

Research Article

Some Properties of Certain Integral Operators on New Subclasses of Analytic Functions with Complex Order

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We define new subclasses of p -valent meromorphic functions with complex order. We prove some properties for certain integral operators on these subclasses.

1. Introduction

Let $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$ be the open unit disc in the complex plane \mathbb{C} , $\mathbb{U}^* = \mathbb{U} \setminus \{0\}$, the punctured open unit disk. Let Σ_p denote the class of meromorphic functions of the form

$$f(z) = \frac{1}{z^p} + \sum_{n=0}^{\infty} a_n z^n, \quad (p \in \mathbb{N}), \quad (1.1)$$

which are analytic and p -valent in \mathbb{U}^* .

For $p = 1$, we obtain the class of meromorphic functions Σ .

We say that a function $f \in \Sigma_p$ is the meromorphic p -valent starlike of order α ($0 \leq \alpha < p$) and belongs to the class $f \in \Sigma_p^*(\alpha)$, if it satisfies the inequality

$$-\Re\left(\frac{zf'(z)}{f(z)}\right) > \alpha. \quad (1.2)$$

A function $f \in \Sigma_p$ is the meromorphic p -valent convex function of order α ($0 \leq \alpha < p$), if f satisfies the following inequality:

$$-\Re\left(1 + \frac{zf''(z)}{f'(z)}\right) > \alpha, \quad (1.3)$$

and we denote this class by $\Sigma K_p(\alpha)$.

Many important properties and characteristics of various interesting subclasses of the class Σ_p of meromorphically p -valent functions were investigated extensively by (among others) Uralegaddi and Somanatha [1, 2], Liu and Srivastava [3, 4], Mogra [5, 6], Srivastava et al. [7], Aouf et al. [8, 9], Joshi and Srivastava [10], Owa et al. [11], and Kulkarni et al. [12].

Now, for $f \in \Sigma_p$, we define the following new subclasses.

Definition 1.1. Let a function $f \in \Sigma_p$ be analytic in \mathbb{U}^* . Then f is in the class $\Sigma_{p,\mathcal{M}}(\beta, b)$ if it satisfies the inequality

$$\Re\left\{p - \frac{1}{b}\left(\frac{zf'(z)}{f(z)} + p\right)\right\} < \beta, \quad (1.4)$$

where

$$p \in \mathbb{N}, \quad b \in \mathbb{C} \setminus \{0\}, \quad \beta > p. \quad (1.5)$$

Definition 1.2. Let a function $f \in \Sigma_p$ be analytic in \mathbb{U}^* . Then f is in the class $\Sigma_{p,\mathcal{N}}(\beta, b)$ if it satisfies the inequality

$$\Re\left\{p - \frac{1}{b}\left(\frac{zf''(z)}{f'(z)} + p + 1\right)\right\} < \beta, \quad (1.6)$$

where

$$p \in \mathbb{N}, \quad b \in \mathbb{C} \setminus \{0\}, \quad \beta > p. \quad (1.7)$$

Definition 1.3. Let a function $f \in \Sigma_p$ be analytic in \mathbb{U}^* . Then f is in the class $\Sigma F_p(\beta, b)$ if it satisfies the inequality

$$\Re\left\{p - \frac{1}{b}\left(\frac{z(zf''(z) + (p+2)f'(z))}{zf'(z) + (p+1)f(z)} + p\right)\right\} < \beta, \quad (1.8)$$

where

$$p \in \mathbb{N}, \quad b \in \mathbb{C} \setminus \{0\}, \quad \beta > p. \quad (1.9)$$

For $p = 1$ in Definitions 1.1, 1.2, and 1.3, we obtain the following new subclasses of meromorphic functions Σ .

