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Research Article

Stability Analysis and Robust H_∞ Control of Switched Stochastic Systems with Time-Varying Delay

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The problems of mean-square exponential stability and robust H_∞ control of switched stochastic systems with time-varying delay are investigated in this paper. Based on the average dwell time method and Gronwall-Bellman inequality, a new mean-square exponential stability criterion of such system is derived in terms of linear matrix inequalities (LMIs). Then, H_∞ performance is studied and robust H_∞ controller is designed. Finally, a numerical example is given to illustrate the effectiveness of the proposed approach.

1. Introduction

Switched systems, a special hybrid system, are composed of a set of continuous-time or discrete-time subsystems and a rule orchestrating the switching between the subsystems. In the last two decades, there has been increasing interest in the stability analysis and control design for such switched systems since many real-world systems such as chemical systems [1], robot control systems [2], traffic systems [3], and networked control systems [4, 5] can be modeled as such systems. The past decades have witnessed an enormous interest in the stability analysis and control synthesis of switched systems [6–11].

It is well known that time delay phenomenon exists in many engineering systems such as networked systems and long-distance transportation systems. Such phenomenon may cause the system unstable if it cannot be handled properly, which motivates many scientists to involve themselves in researching switched systems with time delay. Many results have been reported for stability analysis of switched systems with time delay [12, 13], where the asymptotical stability criteria are given by using common Lyapunov function approach in [12], and the exponential stability criteria under average dwell time switching signals are proposed in [13]. Moreover, the problem of delay-dependent global robust asymptotic stability of switched uncertain Hopfield neural networks with time delay in the leakage term is discussed in [14]. H_∞ control of continuous-time switched systems with time delay and discrete-time switched systems with time delay are investigated in [15, 16], respectively.

On the other hand, stochastic systems have attracted considerable attention during the past decades because stochastic disturbance exists in many actual operations. Many useful results on the stability analysis of stochastic systems are reported in [17–21]. The problem of robust H_∞ control for nonlinear stochastic systems with Markovian jump parameters and interval time-varying delays is considered [22]. Based on the results of stochastic systems and switched systems, the stability analysis and stabilization of switched stochastic systems are investigated in [23, 24]. Furthermore, the problems of reliable control and reliable H_∞ control for switched stochastic systems under asynchronous switching are studied in [25, 26], respectively. Recently, these results are extended to stochastic switched systems with time delay, and the exponential stability criteria are addressed [27, 28]. However, these results are very complex, which make it more difficult for us to solve many issues such as controller design under asynchronous switching and actuator failures. Therefore, there is a lot of work to do in such field. This motivates the present study.

In this paper, we focus on the mean-square exponential stability analysis and robust H_∞ control of switched stochastic systems with time-varying delay. Based on the average dwell time method and Gronwall-Bellman inequality, a new mean-square exponential stability criterion is derived. Moreover, H_∞ performance is studied and H_∞ state feedback controller is proposed. The remainder of the paper is organized as follows. In Section 2, problem statement and some useful lemmas are given. In Section 3, based on the average dwell time method and Gronwall-Bellman inequality, the mean-square exponential stability and H_∞ performance of the switched stochastic systems with time delay are investigated. Then, robust H_∞ controller is designed. In Section 4, a numerical example is given to illustrate the effectiveness of the proposed approach. Finally, concluding remarks are provided in Section 5.

Notation. Throughout this paper, the superscript "T" denotes the transpose, and the symmetric terms in a matrices are denoted by $*$. The notation $X > Y (X \geq Y)$ means that matrix $X - Y$ is positive definite (positive semidefinite, resp.). R^n denotes the n dimensional Euclidean space. $\|x(t)\|$ denotes the Euclidean norm. $L_2[t_0, \infty)$ is the space of square integrable functions on $[t_0, \infty)$. $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ denote the maximum and minimum eigenvalues of matrix P, respectively. I is an identity matrix with appropriate dimension. $\operatorname{diag}\{a_i\}$ denotes diagonal matrix with the diagonal elements a_i, $i = 1, 2, \ldots, n$.

