The case of disconnected seed graphs was studied for two components by Markus and Rall [**4**]. In [**1**] the author presents related results for disconnected seed graphs with more than two components which are either regular or of a given size.

All graphs discussed here are finite, simple, and undirected. We denote the **order** (i.e., the number of vertices) of a graph G by $|G|$ while its **size** (i.e., the number of edges) by $||G||$. The graph induced by a vertex set U will be denoted $\langle U\rangle$. The **disjoint union** $G = G_1 \cup G_2 \cup \cdots \cup G_k$ of graphs $G_1, G_2, \ldots, G_k$ is a graph G with components $G_1, G_2, \ldots, G_k$. We denote nH the graph $H \cup H \cup \cdots \cup H$ with n components isomorphic to H. We also define a **composition** $G[H]$ (sometimes called a **lexicographic product**) of graphs G and H as follows: $V(G[H]) = V(G) \times V(H)$ and two vertices (x_1, y_1) and (x_2, y_2) are adjacent in $G[H]$ if and only if either $x_1x_2 \in E(G)$, or $x_1 = x_2$ and $y_1y_2 \in E(H)$. In other words, take a graph G, take a copy of H in place of every vertex of G and replace each edge of G by $K_{n,n}$, where $n = |H|$. The composition appears to be a useful tool in constructions of isomorphic survivor graphs.

In this article we are interested in disconnected seed graphs with more than two components. The aim of this article is to show that the structure of every seed graph is relatively strictly determined by its smallest component.

2. Preliminaries

We start with some lemmas that will be repeatedly used in proofs leading to our main result. We use the following notation: The intersection of a neighbourhood $N_G(x)$ of a vertex x of G with a subset V of the vertex set of G, $N_G(x) \cap V$, will be denoted by $N_V(x)$. By the **neighbourhood of a subgraph** F of G (denoted $N_G(F)$) we understand the set of all neighbours of vertices of F which do not belong to F.

Lemma 2.1. *Let $F = F_1 \cup F_2 \cup \cdots \cup F_k$ be a seed graph with $k \geq 3$ and G be its realization. Let u be a vertex of G and $S_G(u) = H_1 \cup H_2 \cup \cdots \cup H_k$ be its seed such that $H_i \cong F_i$ for each $i = 1, 2, \ldots, k$. If $V = N_G(u)$ and $|H_i| \leq |H_j|$, then $N_G(x) \cap N_G(u) = N_V(x) \supseteq N_V(y) = N_G(y) \cap N_G(u)$ for each $x \in H_i, y \in H_j$.*

Proof. First we define for every vertex $x \in G$ and every non-negative integer p a function $s(x, p)$ as the number of vertices of all components of $S_G(x)$, whose order is greater than p. As G is an isomorphic survivor graph, it is obvious that $s(x, p) = s(y, p)$ for each pair $x, y \in G$ and every p, namely $s(x, |H_i|) = s(u, |H_i|)$. To prove the assertion, we proceed by contradiction. Suppose that $|H_i| \leq |H_j|$

and there exist vertices $x \in H_i$ and $y \in H_j$ such that $N_V(x) \not\supseteq N_V(y)$. Then there exists a vertex $v \in V = N_G(u)$ which is adjacent to y but not to x. If z is now any vertex of H_k, where $|H_k| > |H_i|$, then z is not adjacent to x and therefore $z \in S_G(x)$. The number of such vertices equals exactly to $s(u, |H_i|)$. Moreover, since v is not adjacent to x but is a neighbour of y, the graph $H' = \langle H_j \cup v \rangle$ belongs to a component of order at least $|H_j| + 1 > |H_i|$. Hence $s(x, |H_i|) \geq s(u, |H_i|) + 1$, which is impossible, as G is an isomorphic survivor graph. Therefore no such a vertex v exists and $N_V(y) \subseteq N_V(x)$. □

The following corollary is an immediate consequence of the Lemma and the proof can therefore be omitted.

