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Research Article

Parametric Extended General Mixed Variational Inequalities

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It is well known that the resolvent equations are equivalent to the extended general mixed variational inequalities. We use this alternative equivalent formulation to study the sensitivity of the extended general mixed variational inequalities without assuming the differentiability of the given data. Since the extended general mixed variational inequalities include extended general variational inequalities, quasi (mixed) variational inequalities and complementarity problems as special cases, results obtained in this paper continue to hold for these problems. In fact, our results can be considered as a significant extension of previously known results.

1. Introduction

In recent years, much attention have been given to investigate the behaviour of the changes of the data of the given problems. The study of these changes is known as the sensitivity analysis. We remark that sensitivity analysis is important for several reasons. First, since estimating problem data often introduces measurement errors, sensitivity analysis helps in identifying sensitive parameters that should be obtained with relatively high accuracy. Second, sensitivity analysis may help to predict the future changes of the equilibrium as a result of changes in the governing systems. Third, from mathematical and engineering points of view, sensitivity analysis can provide new insight regarding problems being studied and can stimulate new ideas for problem solving. Over the last decade, there has been increasing interest in studying the sensitivity analysis of variational inequalities and variational inclusions. Sensitivity analysis for variational inclusions and inequalities has been studied by many authors including Noor et al. [1], Kyparisis [2, 3], Dafermos [4], Qiu and Magnanti [5], Tobin [6], Noor [7–10], Moudafi and Noor [11], M. A. Noor and K. I. Noor [12], and Liu [13] using quite different techniques. The techniques suggested so far vary with the problem being studied. Dafermos [4] used the fixed-point formulation to consider the sensitivity analysis of the classical variational inequalities. This technique has been modified and extended by many authors for studying the sensitivity analysis of other classes of variational inequalities and variational inclusions, see [1, 7, 8, 11, 12, 14–16] and the references therein.

In this paper, we develop a sensitivity framework for the extended general mixed variational inequalities, which were introduced and studied by Noor [17] and Noor et al. [18] in conjunction with the optimality conditions of the differentiable nonconvex functions. This class is quite general and includes the extended general variational inequalities and related optimization problems as special cases. We first establish the equivalence between the extended general mixed variational inequalities and the resolvent equations by using the resolvent operator method. This fixed-point formulation is obtained by a suitable and appropriate rearrangement of the resolvent equations. We would like to point out that the resolvent equations technique is quite general, unified, and flexible and provides us with a new approach to study the sensitivity analysis of variational inclusions and related optimization problems. We use this alternative equivalent formulation to develop sensitivity analysis for the extended general mixed variational inequalities without assuming the differentiability of the given data. Our results can be considered as significant extensions of the results of Dafermos [4], Moudafi and Noor [11], Noor [9], and others in this area.

2. Preliminaries

Let K be a nonempty closed and convex set in a real Hilbert space H, whose inner product and norm are denoted by $\langle\cdot,\cdot\rangle$ and $\|\cdot\|$, respectively. Let $T : H \to H$ be a nonlinear operator and S be a nonexpansive operator. Let P_K be the projection of H onto the convex set K. Let $\varphi : H \to R \cup \{\infty\}$ be a continuous function.

For given nonlinear operators $T, g, h : H \to H$, consider the problem of finding $u \in H$ such that

$$\langle Tu, h(v) - g(u)\rangle + \varphi(h(v)) - \varphi(g(u)) \geq 0, \quad \forall v \in H. \tag{2.1}$$

Inequality of type (2.1) is called the *extended general mixed variational inequality involving four operators* and is quite different than all other classes of variational inequalities. Extended general mixed variational inequalities were introduced by Noor [17]. A wide class of problems arising in pure and applied sciences can be studied via the extended general mixed variational inequalities (2.1), see [18].

Example 2.1 (see [17]). As an application of problem (2.1), we show that the optimality condition for the minimum of sum of differentiable and nondifferentiable nonconvex functions on a nonconvex set K in H can be characterized by the general mixed variational inequality of type (2.1). This result is due to Noor [17]. We include some details to convey an idea of the technique.

For this purpose, we recall the following well-known concepts, see [7].

Definition 2.2. Let K be any set in H. The set K is said to be gh-convex if there exist functions $g, h : H \to H$ such that

$$h(u) + t(g(v) - h(u)) \in K, \quad \forall u, v \in H : h(u), g(v) \in K, \ t \in [0,1]. \tag{2.2}$$

Note that every convex set is gh-convex but the converse is not true, see [7]. If $g = h = I$, then the gh-convex set K is called the convex set.