2. Problem Formulation and Preliminaries

Consider the following stochastic switched systems with time-delay:

$$\begin{gathered} dx(t) = \left[\widehat{A}_{\sigma(t)} x(t) + \widehat{B}_{\sigma(t)} x(t - h(t)) + C_{\sigma(t)} u(t) + G_{\sigma(t)} v(t)\right] dt + \widehat{D}_{\sigma(t)} x(t) dw(t), \\ x(t) = \varphi(t), \qquad t \in [t_0 - h, t_0], \\ z(t) = M_{\sigma(t)} x(t), \end{gathered} \tag{2.1}$$

where $x(t) \in R^n$ is the state vector, $\varphi(t) \in R^n$ is the initial state function, $u(t) \in R^l$ is the control input, $v(t) \in R^p$ is the disturbance input which is assumed belong to $L_2[t_0, \infty]$, $z(t) \in R^q$ is the signal to be estimated, $w(t) \in R$ is a zero-mean Wiener process on a probability space $(\Omega\ F\ P)$ satisfying

$$E\{dw(t)\} = 0, \quad E\left\{dw^2(t) = dt\right\}, \tag{2.2}$$

where Ω is the sample space, F is σ-algebras of subsets of the sample space, P is the probability measure on F, and $E\{\cdot\}$ is the expectation operator. $h(t)$ is the system state delay satisfying

$$0 \le h(t) \le h, \qquad \dot{h}(t) \le h_d < 1, \tag{2.3}$$

where h_d is a known constant. The function $\sigma(t) : [t_0, \infty] \rightarrow \underline{N} = \{1, 2, \ldots, N\}$ is a switching signal which is deterministic, piecewise constant, and right continuous. The switching sequence can be described as $\sigma : \{(t_0, \sigma(t_0)), (t_1, \sigma(t_1)), \ldots, (t_k, \sigma(t_k))\}$, $\sigma(t_k) \in \underline{N}$, $k \in Z$, where t_0 is the initial time and t_k denotes the kth switching instant. Moreover $\sigma(t) = i$ means that the ith subsystem is activated.

For each for all $i \in \underline{N}$, C_i, G_i, and M_i are known real-value matrices with appropriate dimensions, and $\widehat{A}_i$, $\widehat{B}_i$, and $\widehat{D}_i$ are uncertain real matrix with appropriate dimensions, which can be written as

$$\left[\widehat{A}_i\ \widehat{B}_i\ \widehat{D}_i\right] = [A_i\ B_i\ D_i] + H_i F_i(t)[E_{1i}\ E_{2i}\ E_{3i}], \tag{2.4}$$

where A_i, B_i, and D_i are known real-value matrices with appropriate dimensions, and $F_i(t)$ is unknown time-varying matrix that satisfies

$$F_i^T(t) F_i(t) \le I. \tag{2.5}$$

Definition 2.1. System (2.1) is said to be mean-square exponentially stable with under switching signal $\sigma(t)$, if there exist scalars $\kappa > 0$ and $\alpha > 0$, such that the solution $x(t)$ of system (2.1) satisfies $E\{\|x(t)\|^2\} \le \kappa e^{-\alpha(t-t_0)} \sup_{t_0-h\le\theta\le t_0} E\{\|\varphi(\theta)\|^2\}$, for all $t > t_0$. Moreover, α is called the decay rate.

Definition 2.2. For any $T_2 > T_1 \ge t_0$, let $N_\sigma(T_1, T_2)$ denote the switching number of $\sigma(t)$ on an interval (T_1, T_2). If

$$N_\sigma(T_1, T_2) \le N_0 + \frac{T_2 - T_1}{T_\alpha} \tag{2.6}$$

holds for given $N_0 \ge 0$, $T_\alpha > 0$, then the constant T_α is called the average dwell time. As commonly used in the literature, we choose $N_0 = 0$.