Corollary 2.2. *Let $F = F_1 \cup F_2 \cup \cdots \cup F_k$ be a seed graph with $k \geq 3$ and G be its realization. Let u be a vertex of G, $V = N_G(u)$ and $S_G(u) = H_1 \cup H_2 \cup \cdots \cup H_k$ be its seed such that $H_i \cong F_i$ for each $i = 1, 2, \ldots, k$. If $|H_i| \leq |H_j|$ and $x \in H_i$, then $N_V(x) \supseteq N_G(H_j)$ and hence $N_G(H_i) \supseteq N_G(H_j)$.*

Lemma 2.3. *Let F, G, H, u, and V be as in Lemma 2.1. Let H_i and $H_j, i \neq j$, have the property that for each $x \in H_i$ and each $y \in H_j$ it holds that $N_V(x) = N_V(y)$. Then $H_i \cong H_j$ and therefore $F_i \cong F_j$.*

Proof. To obtain the desired result, we compare the seeds of arbitrary vertices $x \in H_i$ and $y \in H_j$. Recall that $N_V(x) = N_G(x) \cap N_G(u)$ and $N_V(y) = N_G(y) \cap N_G(u)$. As $N_V(x) = N_V(y)$, we can see that both $S_G(x)$ and $S_G(y)$ contain all components H_l of $S_G(u)$ such that $l \neq i, j$ and $N_G(H_l) \subseteq N_V(x)$ $(= N_V(y))$. They also both contain a connected graph that includes vertex u, all vertices of $N_G(u) \setminus N_V(x)$ and all vertices of the components H_m of $S_G(u)$ such that $N_V(x) = N_V(y)$ is a proper subset of $N_G(H_m)$. Besides these common components, $S_G(x)$ contains H_j and a graph $S_{H_i}(x)$ (which is empty if x is adjacent to all vertices of H_i). Similarly, $S_G(y)$ contains besides the common components the component H_i and a graph $S_{H_j}(y)$ (which is again empty if y is adjacent to all vertices of H_j). Because G is an isomorphic survivor graph, $S_G(x)$ is isomorphic to $S_G(y)$ and therefore $H_j \cup S_{H_i}(x)$ must be isomorphic to $H_i \cup S_{H_j}(y)$. As $S_{H_i}(x)$ is either empty or a proper subgraph of H_i, it follows that $H_i \cong H_j$, which we wanted to prove. □

Combining Lemmas 2.1 and 2.3, we immediately get the following theorem.

Theorem 2.4. *Let $F = F_1 \cup F_2 \cup \cdots \cup F_k$ be a seed graph with $k \geq 3$ and G be one of its realizations. Let u be a vertex of G, $V = N_G(u)$ and $S_G(u) = H = H_1 \cup H_2 \cup \cdots \cup H_k$ be its seed such that $H_i \cong F_i$ for each $i = 1, 2, \ldots, k$. Then the following conditions are equivalent:*

(i) $|F_i| = |F_j|$,
(ii) $N_V(x) = N_V(y)$ *for each* $x \in H_i$ *and each* $y \in H_j$,
(iii) $F_i \cong F_j$.

Proof. The implication (i) $\Rightarrow$ (ii) follows from Lemma 2.1. If $|H_i| = |H_j|$ then $N_V(x) \subseteq N_V(y)$ and $N_V(y) \subseteq N_V(x)$ for each $x \in H_i$ and each $y \in H_j$. This yields $N_V(x) = N_V(y)$. The second implication, (ii) $\Rightarrow$ (iii), follows from Lemma 2.3, and the last one, (iii) $\Rightarrow$ (i), is trivial and the proof is complete. $\square$

The following corollary is an easy consequence of the Theorem and appears to be useful later.

Corollary 2.5. *Let F, G, H, u, and V be as in Lemma 2.1 and let $|H_i| = |H_j|$. Then for each $x, y \in H_i$ it holds that $N_V(x) = N_V(y)$.*

Proof. Let z be any vertex of H_j. Then, by the Theorem, $N_V(x) = N_V(z)$ and $N_V(y) = N_V(z)$, which yields $N_V(x) = N_V(y)$. $\square$

It seems quite natural now that from the assumption of Corollary 2.5 it also follows that the graph H_i is an isomorphic survivor graph itself. We prove this result in the following section. The last result in this section is another useful tool.

In Corollary 2.2 we have shown that for every H_i, H_j with $|H_i| < |H_j|$ it holds that $N_G(H_j) \subseteq N_G(H_i)$. However, if the seed graph H contains two or more components of the same order $|H_j|$, we can even prove that $N_G(H_j)$ is a proper subset of $N_G(H_i)$. Hence we can state another corollary, which is in certain sense a stronger version of Corollary 2.2.