Definition 2.3 (see [7]). The function $F : K \to H$ is said to be gh-convex if there exist functions g, h such that

$$F\big(g(u) + t\big(h(v) - g(u)\big)\big) \le (1-t)F\big(g(u)\big) + tF(h(v)), \quad \forall u, v \in H : h(u), g(v) \in K,\ t \in [0,1]. \tag{2.3}$$

Clearly every convex function is gh-convex, but the converse is not true. For the properties and various classes of the gh-convex functions, see Noor [7, 8, 19, 20]. We note that if the gh-convex function is differentiable, then

$$F\big(g(v)\big) - F(h(u)) \ge \langle F'(h(u)), g(v) - h(u)\rangle, \quad u, v \in H : h(u), g(v) \in K, \tag{2.4}$$

and conversely, where $F'(h(u))$ is the differential of the gh-convex function at the point $h(u)$.

For a given differentiable gh-convex function F and a nondifferentiable gh-convex function φ, we consider the functional of the type

$$I[v] = F(v) + \varphi(v), \quad \forall v \in K. \tag{2.5}$$

One can prove that the minimum of the functional $I[v]$ on the gh-convex set K can be characterized by a class of variational inequalities (2.1). This result is due to Noor [17].

Lemma 2.4 (see [17, 18]). *Let F be a differentiable gh-convex function and φ be a nondifferentiable gh-convex function on the gh-convex set K. Then $u \in K$ is the minimum of $I[v]$, defined by* (2.5)*, on $K \subset g(H)$ if and only if $u \in H : g(u) \in K$ satisfies the inequality*

$$\langle F'\big(g(u)\big), h(v) - g(u)\rangle + \varphi(h(v)) - \varphi\big(g(u)\big) \ge 0, \quad \forall v \in H : g(v) \in K, \tag{2.6}$$

where $F'(g(u))$ is the differential of F at $g(u) \in K$.

From Lemma 2.4, we see that the extended general mixed variational inequalities arise as a minimization of the sum of the differentiable and nondifferentiable gh-convex functions on the gh-convex set. This shows that the nonconvexity plays an important part in the study of the extended general mixed variational inequalities.

We would like to point out that the extended general mixed variational inequality (2.1) can be written in the equivalent form as: Find $u \in H$ such that

$$\langle \rho Tu + h(u) - g(u), h(v) - g(u)\rangle + \rho\varphi(h(v)) - \rho\varphi\big(g(u)\big) \ge 0, \quad \forall v \in H, \tag{2.7}$$

where $\rho > 0$ is a constant.

This equivalent formulation plays important part in developing iterative methods for solving the general mixed variational inequalities.

If $g = h = I$, the identity operator, then the extended general mixed variational inequalities (2.1) and (2.7) are equivalent to finding that $u \in H$ such that

$$\langle Tu, v - u\rangle + \varphi(v) - \varphi(u) \geq 0, \quad \forall v \in H, \tag{2.8}$$

which is known as the mixed variational inequality introduced or variational inequality of the second type. We note that if the function φ in the extended general mixed variational inequality (2.7) is a proper, convex, and lower semicontinuous, then problem (2.7) is equivalent to finding $u \in H$ such that

$$0 \in \rho Tu + h(u) - g(u) + \rho\partial\varphi\big(g(u)\big), \tag{2.9}$$

which is known as the problem of finding a zero of sum of two (or more) monotone operators. It is well known that a large class of problems arising in industry, ecology, finance, economics, transportation, network analysis and optimization can be formulated and studied in the framework of (2.1) and (2.9), see the references therein.

If φ is an indicator function of a closed convex set K in H, that is,

$$\varphi(u) = I_K(v) = \begin{cases} 0, & \text{if } v \in K \\ +\infty, & \text{otherwise}, \end{cases} \tag{2.10}$$

then the extended general mixed variational inequalities (2.1) are equivalent to finding $u \in H : g(u) \in K$ such that

$$\big\langle \mathrm{Tu}, h(v) - g(u)\big\rangle \geq 0, \quad \forall v \in H : h(v) \in K, \tag{2.11}$$

which is called the extended general variational inequality, introduced and studied by Noor [1, 7–10, 12, 15–21]. From Lemma 2.2, we see that the minimum of a class of differentiable nonconvex function on the nonconvex set can be characterized by the extended general variational inequalities of the type (2.11). For applications, numerical methods and other aspects of the extended general variational inequalities (2.11), see [1, 7–10, 12, 15–21].

