ON THE EXISTENCE OF POSITIVE SOLUTION FOR AN ELLIPTIC EQUATION OF KIRCHHOFF TYPE VIA MOSER ITERATION METHOD

FRANCISCO JÚLIO S. A. CORRÊA AND GIOVANY M. FIGUEIREDO

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Dedicated to our dear friend and collaborator Professor Claudianor O. Alves

We investigate the questions of existence of positive solution for the nonlocal problem $-M(\|u\|^2)\Delta u = f(\lambda,u)$ in Ω and $u = 0$ on $\partial\Omega$, where Ω is a bounded smooth domain of $\mathbb{R}^N$, and M and f are continuous functions.

1. Introduction

In this paper, we study some questions related to the existence of positive solution for the nonlocal elliptic problem

$$\begin{aligned} -M(\|u\|^2)\Delta u &= f(\lambda,u) \quad \text{in } \Omega, \\ u &= 0 \quad \text{on } \partial\Omega, \end{aligned} \tag{$(P)_\lambda$}$$

where Ω is a bounded smooth domain, $M : \mathbb{R}^+ \to \mathbb{R}$ is a function whose behavior will be stated later, $f : \mathbb{R}^+ \times \mathbb{R} \to \mathbb{R}$ is a given nonlinear function, and $\|\cdot\|$ is the usual norm in $H_0^1(\Omega)$ given by

$$\|u\|^2 = \int |\nabla u|^2 \tag{1.1}$$

and finally, through this work, $\int u$ denotes the integral $\int_\Omega u(x)dx$.

The main goal of this paper is to establish conditions on M and f under which problem $(P)_\lambda$ possesses a positive solution.

Problem $(P)_\lambda$ is called nonlocal because of the presence of the term $M(\|u\|^2)$ which implies that the equation in $(P)_\lambda$ is no longer a pointwise identity. This provokes some mathematical difficulties which make the study of such a problem particulary interesting.

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Besides, these kinds of problems have motivations in physics. Indeed, the operator $M(\|u\|^2)\Delta u$ appears in the Kirchhoff equation, by virtue of this $(P)_\lambda$, is called of the Kirchhoff type, which arises in nonlinear vibrations, namely,

$$\begin{gathered} u_{tt} - M(\|u\|^2)\Delta u = f(x,u) \quad \text{in } \Omega \times (0,T), \\ u = 0 \quad \text{on } \partial\Omega \times (0,T), \\ u(x,0) = u_0(x), \qquad u_t(x,0) = u_1(x). \end{gathered} \tag{1.2}$$

Hence, problem $(P)_\lambda$ is the stationary counterpart of the above evolution equation.

Such a hyperbolic equation is a general version of the Kirchhoff equation

$$\rho \frac{\partial^2 u}{\partial t^2} - \left(\frac{P_0}{h} + \frac{E}{2L} \int_0^L \left| \frac{\partial u}{\partial x} \right|^2 dx \right) \frac{\partial^2 u}{\partial x^2} = 0 \tag{1.3}$$

presented by Kirchhoff [14]. This equation extends the classical d'Alembert's wave equation by considering the effects of the changes in the length of the strings during the vibrations. The parameters in (1.3) have the following meanings: L is the length of the string, h is the area of cross-section, E is the Young modulus of the material, ρ is the mass density and P_0 is the initial tension.

Problem (1.2) began to call the attention of several researchers mainly after the work of Lions [15], where a functional analysis approach was proposed to attack it.

The reader may consult [1, 2, 8, 16, 18] and the references therein, for more information on $(P)_\lambda$.

Actually, problem $(P)_\lambda$ is a particular example of a wide class of the so-called nonlocal equations whose study has deserved the attention of many researchers, mainly in recent years.

Let us cite some nonlocal problems in order to emphasize the importance of their studies.

First, we consider the problem

$$\begin{gathered} -a\left(\int |u|^q dx \right) \Delta u = H(x) f(u) \quad \text{in } \Omega, \\ u = 0 \quad \text{on } \partial\Omega, \end{gathered} \tag{1.4}$$

where $a : \mathbb{R}^+ \to \mathbb{R}^+$ is a given function, which does not have variational structure.