Corollary 2.6. *Let F, G, H, u, and V be as in Lemma 2.1. If there exist components H_j, H_m and H_i such that $|H_i| < |H_j| = |H_m|$, then for every $x \in H_i, y \in H_j$ it holds that $N_V(y)$ is a proper subset of $N_V(x)$.*

Proof. Because $|H_i| < |H_j|$, then $H_i \not\cong H_j$ and by Theorem 2.4 there must be vertices $x \in H_i, y \in H_j$ such that $N_V(x) \neq N_V(y)$. From Lemma 2.1 it follows that for each $x \in H_i, y \in H_j$ it holds that $N_V(y) \subseteq N_V(x)$. Therefore there must be a vertex $v \in V$ which is adjacent to x but not to y. But according to Corollary 2.5 all vertices of H_j have the same neighbours in V as y and hence none of them is adjacent to v. Thus the vertex $v \in N_V(x) \subseteq N_G(H_i)$ does not belong to $N_G(H_j)$ and $N_G(H_j)$ is a proper subset of $N_G(H_i)$. $\square$

3. Seed Graphs With Components Of Given Order

In this section we prove that if F is a seed graph with at least three components $F_1, F_2, \ldots, F_k$ with orders $|F_1| \leq |F_2| \leq \cdots \leq |F_k|$, then the structure of a component F_i is more or less determined by the components of smaller orders. If there are two or more components of the same order, then they have interesting properties, as we have already mentioned. They are mutually isomorphic, they are themselves isomorphic survivor graphs and if $|F_{i-1}| < |F_i| = |F_{i+1}|$, then their seeds, $S_{F_i}(x)$, are isomorphic to $F_1 \cup F_2 \cup \cdots \cup F_{i-1}$. Similar results for components of equal size and regularity are proved in [**1**].

First we show that although in the general case the components of a seed graph F do not have to be isomorphic survivor graphs themselves, the seed of every vertex x of a component F_i with respect to F_i, $S_{F_i}(x)$, always contains components that are isomorphic to all components of F which are of smaller order than the component F_i. We split the proof into two lemmas, as the first lemma will be useful later.

Lemma 3.1. *Let $F = F_1 \cup F_2 \cup \cdots \cup F_k$ be a seed graph with $k \geq 3$ and G be its realization. Let u be a vertex of G, $V = N_G(u)$ and $S_G(u) = H = H_1 \cup H_2 \cup \cdots \cup H_k$ be its seed such that*

(i) *$H_i \cong F_i$ for each $i = 1, 2, \ldots, k$,*
(ii) *$N_V(x)$ is a proper subset of $N_G(H_l)$ for each $x \in H_i$ and each $H_l, l = 1, 2, \ldots, i-1$,*
(iii) *$N_V(x) \supseteq N_G(H_l)$ for each $x \in H_i$ and each $H_l, l = i+1, i+2, \ldots, k$.*

Then the seed of every vertex x of the component F_i with respect to F_i, $S_{F_i}(x) = S_G(x) \cap F_i$, contains an induced subgraph isomorphic to $F' = F_1 \cup F_2 \cup \cdots \cup F_{i-1}$.

Proof. Let G, H, u and V be again as above and let x be any vertex of H_i. Then all components $H_{i+1}, H_{i+2}, \ldots, H_k$ of $S_G(u)$ are also components of $S_G(x)$, because $N_V(x) \supseteq N_V(t)$ for each $t \in H_l, l = i+1, i+2, \ldots, k$. Denote now $V_i = N_G(u) \setminus N_G(H_i)$. Because by Corollary 2.6 $N_V(x)$ is for every $z \in H_l, l = 1, 2, \ldots, i-1$, a proper subset of $N_V(z)$, it is clear that each $N_V(z), z \in H_l, l = 1, 2, \ldots, i-1$ contains a vertex of V_i. Hence $\langle H_1 \cup H_2 \cup \cdots \cup H_{i-1} \cup V_i \cup u \rangle$ is a connected graph and therefore a subgraph of some component H^* of $S_G(x)$. Obviously, H^* cannot be isomorphic to any of $H_1, H_2, \ldots, H_{i-1}$ and therefore is isomorphic to H_i. Let us denote $H^* = H_i'$. We can see now that every vertex of $G \setminus H_i$ belongs either to $N_V(x)$ or to one of the components $H_i', H_{i+1}, H_{i+2}, \ldots, H_k$ of $S_G(x)$. It is obvious that all other components of $S_G(x)$, namely those isomorphic to $H_1, H_2, \ldots, H_{i-1}$, must be induced subgraphs of F_i and therefore of $S_{F_i}(x)$, which we wanted to prove. □

Further we prove that a component H_i of $S_G(u)$ in an isomorphic survivor graph G with the property that $|F_1| \leq \cdots \leq |F_{i-1}| < |F_i| \leq |F_{i+1}| \leq \cdots \leq |F_k|$ satisfies the assumptions of Lemma 3.1.