We note that for $h = g$, (2.1) is equivalent to finding $u \in H : g(u) \in K$ such that

$$\big\langle T(u), g(v) - g(u)\big\rangle \geq 0, \quad \forall v \in H : g(v) \in K, \tag{2.12}$$

which is known as the *general variational inequality* and has been studied extensively in recent years. For the formulation, numerical methods, sensitivity analysis, and other aspects of the general variational inequalities, see [1, 7–10, 12, 15–21].

If $g = h = I$, then problems (2.11) and (2.12) reduce to finding $u \in K$ such that

$$\langle Tu, v - u\rangle \geq 0, \quad \forall v \in K, \tag{2.13}$$

which is known as the classical variational inequality, introduced and studied by Stampacchia [22] in 1964. For the numerical methods, formulations and applications of the mixed variational inequalities, readers may consult the the recent state-of-the-art papers [1–29] and the references therein.

We now recall some well-known concepts and results.

Definition 2.5 (see [23]). For any maximal operator T, the resolvent operator associated with T, for any $\rho > 0$, is defined as

$$J_T(u) = (I + \rho T)^{-1}(u), \quad \forall u \in H. \tag{2.14}$$

It is well known that an operator T is maximal monotone if and only if its resolvent operator J_T is defined everywhere. It is single valued and nonexpansive. If $\varphi(.)$ is a proper, convex and lower-semicontinuous function, then its subdifferential $\partial\varphi(.)$ is a maximal monotone operator. In this case, we can define the resolvent operator

$$J_\varphi(u) = (I + \rho\partial\varphi)^{-1}(u), \quad \forall u \in H \tag{2.15}$$

associated with the subdifferential $\partial\varphi(.)$. The resolvent operator J_φ has the following useful characterization.

Lemma 2.6. *For a given $z \in H$, $u \in H$ satisfies the inequality*

$$\langle u - z, v - u\rangle + \rho\varphi(v) - \rho\varphi(u) \geq 0, \quad \forall v \in H \tag{2.16}$$

if and only if

$$u = J_\varphi(z), \tag{2.17}$$

where $J_\varphi = (I + \rho\partial\varphi)^{-1}$ is the resolvent operator.

It is well known the resolvent operator J_φ is nonexpansive, that is,

$$\|J_\varphi u - J_\varphi v\| \leq \|u - v\|, \quad \forall u, v \in H. \tag{2.18}$$

We now consider to the problem of solving the resolvent equations. To be more precise, let $R_\varphi = I - gh^{-1}J_\varphi$, where I is the identity operator, and g, h are given nonlinear operator. For given nonlinear operators T, g, h, we consider the problem of finding $z \in H$ such that

$$Th^{-1}J_\varphi z + \rho^{-1}R_\varphi z = 0, \tag{2.19}$$

which is called the extended general resolvent equation. We note that if $g = h = I$, then one can obtain the original resolvent equations. It has been shown that the resolvent equations have played an important and significant role in developing several numerical techniques for solving extended general mixed variational inequalities and related optimization problems.

If the proper, convex, and lower-semicontinuous function φ is an indicator function of a closed convex set K, then $J_\varphi \equiv P_K$, the projection of H onto the closed convex set K.

Consequently, the extended general resolvent equations (2.19) are equivalent to finding, $z \in H$ such that

$$Th^{-1}P_K z + \rho^{-1} Q_K z = 0, \tag{2.20}$$

which are called the extended general Wiener-Hopf equations, see Noor [8], where $Q_K = I - gh^{-1}P_K$. For $g = h = I$, one can obtain the original Wiener-Hopf equations of Shi [27].

We now consider the parametric versions of the problem (2.7) and (2.19). To formulate the problem, let M be an open subset of H in which the parameter λ takes values. Let $T(u, \lambda)$ be given operator defined on $H \times M$ and take value in H.

From now onward, we denote $T_\lambda(.) \equiv T(., \lambda)$, unless otherwise specified.

The parametric general variational inequality problem is to find $(u, \lambda) \in H \times M$ such that

$$\langle \rho T_\lambda u + h(u) - g(v), g(v) - h(u) \rangle + \varphi(h(v)) - \varphi\big(g(u)\big) \geq 0, \quad \forall v \in H : g(v) \in K. \tag{2.21}$$

We also assume that, for some $\overline{\lambda} \in M$, problem (2.19) has a unique solution $\overline{u}$.