Such a problem appears in some physical situations related, for example, with biology in which u sometimes describes the population of bacteria, in case $q = 1$. In case $q = 2$, we get the well-known Carrier equation which is an appropriate model to study some questions related to nonlinear deflections of beams. See [4–7, 10] and the references therein, for more details related to problem (1.4).

Another relevant nonlocal problem is

$$\begin{gathered} -\Delta u = a(x,u) \|u\|_p^q \quad \text{in } \Omega, \\ u = 0 \quad \text{on } \partial\Omega, \end{gathered} \tag{1.5}$$

where $a : \bar{\Omega} \times \mathbb{R} \to \mathbb{R}^+$ is a known function and $\|\cdot\|_q$ is the usual L^q-norm, and its related system

$$-\Delta u^m = \|v\|_p^\alpha \quad \text{in } \Omega,$$
$$-\Delta v^n = \|u\|_q^\beta \quad \text{in } \Omega, \tag{1.6}$$
$$u = v = 0 \quad \text{on } \partial\Omega$$

comes from a parabolic phenomenon. Such problems arise in the study of the flow of a fluid through a homogeneous isotropic rigid porous medium or in studies of population dynamics. It has been suggested that nonlocal growth terms present a more realistic model of population. See [9, 11, 12, 20] and references therein.

To close this series of examples, we cite the problem

$$\Delta u = \frac{(f(u))^\alpha}{(\int f(u))^\beta} \quad \text{in } \Omega,$$
$$u = 0 \quad \text{on } \partial\Omega, \tag{1.7}$$

which arises in numerous physical models such as: systems of particles in thermodynamical equilibrium via gravitational (Coulomb) potential, 2-D fully turbulent behavior of real flow, thermal runaway in ohmic heating , shear bounds in metal deformed under high strain rates, among others. References to these applications may be found in [21].

After these motivations, let us go back to our original problem $(P)_\lambda$. We impose the following conditions on M and f: M is a continuous function and satisfies

$$M(t) \geq m_0 > 0 \quad \forall t \geq 0, \tag{M_1}$$

$$M(k) < \frac{\mu m_0}{2} \quad \text{for some } 2 < \mu < p, \text{ for any } k > 0, \tag{M_2}$$

$$\max\left\{M(k)^{(2-p+q)/(p-2)}, M(k)^{2/p-2}\right\} \leq \frac{k}{\theta} \tag{M_3}$$

for any $k > 0$, for some $q \leq p$, $2 < p < 2^*$, and $\theta > 0$, where $2^* = 2N/(N-2)$ if $N \geq 3$ and $2^* = \infty$ if $N = 2$. We also suppose that f is a continuous function and satisfies

$$\frac{f(\lambda,t) - |t|^{p-2}t}{\lambda} := g(t) \quad \text{with } g(t) \geq 0. \tag{f_1}$$

Note that by (f_1), $f(\lambda,t) \geq 0$, for all $\lambda > 0$ and assume that for all $t \geq 0$,

$$\lim_{t \to 0^+} \frac{g(t)}{t} = 0. \tag{g_1}$$

Moreover, we require that there exists $2 < \mu < p$ such that

$$0 < \mu G(t) = \int_0^t g(s)ds \leq g(t)t \quad \forall t > 0. \tag{g_2}$$

Our main result is as follows.

Theorem 1.1. *Let us suppose that the function M satisfies (M_1), (M_2), and (M_3), f satisfies (f_1), and g satisfies (g_1) and (g_2). Then there exists $\lambda_0 > 0$ such that problem $(P)_\lambda$ possesses a positive solution for each $\lambda \in [0,\lambda_0]$.*

We point out that the function $g(t) = |t|^{s-2}t$ with $s \geq 2^*$ satisfies assumptions (g_1) and (g_2).