Definition 1.4. Let a function $f \in \Sigma$ be analytic in \mathbb{U}^* . Then f is in the class $\Sigma_{\mathcal{M}}(\beta, b)$ if it satisfies the inequality

$$\Re \left\{ 1 - \frac{1}{b} \left(\frac{zf'(z)}{f(z)} + 1 \right) \right\} < \beta, \quad (1.10)$$

where

$$b \in \mathbb{C} \setminus \{0\}, \quad \beta > 1. \quad (1.11)$$

Definition 1.5. Let a function $f \in \Sigma$ be analytic in \mathbb{U}^* . Then f is in the class $\Sigma_{\mathcal{N}}(\beta, b)$ if it satisfies the inequality

$$\Re \left\{ 1 - \frac{1}{b} \left(\frac{zf''(z)}{f'(z)} + 2 \right) \right\} < \beta, \quad (1.12)$$

where

$$b \in \mathbb{C} \setminus \{0\}, \quad \beta > 1. \quad (1.13)$$

For $b = 1$ in Definition 1.5, we obtain $\Sigma_N(\beta)$, the class of meromorphic function, introduced and studied by Wang et al. [13–15] (see [16–18]).

Definition 1.6. Let a function $f \in \Sigma$ be analytic in \mathbb{U}^* . Then f is in the class $\Sigma F(\beta, b)$ if it satisfies the inequality

$$\Re \left\{ 1 - \frac{1}{b} \left(\frac{z(zf''(z) + 3f'(z))}{zf'(z) + 2f(z)} + 1 \right) \right\} < \beta, \quad (1.14)$$

where

$$b \in \mathbb{C} \setminus \{0\}, \quad \beta > 1. \quad (1.15)$$

Most recently, Mohammed and Darus [19] introduced the following two general integral operators of p -valent meromorphic functions Σ_p :

$$\mathcal{F}_{p, \gamma_1, \dots, \gamma_n}(z) = \frac{1}{z^{p+1}} \int_0^z (u^p f_1(u))^{\gamma_1} \cdots (u^p f_n(u))^{\gamma_n} du, \quad (1.16)$$

$$\mathcal{Z}_{p, \gamma_1, \dots, \gamma_n}(z) = \frac{1}{z^{p+1}} \int_0^z \left(\frac{-u^{p+1}}{p} f_1'(u) \right)^{\gamma_1} \cdots \left(\frac{-u^{p+1}}{p} f_n'(u) \right)^{\gamma_n} du, \quad (1.17)$$

where

$$n, p \in \mathbb{N}, \quad j \in \{1, 2, 3, \dots, n\}, \quad \gamma_j > 0. \quad (1.18)$$

For $p = 1$ in (1.16) and (1.17), respectively, we obtain the general integral operators $\mathcal{F}_{1, \gamma_1, \dots, \gamma_n}(z) = \mathcal{L}(z)$ and $\mathcal{J}_{1, \gamma_1, \dots, \gamma_n}(z) = \mathcal{L}_{\gamma_1, \dots, \gamma_n}(z)$, introduced by the authors in [16, 20].

2. Main Results

In this section, considering the above new subclasses we obtain for the integral operators $F_{1, \gamma_1, \dots, \gamma_n}(z)$ and $\mathcal{J}_{1, \gamma_1, \dots, \gamma_n}(z)$ some sufficient conditions for a family of functions f_i to be in the above new subclasses.

Theorem 2.1. *Let $f_j \in \Sigma_p$. If $f_j \in \Sigma_{p, \mathcal{M}}(\beta_j, b)$, then*

$$\mathcal{F}_{p, \gamma_1, \dots, \gamma_n}(z) \in \Sigma F_p(\mu, b), \quad (2.1)$$

where

$$\begin{aligned} n, p \in \mathbb{N}, \quad j \in \{1, \dots, n\}, \quad \gamma_j > 0, \quad b \in \mathbb{C} \setminus \{0\}, \quad \beta_j > p, \\ \mu = p + \sum_{j=1}^n \gamma_j (\beta_j - p). \end{aligned} \quad (2.2)$$