Definition 2.3. Let $\gamma > 0$ be a positive constant, for system (2.1), if there exists a controller $u(t)$ and a switching signal $\sigma(t)$, such that

(1) when $v(t) = 0$, system (2.1) is mean-square exponentially stable;

(2) under zero initial condition $x(t) = 0$, $t \in [t_0 - h, t_0]$, the output $z(t)$ satisfies

$$E\left\{\int_{t_0}^{\infty} e^{-\lambda(s-t_0)} z^T(s)z(s)ds\right\} \leq \gamma^2 \int_{t_0}^{\infty} v^T(s)v(s)ds, \quad \forall v(t) \in L_2[t_0, \infty). \tag{2.7}$$

Then system (2.1) is said to be robustly exponentially stabilizable with a prescribed weighted H_∞ performance, where $\lambda > 0$.

The following lemmas play an important role in the later development.

Lemma 2.4 (see [29] (Gronwall-Bellman Inequality)). *Let $x(t)$ and $y(t)$ be real-valued nonnegative continuous functions with domain $\{t \mid t \geq t_0\}$; a is a nonnegative scalar; if the following inequality*

$$x(t) \leq a + \int_{t_0}^{t} x(s)y(s)ds \tag{2.8}$$

holds, for $t \geq t_0$, then $x(t) \leq a\exp(\int_{t_0}^{t} y(s)ds)$.

Lemma 2.5 (see [30]). *Let U, V, W, and X be constant matrices of appropriate dimensions with X satisfying $X = X^T$, then for all $V^TV \leq I$, $X + UVW + W^TV^TU^T < 0$, if and only if there exists a scalar $\varepsilon > 0$ such that $X + \varepsilon UU^T + \varepsilon^{-1}W^TW < 0$.*

3. Main Results

3.1. Stability Analysis

In this subsection, we will focus on the exponential stability analysis of switched stochastic systems with time-varying delay.

Consider the following switched stochastic system:

$$\begin{gathered} dx(t) = \left[A_{\sigma(t)}x(t) + B_{\sigma(t)}x(t-h(t))\right]dt + D_{\sigma(t)}x(t)dw(t), \\ x(t) = \varphi(t), \quad t \in [t_0 - h, t_0]. \end{gathered} \tag{3.1}$$

Theorem 3.1. *Considering system* (3.1), *for a given scalar $\alpha > 0$, if there exist symmetric positive definite matrices P_i, $Q_i > 0$ satisfying*

$$\begin{bmatrix} A_i^TP_i + P_iA_i + \alpha P_i + Q_i & P_iB_i & D_i^TP_i \\ * & -(1-h_d)Q_i & 0 \\ * & * & -P_i \end{bmatrix} < 0, \tag{3.2}$$

for all $i \in \underline{N}$, then system (3.1) *is mean-square exponentially stable under arbitrary switching signal with the average dwell time:*

$$T_\alpha \geq T_\alpha^* = \frac{\ln \mu}{\alpha}, \tag{3.3}$$

where $\mu \geq 1$ satisfies

$$P_i \leq \mu P_j, \quad Q_i \leq \mu Q_j, \quad \forall i, j \in \underline{N}. \tag{3.4}$$

Proof. Consider the following Lyapunov functional for the ith subsystem:

$$V_i(t, x(t)) = V_{1,i}(t, x(t)) + V_{2,i}(t, x(t)), \tag{3.5}$$

where

$$V_{1,i}(t, x(t)) = x^T(t) P_i x(t), \qquad V_{2,i}(t, x(t)) = \int_{t-h(t)}^{t} x^T(s) Q_i x(s) ds. \tag{3.6}$$

For the sake of simplicity, $V_i(t, x(t))$ is written as $V_i(t)$ in this paper.