Related to the parametric extended general mixed variational inequality (2.21), we consider the parametric resolvent equations. We consider the problem of finding (z, λ), $(u, \lambda) \in H \times M$, such that

$$T_\lambda h^{-1} J_\varphi z + \rho^{-1} R_\varphi z = 0, \tag{2.22}$$

where $\rho > 0$ is a constant and J_φ is defined on the set of (z, λ) with $\lambda \in M$ and takes values in H. The equations of the type (2.22) are called the parametric resolvent equations.

One can establish the equivalence between the problems (2.21) and (2.22) by using the resolvent operator technique, see Noor [9, 10].

Lemma 2.7. *The parametric extended general mixed variational inequality* (2.21) *has a solution* $(u, \lambda) \in H \times M$ *if and only if the parametric resolvent equations* (2.22) *have a solution* $(z, \lambda), (u, \lambda) \in H \times M$, *where*

$$\begin{gathered} h(u) = J_\varphi, \\ z = g(u) - \rho T_\lambda(u). \end{gathered} \tag{2.23}$$

From Lemma 2.7, we see that the parametric extended general mixed variational inequalities (2.21) and the parametric resolvent equations (2.22) are equivalent. We use this equivalence to study the sensitivity analysis of the extended general mixed variational inequalities. We assume that for some $\overline{\lambda} \in M$, problem (2.22) has a solution $\overline{z}$, and X is a closure of a ball in H centered at $\overline{z}$. We want to investigate those conditions under which, for each λ in a neighborhood of $\overline{\lambda}$, problem (2.22) has a unique solution $z(\lambda)$ near $\overline{z}$, and the function $z(\lambda)$ is (Lipschitz) continuous and differentiable.

Definition 2.8. Let $T_\lambda(.)$ be an operator on $X \times M$. Then, the operator $T_\lambda(.)$ is said to the following:

(a) *locally strongly monotone* if there exists a constant $\alpha > 0$ such that

$$\langle T_\lambda(u) - T_\lambda(v), u - v\rangle \geq \alpha\|u - v\|^2, \quad \forall\lambda \in M,\ u, v \in X, \tag{2.24}$$

(b) *locally Lipschitz continuous* if there exists a constant $\beta > 0$ such that

$$\|T_\lambda(u) - T_\lambda(v)\| \leq \beta\|u - v\|, \quad \forall\lambda \in M, u, v \in X. \tag{2.25}$$

3. Main Results

We consider the case, when the solutions of the parametric resolvent equations (2.22) lie in the interior of X. Following the ideas of Dafermos [4] and Noor [8, 10], we consider the map

$$\begin{aligned} F_\lambda(z) &= J_\varphi z - \rho T_\lambda(u), \quad \forall(z, \lambda) \in X \times M \\ &= g(u) - \rho T_\lambda(u), \end{aligned} \tag{3.1}$$

where

$$h(u) = P_K z. \tag{3.2}$$

We have to show that the map $F_\lambda(z)$ has a fixed point, which is a solution of the resolvent equations (2.13). First of all, we prove that the map $F_\lambda(z)$, defined by (3.1), is a contraction map with respect to z uniformly in $\lambda \in M$.

Lemma 3.1. *Let $T_\lambda(.)$ be a locally strongly monotone with constant $\alpha > 0$ and locally Lipschitz continuous with constant $\beta > 0$. If that the operators g, h are strongly monotone with constants $\sigma > 0$, $\mu > 0$ and Lipschitz continuous with constants $\delta > 0$, $\eta > 0$, respectively, then, for all $z_1, z_2 \in X$ and $\lambda \in M$, we have*

$$\|F_\lambda(z_1) - F_\lambda(z_2)\| \leq \theta\|z_1 - z_2\|, \tag{3.3}$$

where

$$\theta = \frac{\sqrt{1 - 2\sigma + \delta^2} + \sqrt{1 - 2\alpha\rho + \beta^2\rho^2}}{1 - \sqrt{1 - 2\mu + \eta^2}} \tag{3.4}$$

for

$$\left|\rho - \frac{\alpha}{\beta^2}\right| < \frac{\sqrt{\alpha^2 - \beta^2 k(2-k)}}{\beta^2}, \quad \alpha > \beta\sqrt{k(2-k)},\ k < 1, \tag{3.5}$$