In the present paper, we continue the study from [2], because we consider supercritical nonlinearities. In [2], the authors only consider nonlinearities with subcritical growth and so they are able to use a combination of the mountain pass theorem and an appropriate truncation of the function M to attack problem $(P)_\lambda$.

In order to solve problem $(P)_\lambda$, we first consider a truncated problem which involves only a subcritical Sobolev exponent. We show that positive solution of truncated problem is a positive solution of $(P)_\lambda$.

In Sections 2 and 3, we study the truncated problem and in Section 4, we prove an existence result for problem $(P)_\lambda$.

2. The truncated problem

First of all, we have to note that because f has a supercritical growth, we cannot use directly the variational techniques, due to the lack of compactness of the Sobolev immersions.

So we construct a suitable truncation of f in order to use variational methods or, more precisely, the mountain pass theorem. This truncation was used in the paper [19] (see [3, 13]).

Let $K > 0$ be a real number, whose precise value will be fixed later, and consider the function $g_K : \mathbb{R} \to \mathbb{R}$ given by

$$g_K(t) = \begin{cases} 0 & \text{if } t < 0, \\ g(t) & \text{if } 0 \leq t \leq K, \\ \dfrac{g(K)}{K^{p-1}} t^{p-1} & \text{if } t \geq K. \end{cases} \tag{2.1}$$

We also study the associated truncated problem

$$\begin{aligned} -M(\|u\|^2)\Delta u = f_K(u) \quad &\text{in } \Omega, \\ u = 0 \quad \text{on } \partial\Omega&, \end{aligned} \tag{$(T)_\lambda$}$$

where $f_K(t) = (t_+)^{p-1} + \lambda g_K(t)$. Such a function enjoys the following conditions:

$$f_K(t) = o(t) \quad (\text{as } t \longrightarrow 0), \tag{$f_{K,1}$}$$

$$0 < \mu \int F_K(u) \leq \int f_K(u)u \quad \forall u \in H_0^1(\Omega),\ u > 0, \tag{$f_{K,2}$}$$

where $\mu > 2$ and $F_K(t) = \int_0^t f_K(s)ds$;

$$\lim_{t\to\infty} \frac{f_K(t)}{t^{p-1}} = 1 + \lambda \frac{g(K)}{K^{p-1}}. \tag{$f_{K,3}$}$$

3. Existence of solution for the truncated problem

First, we note that

$$|f_K(t)| \le C_1|t|^{q-1} + C_2|t|^{p-1}, \tag{$f_{K,4}$}$$

where $C_1 \ge 0$, $C_2 > 0$, and for all $q \ge 1$. This is an immediate consequence of the definition of f_K.

Hence, by using $(f_{K,3})$, $(f_{K,4})$, and (M_1), we conclude from [2, Lemma 2] that there exists $\theta > 0$ such that

$$||u_\lambda||^2 \le \max\left\{M(||u_\lambda||)^{(2-p+q)/(p-2)}, M\left(||u_\lambda||^2\right)^{2/p-2}\right\}\theta \tag{3.1}$$

for all classical solutions u_λ of $(T)_\lambda$.

We now use $(f_{K,1})$, $(f_{K,2})$, $(f_{K,3})$, (M_1), (M_2) (with $\mu > 2$ obtained from condition $(f_{K,2})$) and (M_3) (with $\theta > 0$ obtained in (3.1)) to obtain, thanks to [2, Theorem 5], a positive solution u_λ of T_0 such that $I_\lambda(u_\lambda) = c_\lambda$, where c_λ is the mountain pass level associated to the functional

$$I_\lambda(u_\lambda) = \frac{1}{2}\widehat{M}\left(||u_\lambda||^2\right) - \frac{1}{p}\int F_K(u_\lambda) \tag{3.2}$$

which is related to the problem T_0, where $\widehat{M}(t) = \int_0^t M(s)ds$.