According to Itô formula, along the trajectory of system (3.1), we have

$$dV_i(t) = \mathcal{L}V_i(t)dt + 2x^T(t) P_i D_i x(t) dw(t), \tag{3.7}$$

where

$$\begin{aligned} \mathcal{L}V_i(t) = {} & 2x^T(t) P_i (A_i x(t) + B_i x(t - h(t))) + x^T(t) Q_i x(t) + x^T(t) D_i^T P_i D_i x(t) \\ & - (1 - \dot{h}(t)) x^T(t - h(t)) Q_i x(t - h(t)). \end{aligned} \tag{3.8}$$

According to (2.3), we can obtain that

$$\begin{aligned} \mathcal{L}V_i(t) & \leq 2x^T(t) P_i (A_i x(t) + B_i x(t - h(t))) + x^T(t) Q_i x(t) + x^T(t) D_i^T P_i D_i x(t) \\ & \quad - (1 - h_d) x^T(t - h(t)) Q_i x(t - h(t)) \\ & = \xi^T(t) \Theta_i \xi(t), \end{aligned} \tag{3.9}$$

where

$$\xi(t) = \begin{bmatrix} x(t) \\ x(t - h(t)) \end{bmatrix}, \qquad \Theta_i = \begin{bmatrix} A_i^T P_i + P_i A_i + Q_i + D_i^T P_i D_i & P_i B_i \\ * & -(1 - h_d) Q_i \end{bmatrix}. \tag{3.10}$$

Using Schur complement, it is not difficult to get that if inequality (3.2) is satisfied, the following inequality can be obtained:

$$\mathcal{L}V_i(t) < -\alpha V_{1,i}(t) < 0. \tag{3.11}$$

Combining (3.7) with (3.11) leads to

$$dV_i(t) = \mathcal{L}V_i(t)dt + 2x^T(t)P_iD_ix(t)dw(t) < -\alpha V_{1,i}(t)dt + 2x^T(t)P_iD_ix(t)dw(t). \tag{3.12}$$

Noticing (2.2) and taking the expectation to (3.12), we have

$$E\left\{\frac{dV_i(t)}{dt}\right\} = E\{\mathcal{L}V_i(t)\} < -\alpha E\{V_{1,i}(t)\} < 0. \tag{3.13}$$

According to (3.4)–(3.6), we have

$$E\{V_p(t)\} \le \mu E\{V_q(t)\} = \mu E\{V_q(t^-)\}, \tag{3.14}$$

$$E\{V_{1,p}(t)\} \le \mu E\{V_{1,q}(t)\}, \quad \forall p, q \in \underline{N}. \tag{3.15}$$

Assume that the ith subsystem is activated during $[t_k, t_{k+1})$ and jth subsystem is activated during $[t_{k-1}, t_k)$, respectively. Using Itô formula and according to (3.13)–(3.15), we have, for any $t \in [t_k, t_{k+1})$,

$$\begin{aligned}
E\{V_{1,i}(t)\} &\le E\{V_i(t)\} = E\{V_i(t_k)\} + E\left\{\int_{t_k}^{t} \mathcal{L}V_i(s)ds\right\} \\
&< \mu E\{V_j(t_k^-)\} - \alpha E\left\{\int_{t_k}^{t} V_{1,i}(s)ds\right\} \\
&= \mu E\{V_j(t_{k-1})\} + \mu E\left\{\int_{t_{k-1}}^{t_k} \mathcal{L}V_{1,j}(s)ds\right\} - \alpha E\left\{\int_{t_k}^{t} V_{1,i}(s)ds\right\} \\
&< \mu E\{V_j(t_{k-1})\} - \alpha\mu E\left\{\int_{t_{k-1}}^{t_k} V_{1,j}(s)ds\right\} - \alpha E\left\{\int_{t_k}^{t} V_{1,i}(s)ds\right\} \\
&\le \mu E\{V_j(t_{k-1})\} - \alpha E\left\{\int_{t_{k-1}}^{t} V_{1,i}(s)ds\right\} \\
&\le \cdots \\
&\le \mu^{N_\sigma(t_0,t)} E\{V(t_0)\} - \alpha E\left\{\int_{t_0}^{t} V_{1,i}(s)ds\right\}.
\end{aligned} \tag{3.16}$$