Proof. A differentiation of $\mathcal{F}_{p, \gamma_1, \dots, \gamma_n}(z)$ which is defined in (1.16), we get

$$\begin{aligned} z^{p+1} \mathcal{F}'_{p, \gamma_1, \dots, \gamma_n}(z) + (p+1) z^p \mathcal{F}_{p, \gamma_1, \dots, \gamma_n}(z) &= (z^p f_1(z))^{\gamma_1} \cdots (z^p f_n(z))^{\gamma_n}, \\ z^{p+1} \mathcal{F}''_{p, \gamma_1, \dots, \gamma_n}(z) + 2(p+1) z^p \mathcal{F}'_{p, \gamma_1, \dots, \gamma_n}(z) + p(p+1) z^{p-1} \mathcal{F}_{p, \gamma_1, \dots, \gamma_n}(z) & \\ = \sum_{j=1}^n \gamma_j \left(\frac{z^p f'_j(z) + p z^{p-1} f_j(z)}{z^p f_j(z)} \right) [(z^p f_1(z))^{\gamma_1} \cdots (z^p f_n(z))^{\gamma_n}], & \end{aligned} \quad (2.3)$$

Then from (2.3), we obtain

$$\begin{aligned} \frac{z^{p+1} \mathcal{F}''_{p, \gamma_1, \dots, \gamma_n}(z) + 2(p+1) z^p \mathcal{F}'_{p, \gamma_1, \dots, \gamma_n}(z) + p(p+1) z^{p-1} \mathcal{F}_{p, \gamma_1, \dots, \gamma_n}(z)}{z^{p+1} \mathcal{F}'_{p, \gamma_1, \dots, \gamma_n}(z) + (p+1) z^p \mathcal{F}_{p, \gamma_1, \dots, \gamma_n}(z)} & \\ = \sum_{j=1}^n \gamma_j \left(\frac{f'_j(z)}{f_j(z)} + \frac{p}{z} \right). & \end{aligned} \quad (2.4)$$

By multiplying (2.4) with z yield,

$$\begin{aligned} & \frac{z^{p+1}\mathfrak{F}_{p,\gamma_1,\dots,\gamma_n}''(z) + 2(p+1)z^p\mathfrak{F}_{p,\gamma_1,\dots,\gamma_n}'(z) + p(p+1)z^{p-1}\mathfrak{F}_{p,\gamma_1,\dots,\gamma_n}(z)}{z^p\mathfrak{F}_{p,\gamma_1,\dots,\gamma_n}'(z) + (p+1)z^{p-1}\mathfrak{F}_{p,\gamma_1,\dots,\gamma_n}(z)} \\ &= \sum_{j=1}^n \gamma_j \left(\frac{zf_j'(z)}{f_j(z)} + p \right). \end{aligned} \quad (2.5)$$

That is equivalent to

$$\begin{aligned} & \frac{z^{p+1}\mathfrak{F}_{p,\gamma_1,\dots,\gamma_n}''(z) + (p+2)z^p\mathfrak{F}_{p,\gamma_1,\dots,\gamma_n}'(z)}{z^p\mathfrak{F}_{p,\gamma_1,\dots,\gamma_n}'(z) + (p+1)z^{p-1}\mathfrak{F}_{p,\gamma_1,\dots,\gamma_n}(z)} + p = \sum_{j=1}^n \gamma_j \left(\frac{zf_j'(z)}{f_j(z)} + p \right); \\ & \frac{z(z\mathfrak{F}_{p,\gamma_1,\dots,\gamma_n}''(z) + (p+2)\mathfrak{F}_{p,\gamma_1,\dots,\gamma_n}'(z))}{z\mathfrak{F}_{p,\gamma_1,\dots,\gamma_n}'(z) + (p+1)\mathfrak{F}_{p,\gamma_1,\dots,\gamma_n}(z)} + p = \sum_{j=1}^n \gamma_j \left(\frac{zf_j'(z)}{f_j(z)} + p \right). \end{aligned} \quad (2.6)$$