Furthermore,

$$\begin{aligned} I_\lambda(u_\lambda) - \frac{1}{\mu}I'_\lambda(u_\lambda)u_\lambda &\ge \left(\frac{m_0}{2} - \frac{M(||u_\lambda||^2)}{\mu}\right)||u_\lambda||^2 + \int \frac{1}{\mu}[f_K(u_\lambda)u_\lambda - F_K(u_\lambda)] \\ &\ge \frac{m_0}{2}||u_\lambda||^2 + \int \frac{1}{\mu}[f_K(u_\lambda)u_\lambda - F_K(u_\lambda)]. \end{aligned} \tag{3.3}$$

4. Proof of Theorem 1.1

In the proof of Theorem 1.1, we need the following estimate.

Lemma 4.1. *If u_λ is a solution (positive) of problem T_0, then $\|u_\lambda\| \le \overline{C}$ for all $\lambda \ge 0$, where $\overline{C} > 0$ is a constant that does not depend on λ.*

Proof. Since $F_k(t) \ge t_+^p/p$, one has $c_\lambda \le c_0$, where c_0 is the mountain pass level related to the functional

$$I_0(u) = \frac{1}{2}\widehat{M}(\|u\|^2) - \frac{1}{p}\int |u|^p \tag{4.1}$$

which is associated to the problem

$$\begin{aligned} -M(\|u\|^2)\Delta u &= |u|^{p-2}u \quad \text{in } \Omega, \\ u &= 0 \quad \text{on } \partial\Omega. \end{aligned} \tag{T_0}$$

Furthermore,

$$c_0 \geq c_\lambda = I_\lambda(u_\lambda) = I_\lambda(u_\lambda) - \frac{1}{\mu} I'_\lambda(u_\lambda)u_\lambda \tag{4.2}$$

and from (3.3),

$$c_0 \geq \frac{m_0}{2}||u_\lambda||^2 + \int \left[\frac{1}{\mu} f_K(u_\lambda)u_\lambda - F_K(u_\lambda)\right]. \tag{4.3}$$

From $(f_{K,2})$, we get

$$||u_\lambda|| \leq \sqrt{\frac{2c_0}{m_0}} := \overline{C} \tag{4.4}$$

for all $\lambda \geq 0$. □

Next, we are going to use the Moser iteration method [17](see [3, 13]).

Proof of Theorem 1.1. Let u_λ be a solution of problem T_0. We will show that there is K_0 such that for all $K > K_0$, there exists a corresponding λ_0 for which

$$|u_\lambda|_{L^\infty(\Omega)} \leq K \quad \forall \lambda \in [0,\lambda_0]. \tag{4.5}$$

If this is the case, one has $f_K(u_\lambda) = u_\lambda^{p-1} + \lambda g(u_\lambda)$ and so u_λ is a solution of problem $(P)_\lambda$ for all $\lambda \in [0,\lambda_0]$.

For the sake of simplicity, we will use the following notation:

$$u_\lambda := u. \tag{4.6}$$

For $L > 0$, let us define the following functions:

$$u_L = \begin{cases} u & \text{if } u \leq L, \\ L & \text{if } u > L, \end{cases} \qquad z_L = u_L^{2(\beta-1)}u, \qquad w_L = uu_L^{\beta-1}, \tag{4.7}$$

where $\beta > 1$ will be fixed later. Let us use z_L as a test function, that is,

$$M(\|u\|^2)\int \nabla u \nabla z_L = \int f_K(u)z_L \tag{4.8}$$

which implies

$$M(\|u\|^2)\int u_L^{2(\beta-1)}|\nabla u|^2 = -2(\beta-1)\int u_L^{2\beta-3}u\nabla u \nabla u_L + \int f_K(u)uu_L^{2(\beta-1)}. \tag{4.9}$$

Because of the definition of u_L, we have

$$2(\beta-1)\int u_L^{2\beta-3}u\nabla u\nabla u_L = 2(\beta-1)\int_{\{u\leq L\}} u^{2(\beta-1)}|\nabla u|^2 \geq 0 \tag{4.10}$$

and using the fact

$$f_K(u) \le \left(1+\lambda\frac{g(u)}{K^{p-1}}\right)|u|^{p-1} \tag{4.11}$$

together with (M_1)

$$\int u_L^{2(\beta-1)}|\nabla u|^2 \le \left(1+\lambda\frac{g(K)}{K^{p-1}}\right)\frac{1}{m_0}\int u^p u_L^{2(\beta-1)}, \tag{4.12}$$

we obtain

$$\int u_L^{2(\beta-1)}|\nabla u|^2 \le C_{\lambda,K}\int u^p u_L^{2(\beta-1)}, \tag{4.13}$$

where $C_{\lambda,K} = (1+\lambda(g(u)/K^{p-1}))(1/m_0)$.