Lemma 3.2. *Let G be an isomorphic survivor graph and let $S_G(u) = H = H_1 \cup H_2 \cup \cdots \cup H_k$ with $k \geq 3$ and $|H_1| \leq \cdots \leq |H_{i-1}| < |H_i| \leq |H_{i+1}| \leq \cdots \leq |H_k|$. Then H_i satisfies assumptions* (ii) *and* (iii) *of Lemma* 3.1.

Proof. The condition (iii) is satisfied by Corollary 2.2. Without loss of generality we can assume that H contains m copies of H_i, namely $H_i, \ldots H_{i+m-1}$, where $1 \leq m \leq k - i + 1$. Suppose, to the contrary, that there is a vertex $y \in H_i$ such that $N_V(y)$ is not a proper subset of $N_V(H_j)$ for some $j < i$. Then, by Lemma 2.1, $N_V(y) = N_V(x)$ for every $x \in H_j$ and also for every $x \in H_s, s = j+1, \ldots, i-1$.

There must be a vertex $z \in H_i$ and $v \in N_V(y)$ such that v is not adjacent to z, otherwise for every vertex t of H_i we have $N_V(t) = N_V(x)$ and by Lemma 2.3 $H_j \cong H_i$, which is impossible. Because by Lemma 2.1 $N_V(z) \supseteq N_V(H_l)$ for each $l > i$, $S_G(z)$ contains the components $H_{i+1}, H_{i+2}, \ldots, H_k$. On the other hand, all vertices of $H_1, H_2, \ldots, H_{i-1}$ are adjacent to v and therefore they must all belong to the same component H^* of $S_G(z)$. Hence it is clear that $H^* \cong H_i$ and $|H_i| = |H^*| \geq 1 + |N_G(u)| - |N_V(z)| + |H_1| + |H_2| + \cdots + |H_{i-1}|$. At the same time $S_G(y)$ also contains for the same reasons the components $H_{i+1}, H_{i+2}, \ldots, H_k$. $S_G(y)$ also contains the components $H_j, H_{j+1}, \ldots, H_{i-1}$ that are all of smaller order than H_i and are not isomorphic to H_i. Because the component $H_i' \cong H_i$ of $S_G(y)$ cannot obviously be a subgraph of H_i, H_i' must be the one containing the vertex u. The order of H_i' is at most $1+|N_G(u)|-|N_V(y)|+|H_1|+|H_2|+\cdots+|H_{j-1}|$. This is of course less than $|H_i|$, because $|N_V(z)| < |N_V(y)|$. Thus $S_G(y)$ contains at most $m - 1$ components isomorphic to H_i, which is a contradiction. Hence there must be a vertex non-adjacent to y in every $N_G(H_j)$, $j = 1, 2, \ldots i - 1$, the condition (ii) is also satisfied and the proof is complete. □

The lemmas immediately yield the following theorem.

Theorem 3.3. *Let $F = F_1 \cup F_2 \cup \cdots \cup F_k$ with $k \geq 3$ components be a seed graph such that $|F_1| \leq \cdots \leq |F_{i-1}| < |F_i| \leq |F_{i+1}| \leq \cdots \leq |F_k|$. Then for every $x \in F_i$ it holds that $S_{F_i}(x)$ contains an induced subgraph isomorphic to $F' = F_1 \cup F_2 \cup \cdots \cup F_{i-1}$.*

To present an example of such a seed graph F, we use the recursive composition of a sequence of isomorphic survivor graphs.