Equivalently, the above can be written as

$$\begin{aligned} & p - \frac{1}{b} \left(\frac{z(z\mathfrak{F}_{p,\gamma_1,\dots,\gamma_n}''(z) + (p+2)\mathfrak{F}_{p,\gamma_1,\dots,\gamma_n}'(z))}{z\mathfrak{F}_{p,\gamma_1,\dots,\gamma_n}'(z) + (p+1)\mathfrak{F}_{p,\gamma_1,\dots,\gamma_n}(z)} + p \right) \\ &= \sum_{j=1}^n \gamma_j \left\{ p - \frac{1}{b} \left(\frac{zf_j'(z)}{f_j(z)} + p \right) \right\} + p - p \sum_{j=1}^n \gamma_j. \end{aligned} \quad (2.7)$$

Taking the real part of both terms of (2.7), we have

$$\begin{aligned} & \Re \left\{ p - \frac{1}{b} \left(\frac{z(z\mathfrak{F}_{p,\gamma_1,\dots,\gamma_n}''(z) + (p+2)\mathfrak{F}_{p,\gamma_1,\dots,\gamma_n}'(z))}{z\mathfrak{F}_{p,\gamma_1,\dots,\gamma_n}'(z) + (p+1)\mathfrak{F}_{p,\gamma_1,\dots,\gamma_n}(z)} + p \right) \right\} \\ &= \sum_{j=1}^n \gamma_j \Re \left\{ p - \frac{1}{b} \left(\frac{zf_j'(z)}{f_j(z)} + p \right) \right\} + p - p \sum_{j=1}^n \gamma_j. \end{aligned} \quad (2.8)$$

Sine $f_j \in \Sigma_{p,\mathcal{M}}(\beta_j, b)$, we get

$$\Re \left\{ p - \frac{1}{b} \left(\frac{z(z\mathfrak{F}_{p,\gamma_1,\dots,\gamma_n}''(z) + (p+2)\mathfrak{F}_{p,\gamma_1,\dots,\gamma_n}'(z))}{z\mathfrak{F}_{p,\gamma_1,\dots,\gamma_n}'(z) + (p+1)\mathfrak{F}_{p,\gamma_1,\dots,\gamma_n}(z)} + p \right) \right\} < \sum_{j=1}^n \gamma_j \beta_j + p - p \sum_{j=1}^n \gamma_j. \quad (2.9)$$

That is,

$$\Re \left\{ p - \frac{1}{b} \left(\frac{z(z\mathcal{F}_{p,\gamma_1,\dots,\gamma_n}''(z) + (p+2)\mathcal{F}_{p,\gamma_1,\dots,\gamma_n}'(z))}{z\mathcal{F}_{p,\gamma_1,\dots,\gamma_n}'(z) + (p+1)\mathcal{F}_{p,\gamma_1,\dots,\gamma_n}(z)} + p \right) \right\} < p + \sum_{j=1}^n \gamma_j (\beta_j - p). \quad (2.10)$$

Then

$$\mathcal{F}_{p,\gamma_1,\dots,\gamma_n}(z) \in \Sigma F_p(\mu, b), \quad \mu = p + \sum_{j=1}^n \gamma_j (\beta_j - p). \quad (2.11)$$

This completes the proof. \square

Theorem 2.2. Let $f_j \in \Sigma_p$. If $f_j \in \Sigma_{p,\mathcal{A}}(\beta_j, b)$, then

$$\mathcal{J}_{p,\gamma_1,\dots,\gamma_n}(z) \in \Sigma F_p(\mu, b), \quad (2.12)$$

where

$$\begin{aligned} n, p \in \mathbb{N}, \quad j \in \{1, \dots, n\}, \quad \gamma_j > 0, \quad b \in \mathbb{C} \setminus \{0\}, \quad \beta_j > p, \\ \mu = p + \sum_{j=1}^n \gamma_j (\beta_j - p). \end{aligned} \quad (2.13)$$