On the other hand, from the continuous Sobolev immersion, one gets

$$|w_L|_{2^*}^2 \le C_1\int |\nabla w_L|^2 = C_1\int \left|\nabla\left(uu_L^{\beta-1}\right)\right|^2. \tag{4.14}$$

Consequently,

$$|w_L|_{2^*}^2 \le C_1\int u_L^{2(\beta-1)}|\nabla u|^2 + C_1(\beta-1)^2\int u_L^{2(\beta-2)}u^2|\nabla u_L|^2 \tag{4.15}$$

which gives

$$|w_L|_{2^*}^2 \le C_2\beta^2\int u_L^{2(\beta-1)}|\nabla u|^2. \tag{4.16}$$

From (4.13) and (4.16), we get

$$|w_L|_{2^*}^2 \le C_2\beta^2 C_{\lambda,K}\int u^p u_L^{2(\beta-1)} \tag{4.17}$$

and hence,

$$|w_L|_{2^*}^2 \le C_2\beta^2 C_{\lambda,K}\int u^{p-2}\left(uu_L^{\beta-1}\right)^2 = C_2\beta^2 C_{\lambda,K}\int u^{p-2}w_L^2. \tag{4.18}$$

We now use Hölder inequality, with exponents $2^*/[p-2]$ and $2^*/[2^*-(p-2)]$, to obtain

$$|w_L|_{2^*}^2 \le C_2\beta^2 C_{\lambda,K}\left(\int u^{2^*}\right)^{(p-2)/2^*}\left(\int w_L^{2.2^*/[2^*-(p-2)]}\right)^{[2^*-(p-2)]/2^*}, \tag{4.19}$$

where $2 < 2.2^*/(2^*-(p-2)) < 2^*$. Considering the continuous Sobolev immersion $H_0^1(\Omega) \hookrightarrow L^q(\Omega)$, $1 \le q \le 2^*$, we obtain

$$|w_L|_{2^*}^2 \le C_2'\beta^2 C_{\lambda,K}\|u\|^{p-2}|w_L|_{\alpha^*}^2, \tag{4.20}$$

where $\alpha^* = 2.2^*/(2^* - (p-2))$. Using Lemma 4.1, we get

$$|w_L|_{2^*}^2 \leq C_3\beta^2 C_{\lambda,K}\overline{C}^{p-2}|w_L|_{\alpha^*}^2. \tag{4.21}$$

Since $w_L = uu_L^{\beta-1} \leq u^\beta$ and supposing that $u^\beta \in L^{\alpha^*}(\Omega)$, we have from (4.21) that

$$\left(\int |uu_L^{\beta-1}|^{2^*}\right)^{2/2^*} \leq C_4\beta^2 C_{\lambda,K}\left(\int u^{\beta\alpha^*}\right)^{2/\alpha^*} < +\infty. \tag{4.22}$$

We now apply Fatou's lemma with respect to the variable L to obtain

$$|u|_{\beta\cdot 2^*}^{2\beta} \leq C_4 C_{\lambda,K}\beta^2|u|_{\beta\alpha^*}^{2\beta} \tag{4.23}$$

so

$$|u|_{\beta.2^*} \leq (C_4 C_{\lambda,K})^{1/\beta 2}\beta^{1/\beta}|u|_{\beta\alpha^*}. \tag{4.24}$$

Furthermore, by considering $\chi = 2^*/\alpha^*$, we have $2^* = \chi\alpha^*$ and $\beta\chi\alpha^* = 2^*\cdot\beta$ for all $\beta > 1$ verifying $u^\beta \in L^{\alpha^*}(\Omega)$.