According to Lemma 2.4 and when (3.3) holds, we have

$$E\{V_{1,i}(t)\} \leq \mu^{N_\sigma(t_0,t)} e^{-\alpha(t-t_0)} E\{V(t_0)\} \leq \kappa e^{-(\alpha-(\ln\mu/T_\alpha))(t-t_0)} E\{V(t_0)\}. \tag{3.17}$$

Moreover, we can obtain

$$E\left\{\|x(t)\|^2\right\} \leq \kappa e^{-\lambda(t-t_0)} \sup_{-h\leq\theta\leq 0} E\left\{\|x(\theta)\|^2\right\}, \tag{3.18}$$

where $\kappa = \sqrt{\max_{i\in\underline{N}}(\lambda_{\max}(P_i) + h\lambda_{\max}(Q_i))/\min_{i\in\underline{N}}\lambda_{\min}(P_i)}$, and $\lambda = (1/2)(\alpha - (\ln\mu/T_\alpha))$ is the decay rate.

The proof is completed. □

Remark 3.2. The exponential stability criterion of stochastic switched systems with time-varying delay is given in Theorem 3.1. When $w(t) = 0$, system (3.1) is degenerated to the switched system with time-varying delay, which can be described as

$$\begin{aligned} \dot{x}(t) &= A_{\sigma(t)}x(t) + B_{\sigma(t)}x(t-h(t)), \\ x(t) &= \varphi(t), \quad t \in [t_0 - h, t_0]. \end{aligned} \tag{3.19}$$

Using the same method, we can obtain the following exponential stability criterion of switched system (3.19).

Corollary 3.3. *Considering system* (3.19), *for a given scalar* $\alpha > 0$, *if there exist symmetric positive definite matrices* $P_i,\ Q_i > 0$ *satisfying*

$$\begin{bmatrix} A_i^T P_i + P_i A_i + \alpha P_i + Q_i & P_i B_i \\ * & -(1-h_d)Q_i \end{bmatrix} < 0, \tag{3.20}$$

for all $i \in \underline{N}$, *system* (3.19) *is exponentially stable under arbitrary switching signal with average dwell time satisfying* (3.3).

3.2. H_∞ *Performance Analysis*

In this subsection, we will investigate the H_∞ performance of switched stochastic systems with time-varying delay.

Consider the following switched stochastic system:

$$\begin{aligned} dx(t) &= \left[A_{\sigma(t)}x(t) + B_{\sigma(t)}x(t-h(t)) + G_{\sigma(t)}v(t)\right]dt + D_{\sigma(t)}x(t)dw(t), \\ x(t) &= \varphi(t), \quad t \in [t_0 - h, t_0], \\ z(t) &= M_{\sigma(t)}x(t). \end{aligned} \tag{3.21}$$

Theorem 3.4. *Considering system* (3.21), *for a given scalar* $\alpha > 0$, *if there exist symmetric positive definite matrices* $P_i,\ Q_i > 0$ *such that*

$$\begin{bmatrix} A_i^T P_i + P_i A_i + \alpha P_i + Q_i & P_i B_i & P_i G_i & D_i^T P_i & M_i^T \\ * & -(1-h_d)Q_i & 0 & 0 & 0 \\ * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & -P_i & 0 \\ * & * & * & * & -I \end{bmatrix} < 0 \tag{3.22}$$

hold for all $i \in \underline{N}$, *system* (3.21) *is said to have weighted* H_∞ *performance* γ *under arbitrary switching signal with the average dwell time:*

$$T_\alpha \geq T_\alpha^* = \frac{\ln \mu}{\alpha}, \tag{3.23}$$

where $\mu \geq 1$ *satisfies*

$$P_i \leq \mu P_j, \quad Q_i \leq \mu Q_j, \quad \forall i, j \in \underline{N}. \tag{3.24}$$

Proof. By Theorem 3.1, we can readily obtain that system (3.21) is mean-square exponential stable when $v(t) = 0$.