Example 3.4. Let $G_1 = C_6$. We construct an isomorphic survivor graph $G = G_k$ with a seed graph F with k components recursively. For $i = 2, 3, \ldots, k$ we define $G_i = C_6[G_{i-1}]$. Then $F \cong F_1 \cup F_2 \cup \cdots \cup F_k$ with $|F_1| < |F_2| < \cdots < |F_k|$, where $F_1 \cong P_3, F_2 \cong P_3[C_6] = P_3[G_1], F_3 \cong P_3[C_6[C_6]] = P_3[G_2], \ldots F_i \cong P_3[G_{i-1}], \ldots, F_k \cong P_3[G_{k-1}]$. One can check that for every $i = 2, 3, \ldots, k$ and every $x \in F_i$ the seed of x in the graph F_i, $S_{F_i}(x)$, contains an induced subgraph isomorphic to $F_1 \cup F_2 \cup \cdots \cup F_{i-1}$. However, there are two classes of vertices in every $F_i, 2 \leq i \leq k$. For instance in F_2 there are 12 vertices $x_{21}, x_{22}, \ldots, x_{2\ 12}$ with $S_{F_2}(x_{2i}) \cong P_3 \cup C_6 \cong F_1 \cup G_1$ for each $i = 1, 2, \ldots, 12$ and 6 vertices $x'_{21}, x'_{22}, \ldots, x'_{26}$ with $S_{F_2}(x'_{2j}) \cong P_3 \cong F_1$ for each $j = 1, 2, \ldots, 6$. Similarly in F_3 there are 72 vertices $x_{31}, x_{32}, \ldots, x_{3\ 72}$ with $S_{F_3}(x_{3i}) \cong P_3 \cup P_3[C_6] \cup C_6[C_6] \cong F_1 \cup F_2 \cup G_2$ for each $i = 1, 2, \ldots, 72$ and 36 vertices $x'_{31}, x'_{32}, \ldots, x'_{3\ 36}$ with $S_{F_3}(x'_{3j}) \cong P_3 \cup P_3[C_6] \cong F_1 \cup F_2$ for each $j = 1, 2, \ldots, 36$. In general, every $F_l, l = 2, 3, \ldots, k$ contains vertices x_{li} with $S_{F_l}(x_{li}) \cong F_1 \cup F_2 \cup \cdots \cup F_{l-1} \cup G_{l-1}$ and vertices x'_{lj} with $S_{F_l}(x'_{lj}) \cong F_1 \cup F_2 \cup \cdots \cup F_{l-1}$.

We can also use the same idea to construct more general seed graphs with $k \geq 3$ components of different orders. Let $G_1, G_2, \ldots, G_k$ be arbitrary isomorphic survivor graphs with connected seed graphs $F_1, F_2, \dots, F_k$, respectively, and let $G'_1 = G_1$. We then define $G'_i = G_i[G'_{i-1}]$ for $i = 2, 3 \dots, k$ to obtain an isomorphic survivor graph $G = G'_k$ with the seed $F' = F'_1 \cup F'_2 \cup \dots \cup F'_k$, where $F'_1 \cong F_1, F'_2 \cong F_2[G_1] \cong F_2[G'_1], \dots, F'_i \cong F_i[G'_{i-1}], \dots, F'_k \cong F_k[G'_{k-1}]$.

Finally we show that if F_i, F_j are components of the smallest order among all components of F, then $F_i \cong F_j \cong K_p$ for $p = |F_i|$.

Theorem 3.5. *Let $F = F_1 \cup F_2 \cup \dots \cup F_k$, $k \geq 3$ be a seed graph with $|F_1| = |F_2| = \dots = |F_m| \leq \dots \leq |F_k|$, where $2 \leq m \leq k$. Then $F_1 \cong F_2 \cong \dots \cong F_m$ is a complete graph.*

Proof. Let G be a realization of F and u a vertex of G. Let $S_G(u) = H = H_1 \cup H_2 \cup \dots \cup H_k$ and $H_i \cong F_i$ for $i = 1, 2, \dots, k$. If $x \in H_1$, then according to Lemma 2.1 and Theorem 2.4 $N_G(H_l) \subseteq N_V(x)$ for each $l = 2, 3, \dots, k$ and hence $S_G(x)$ contains $k - 1$ components $H_2, H_3, \dots, H_k$. Moreover, $S_G(x)$ contains the vertex u which therefore belongs to a component $H'_1 \cong H_1 \cong F_1$. If there was a vertex $y \in H_1$ nonadjacent to x, then $S_G(x)$ would have more than k components, because by Corollary 2.5 $N_V(y) = N_G(x)$ and hence y does not belong to any of the components $H'_1, H_2, \dots, H_k$. This is absurd. Therefore x is adjacent to all vertices of H_1 and $H_1 \cong K_p$. By Theorem 2.4 $H_1 \cong H_2 \cong \dots \cong H_m$ and the proof is complete. □

The following corollary follows directly from the previous theorem and treats the special case when all components have the same order.

Corollary 3.6. *Let $F = F_1 \cup F_2 \cup \dots \cup F_k$ be a seed graph with $|F_1| = |F_2| = \dots = |F_k|$ and $k \geq 3$. Then $F_1 \cong F_2 \cong \dots \cong F_k \cong K_p$ for $p = |F_1|$.*

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