Let us consider two cases.

Case 1. First, we consider $\beta = 2^*/\alpha^*$ and note that

$$u^\beta \in L^{\alpha^*}(\Omega). \tag{4.25}$$

Hence, from the Sobolev immersions, Lemma 4.1, and inequality (4.24), we get

$$|u|_{(2^*)^2/\alpha^*} \leq (C_4 C_{\lambda,K})^{1/2\beta}\beta^{1/\beta}\overline{C}C_5, \tag{4.26}$$

so

$$|u|_{\chi^2\alpha^*} \leq C_6(C_{\lambda,K})^{1/\chi 2}\chi^{1/\chi}. \tag{4.27}$$

Case 2. We now consider $\beta = (2^*/\alpha^*)^2$ and note again that

$$u^\beta \in L^{\alpha^*}(\Omega). \tag{4.28}$$

From inequality (4.24), we obtain

$$|u|_{(2^*)^3/(\alpha^*)^2} \leq C_6(C_{\lambda,K})^{1/\beta 2}\beta^{1/\beta}|u|_{(2^*)^{2/\alpha^*}}, \tag{4.29}$$

which implies

$$|u|_{\chi^3\alpha^*} \leq C_6(C_{\lambda,K})^{1/\chi^2}(\chi^2)^{1/\chi^2}|u|_{\chi^2\alpha^*} \tag{4.30}$$

or

$$|u|_{\chi^3\alpha^*} \leq C_7(C_{\lambda,K})^{1/\chi^2+1/\chi^2}(\chi^2)^{2/\chi^2+1/\chi}. \tag{4.31}$$

An iterative process leads to

$$|u|_{\chi^{(m+1)}\alpha^*} \leq C_8\left(C_{\lambda,K}\right)^{\sum_{i=1}^{m}\chi^{2(-i)}}\chi^{2\sum_{i=1}^{m} i\chi^{-i}}. \tag{4.32}$$

Taking limit as $m \to \infty$, we obtain

$$|u|_{L^\infty(\Omega)} \leq C_8\left(C_{\lambda,K}\right)^{\sigma_1}\chi^{\sigma_2}, \tag{4.33}$$

where $\sigma_1 = \sum_{i=1}^{\infty}\chi^{2(-i)}$ and $\sigma_2 = 2\sum_{i=1}^{\infty} i\chi^{-i}$.

In order to choose λ_0, we consider the inequality

$$C_8\left(C_{\lambda,K}^{\sigma_1}\right)\chi^{\sigma_2} = C_8\left[\left(1+\lambda\frac{g(K)}{K^{p-1}}\right)\frac{1}{m_0}\right]^{\sigma_1}\chi^{\sigma_2} \leq K, \tag{4.34}$$

from which

$$\left(1+\frac{\lambda g(K)}{K^{p-1}}\right)^{\sigma_1} \leq \frac{Km_0^{\sigma_1}}{\chi^{\sigma_2}C_8}. \tag{4.35}$$

Choosing λ_0, verifying the inequality

$$\lambda_0 \leq \left[\frac{K^{1/\sigma_1}m_0}{C_9} - 1\right]\frac{K^{p-1}}{g(K)}, \tag{4.36}$$

and fixing K such that

$$\left[\frac{K^{1/\sigma_1}m_0}{C_9} - 1\right] > 0, \tag{4.37}$$

we obtain

$$\left|u_\lambda\right|_{L^\infty(\Omega)} \leq K \quad \forall\lambda \in [0,\lambda_0], \tag{4.38}$$

which concludes the proof. □

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Francisco Júlio S. A. Corrêa: Departamento de Matemática-CCEN, Universidade Federal do Pará, 66.075-110 Belém-Pará, Brazil
E-mail address: fjulio@ufpa.br

Giovany M. Figueiredo: Departamento de Matemática-CCEN, Universidade Federal do Pará, 66.075-110 Belém-Pará, Brazil
E-mail address: giovany@ufpa.br