Proof. A differentiation of $\mathcal{J}_{p,\gamma_1,\dots,\gamma_n}(z)$, which is defined in (1.17), we get

$$\begin{aligned} z^{p+1} \mathcal{J}_{p,\gamma_1,\dots,\gamma_n}'(z) + (p+1)z^p \mathcal{J}_{p,\gamma_1,\dots,\gamma_n}(z) &= \left(\frac{-z^{p+1}}{p} f_1'(z) \right)^{\gamma_1} \cdots \left(\frac{-z^{p+1}}{p} f_n'(z) \right)^{\gamma_n}, \\ z^{p+1} \mathcal{J}_{p,\gamma_1,\dots,\gamma_n}''(z) + 2(p+1)z^p \mathcal{J}_{p,\gamma_1,\dots,\gamma_n}'(z) + p(p+1)z^{p-1} \mathcal{J}_{p,\gamma_1,\dots,\gamma_n}(z) & \\ = \sum_{j=1}^n \gamma_j \left(\frac{z^{p+1} f_j''(z) + (p+1)z^p f_j'(z)}{z^{p+1} f_j'(z)} \right) \left[\left(\frac{-z^{p+1}}{p} f_1'(z) \right)^{\gamma_1} \cdots \left(\frac{-z^{p+1}}{p} f_n'(z) \right)^{\gamma_n} \right]. \end{aligned} \quad (2.14)$$

Then from (2.14), we obtain

$$\begin{aligned} & \frac{z^{p+1} \mathcal{J}_{p,\gamma_1,\dots,\gamma_n}''(z) + 2(p+1)z^p \mathcal{J}_{p,\gamma_1,\dots,\gamma_n}'(z) + p(p+1)z^{p-1} \mathcal{J}_{p,\gamma_1,\dots,\gamma_n}(z)}{z^p \mathcal{J}_{p,\gamma_1,\dots,\gamma_n}'(z) + (p+1)z^{p-1} \mathcal{J}_{p,\gamma_1,\dots,\gamma_n}(z)} \\ &= \sum_{j=1}^n \gamma_j \left(\frac{z f_j''(z)}{f_j'(z)} + p + 1 \right). \end{aligned} \quad (2.15)$$

That is equivalent to

$$\frac{z(z\mathcal{D}_{p,\gamma_1,\dots,\gamma_n}''(z) + (p+2)\mathcal{D}_{p,\gamma_1,\dots,\gamma_n}'(z))}{z\mathcal{D}_{p,\gamma_1,\dots,\gamma_n}'(z) + (p+1)\mathcal{D}_{p,\gamma_1,\dots,\gamma_n}(z)} + p = \sum_{j=1}^n \gamma_j \left(\frac{zf_j''(z)}{f_j'(z)} + p + 1 \right). \quad (2.16)$$

Equivalently, the above can be written as

$$\begin{aligned} & p - \frac{1}{b} \left(\frac{z(z\mathcal{D}_{p,\gamma_1,\dots,\gamma_n}''(z) + (p+2)\mathcal{D}_{p,\gamma_1,\dots,\gamma_n}'(z))}{z\mathcal{D}_{p,\gamma_1,\dots,\gamma_n}'(z) + (p+1)\mathcal{D}_{p,\gamma_1,\dots,\gamma_n}(z)} + p \right) \\ &= \sum_{j=1}^n \gamma_j \left\{ p - \frac{1}{b} \left(\frac{zf_j''(z)}{f_j'(z)} + p + 1 \right) \right\} + p - p \sum_{j=1}^n \gamma_j. \end{aligned} \quad (2.17)$$