where

$$k = \sqrt{1 - 2\sigma + \delta^2} + \sqrt{1 - 2\mu + \eta^2}, \tag{3.6}$$

Proof. For all $z_1, z_2 \in X$, $\lambda \in M$, we have, from (3.1),

$$\begin{aligned}\|F_\lambda(z_1) - F_\lambda(z_2)\| &= \|g(u_1) - g(u_2) - \rho(T_\lambda(u_1) - T_\lambda(u_2))\| \\ &\leq \|u_1 - u_2 - (g(u_1) - g(u_2))\| \\ &\quad + \|u_1 - u_2 - \rho(T_\lambda(u_1) - T_\lambda(u_2))\|.\end{aligned} \tag{3.7}$$

Using the strongly monotonicity and Lipschitz continuity of the operator g, we have

$$\begin{aligned}\|u_1 - u_2 - (g(u_1) - g(u_2))\|^2 &\leq \|u_1 - u_2\|^2 - 2\langle u_1 - u_2, g(u_1) - g(u_2)\rangle \\ &\quad + \|g(u_1) - g(u_2)\|^2 \\ &\leq \left(1 - 2\sigma + \delta^2\right)\|u_1 - u_2\|^2.\end{aligned} \tag{3.8}$$

In a similar way, we have

$$\|u_1 - u_2 - \rho(T_\lambda(u_1) - T_\lambda(u_2))\|^2 \leq \left(1 - 2\rho\alpha + \beta^2\rho^2\right)\|u_1 - u_2\|^2, \tag{3.9}$$

where $\alpha > 0$ is the strongly monotonicity constant and $\beta > 0$ is the Lipschitz continuity constant of the operator T_λ respectively.

From (3.7), (3.8), and (3.9), we have

$$\|F_\lambda(z_1) - F_\lambda(z_2)\| \leq \left\{\sqrt{1 - 2\sigma + \delta^2} + \sqrt{1 - 2\alpha\rho + \beta^2\rho^2}\right\}\|u_1 - u_2\|. \tag{3.10}$$

From (3.2) and using the nonexpansivity of the operator J_φ, we have

$$\begin{aligned}\|u_1 - u_2\| &\leq \|u_1 - u_2 - (h(u_1) - h(u_2))\| + \|J_\varphi z_1 - J_\varphi z_2\| \\ &\leq \left\{\sqrt{1 - 2\mu + \eta^2}\|u_1 - u_2\| + \|z_1 - z_2\|,\right.\end{aligned} \tag{3.11}$$

from which we obtain that

$$\|u_1 - u_2\| \leq \left\{ \frac{1}{1 - \sqrt{1 - 2\mu + \eta^2}} \right\} \|z_1 - z_2\|, \tag{3.12}$$

where $\mu > 0$ is the strongly monotonicity constant, and $\eta > 0$ is the Lipschitz continuity constant of the operator h, respectively.

Combining (3.10) and (3.12), we have

$$\begin{aligned} \|F_\lambda(z_1) - F_\lambda(z_2)\| &\leq \frac{\sqrt{1 - 2\sigma + \delta^2} + \sqrt{1 - 2\rho\alpha + \rho^2\beta^2}}{1 - \sqrt{1 - 2\mu + \eta^2}} \|z_1 - z_2\| \\ &= \theta\|z_1 - z_2\|, \end{aligned} \tag{3.13}$$

where $\theta = (\sqrt{1 - 2\sigma + \delta^2} + \sqrt{1 - 2\alpha\rho + \beta^2\rho^2})/1 - \sqrt{1 - 2\mu + \eta^2}$.

Now consider $\theta < 1$. Using (3.5), we have

$$k + \sqrt{1 - 2\rho\alpha + \rho^2\beta^2} < 1, \tag{3.14}$$

which shows that (3.5) holds. Consequently, from (3.5), it follows that $\theta < 1$ and consequently the map $F_\lambda(z)$ defined by (3.1) is a contraction map and has a fixed point $z(\lambda)$, which is the solution of the resolvent equations (2.22). □

Remark 3.2. From Lemma 3.1, we see that the map $F_\lambda(z)$ defined by (3.1) has a unique fixed point $z(\lambda)$, that is, $z(\lambda) = F_\lambda(z)$. Also, by assumption, the function $\overline{z}$, for $\lambda = \overline{\lambda}$ is a solution of the parametric resolvent equations (2.22). Again using Lemma 3.1, we see that $\overline{z}$, for $\lambda = \overline{\lambda}$, is a fixed point of $F_\lambda(z)$ and it is also a fixed point of $F_{\overline{\lambda}}(z)$. Consequently, we conclude that

$$z\left(\overline{\lambda}\right) = \overline{z} = F_{\overline{\lambda}}\left(z\left(\overline{\lambda}\right)\right). \tag{3.15}$$

Using Lemma 3.1, we can prove the continuity of the solution $z(\lambda)$ of the parametric resolvent equations (2.22) using the technique of Noor [9, 10]. However, for the sake of completeness and to convey an idea of the techniques involved, we give its proof.

Lemma 3.3. *Assume that the operator $T_\lambda(.)$ is locally Lipschitz continuous with respect to the parameter λ. If the operator $T_\lambda(.)$ is locally Lipschitz continuous and the map $\lambda \longrightarrow J_{\varphi_\lambda} z$ is continuous (or Lipschitz continuous), then the function $z(\lambda)$ is (Lipschitz) continuous at $\lambda = \overline{\lambda}$.*

Proof. For all $\lambda \in M$, invoking Lemma 3.1 and the triangle inequality, we have

$$\begin{aligned} \left\|z(\lambda) - z\left(\overline{\lambda}\right)\right\| &\leq \left\|F_\lambda(z(\lambda)) - F_{\overline{\lambda}}\left(z\left(\overline{\lambda}\right)\right\| + \|F_\lambda\left(z\left(\overline{\lambda}\right)\right) - F_{\overline{\lambda}}\left(z\left(\overline{\lambda}\right)\right)\| \\ &\leq \theta\|z(\lambda) - z\left(\overline{\lambda}\right)\| + \|F_\lambda\left(z\left(\overline{\lambda}\right)\right) - F_{\overline{\lambda}}\left(z\left(\overline{\lambda}\right)\right)\|. \end{aligned} \tag{3.16}$$

From (3.16) and the fact that the operator T_λ is a Lipschitz continuous with respect to the parameter λ, we have

$$\begin{aligned}\left\|F_\lambda\left(z\left(\overline{\lambda}\right)\right)-F_{\overline{\lambda}}\left(z\left(\overline{\lambda}\right)\right)\right\| &= \left\|u\left(\overline{\lambda}\right)-u\left(\overline{\lambda}\right)+\rho\left(T_\lambda\left(u\left(\overline{\lambda}\right),u\left(\overline{\lambda}\right)\right)-T_{\overline{\lambda}}\left(u\left(\overline{\lambda}\right),u\left(\overline{\lambda}\right)\right)\right)\right\| \\ &\leq \rho\mu\left\|\lambda-\overline{\lambda}\right\|.\end{aligned} \tag{3.17}$$

Combining (3.16) and (3.17), we obtain

$$\left\|z(\lambda)-z\left(\overline{\lambda}\right)\right\| \leq \frac{\rho\mu}{1-\theta}\left\|\lambda-\overline{\lambda}\right\|, \quad \forall\lambda,\ \overline{\lambda}\in M, \tag{3.18}$$

from which the required result follows. □

We now state and prove the main result of this paper and is the motivation of our next result.

Theorem 3.4. *Let $\overline{u}$ be the solution of the parametric extended generalized variational inequality* (2.21) *and $\overline{z}$ be the solution of the parametric resolvent equations* (2.22) *for $\lambda=\overline{\lambda}$. Let $T_\lambda(u)$ be the locally strongly monotone Lipschitz continuous operator for all $u,v\in X$. If the the map $\lambda\longrightarrow J_\varphi$ is (Lipschitz) continuous at $\lambda=\overline{\lambda}$, then there exists a neighborhood $N\subset M$ of $\overline{\lambda}$ such that, for $\lambda\in N$, the parametric resolvent equations* (2.22) *have a unique solution $z(\lambda)$ in the interior of X, $z(\overline{\lambda})=\overline{z}$ and $z(\lambda)$ is (Lipschitz) continuous at $\lambda=\overline{\lambda}$.*

Proof. Its proof follows from Lemmas 3.1, 3.3, and Remark 3.2. □

4. Conclusion

In this paper, we have developed a general framework of the sensitivity analysis for the extended general mixed variational inequalities. Several special cases are also discussed. Results proved in this paper may be extended for the multivalued general variational inequalities and related optimization problems. This is an interesting and fascinating problem for future research.

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