Taking the real part of both terms of (2.17), we have

$$\begin{aligned} & \Re \left\{ p - \frac{1}{b} \left(\frac{z(z\mathcal{D}_{p,\gamma_1,\dots,\gamma_n}''(z) + (p+2)\mathcal{D}_{p,\gamma_1,\dots,\gamma_n}'(z))}{z\mathcal{D}_{p,\gamma_1,\dots,\gamma_n}'(z) + (p+1)\mathcal{D}_{p,\gamma_1,\dots,\gamma_n}(z)} + p \right) \right\} \\ &= \sum_{j=1}^n \gamma_j \Re \left\{ p - \frac{1}{b} \left(\frac{zf_j''(z)}{f_j'(z)} + p + 1 \right) \right\} + p - p \sum_{j=1}^n \gamma_j. \end{aligned} \quad (2.18)$$

Sine $f_j \in \Sigma_{p,\mathcal{N}}(\beta_j, b)$, we get

$$\Re \left\{ p - \frac{1}{b} \left(\frac{z(z\mathcal{D}_{p,\gamma_1,\dots,\gamma_n}''(z) + (p+2)\mathcal{D}_{p,\gamma_1,\dots,\gamma_n}'(z))}{z\mathcal{D}_{p,\gamma_1,\dots,\gamma_n}'(z) + (p+1)\mathcal{D}_{p,\gamma_1,\dots,\gamma_n}(z)} + p \right) \right\} < \sum_{j=1}^n \gamma_j \beta_j + p - p \sum_{j=1}^n \gamma_j. \quad (2.19)$$

That is,

$$\Re \left\{ p - \frac{1}{b} \left(\frac{z(z\mathcal{D}_{p,\gamma_1,\dots,\gamma_n}''(z) + (p+2)\mathcal{D}_{p,\gamma_1,\dots,\gamma_n}'(z))}{z\mathcal{D}_{p,\gamma_1,\dots,\gamma_n}'(z) + (p+1)\mathcal{D}_{p,\gamma_1,\dots,\gamma_n}(z)} + p \right) \right\} < p + \sum_{j=1}^n \gamma_j (\beta_j - p). \quad (2.20)$$

Then

$$\mathcal{D}_{p,\gamma_1,\dots,\gamma_n}(z) \in \Sigma F_p(\mu, b), \quad \mu = p + \sum_{j=1}^n \gamma_j (\beta_j - p). \quad (2.21)$$

This completes the proof. \square

Putting $p = 1$ in Theorem 2.1, we have the following.

Theorem 2.3. Let $f_j \in \Sigma$. If $f_j \in \Sigma_{\mathcal{M}}(\beta_j, b)$, then

$$\mathcal{H}_n(z) \in \Sigma F(\mu, b), \quad (2.22)$$

where

$$\begin{aligned} n \in \mathbb{N}, \quad j \in \{1, \dots, n\}, \quad \gamma_j > 0, \quad b \in \mathbb{C} \setminus \{0\}, \quad \beta_j > 1, \\ \mu = 1 + \sum_{j=1}^n \gamma_j (\beta_j - 1). \end{aligned} \quad (2.23)$$

Putting $p = 1$ in Theorem 2.2, we have the following.

Theorem 2.4. Let $f_j \in \Sigma$. If $f_j \in \Sigma_{\mathcal{N}}(\beta_j, b)$, then

$$\mathcal{H}_{\gamma_1, \dots, \gamma_n}(z) \in \Sigma F(\mu, b), \quad (2.24)$$

where

$$\begin{aligned} n \in \mathbb{N}, \quad j \in \{1, \dots, n\}, \quad \gamma_j > 0, \quad b \in \mathbb{C} \setminus \{0\}, \quad \beta_j > 1, \\ \mu = 1 + \sum_{j=1}^n \gamma_j (\beta_j - 1). \end{aligned} \quad (2.25)$$

for other work that we can look at regarding integral operators see [17, 21, 22].

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