Assume that the ith subsystem is activated during $[t_k, t_{k+1})$. Choose the following Lyapunov functional candidate for the ith subsystem:

$$V_i(t) = V_{1,i}(t) + V_{2,i}(t), \tag{3.25}$$

where

$$V_{1,i}(t) = x^T(t) P_i x(t), \quad V_{2,i}(t) = \int_{t-h(t)}^{t} x^T(s) Q_i x(s) ds. \tag{3.26}$$

Using Itô formula, along the trajectory of system (3.21); we have

$$dV_i(t) = \mathcal{L}V_i(t)dt + 2x^T(t) P_i D_i x(t) dw(t), \tag{3.27}$$

where

$$\begin{aligned} \mathcal{L}V_i(t) = {} & 2x^T(t) P_i (A_i x(t) + B_i x(t-h(t)) + G_i v(t)) + x^T(t) Q_i x(t) + x^T(t) D_i^T P_i D_i x(t) \\ & - \left(1 - \dot{h}(t)\right) x^T(t-h(t)) Q_i x(t-h(t)) \end{aligned} \tag{3.28}$$

Let $\Gamma(t) = z^T(t) z(t) - \gamma^2 v^T(t) v(t)$; we have

$$\mathcal{L}V_i(t) + \Gamma(t) = \varsigma^T(t) \Phi_i \varsigma(t), \tag{3.29}$$

where $\varsigma(t) = [x^T(t)\ x^T(t-h(t))\ v^T(t)]^T$, and

$$\Phi_i = \begin{bmatrix} A_i^T P_i + P_i A_i + Q_i + M_i^T M_i + D_i^T P_i D_i & P_i B_i & P_i G_i \\ * & -(1-h_d)Q_i & 0 \\ * & * & -\gamma^2 I \end{bmatrix}. \tag{3.30}$$

Combining (3.22) with (3.29)–(3.30), and using Schur complement, we have

$$\mathcal{L}V_i(t) + \Gamma(t) < -\alpha V_{1,i}(t) < 0. \tag{3.31}$$

Noticing (2.2) and taking the expectation to (3.27), we have

$$E\left\{\frac{dV_i(t)}{dt}\right\} = E\{\mathcal{L}V_i(t)\}. \tag{3.32}$$

According to (3.25)–(3.27), we have

$$E\{V_p(t)\} \le \mu E\{V_q(t^-)\}, \qquad \forall p, q \in \underline{N}. \tag{3.33}$$

Using Itô formula, we have, for any $t \in [t_k, t_{k+1})$,

$$\begin{aligned}
E\{V_i(t)\} &= E\{V_i(t_k)\} + \int_{t_k}^{t} E\{\mathcal{L}V_i(s) + \Gamma(s)\}ds - E\left\{\int_{t_k}^{t} \Gamma(s)ds\right\} \\
&< \mu E\{V_{i-1}(t_k^-)\} - E\left\{\int_{t_k}^{t} \Gamma(s)ds\right\} \\
&= \mu E\{V_{i-1}(t_{k-1})\} + \mu E\left\{\int_{t_{k-1}}^{t_k} (\mathcal{L}V_{i-1}(s) + \Gamma(s))ds\right\} - \mu E\left\{\int_{t_{k-1}}^{t_k} \Gamma(s)ds\right\} \\
&\quad - E\left\{\int_{t_k}^{t} \Gamma(s)ds\right\} \\
&< \mu E\{V_{i-1}(t_{k-1})\} - \mu E\left\{\int_{t_{k-1}}^{t_k} \Gamma(s)ds\right\} - E\left\{\int_{t_k}^{t} \Gamma(s)ds\right\} \\
&< \cdots \\
&< \mu^{N_\sigma(t_0,t)} E\{V(t_0)\} - \mu^{N_\sigma(t_0,t)} E\left\{\int_{t_0}^{t_1} \Gamma(s)ds\right\} - \cdots - E\left\{\int_{t_k}^{t} \Gamma(s)ds\right\} \\
&= \mu^{N_\sigma(t_0,t)} E\{V(t_0)\} - E\left\{\int_{t_0}^{t} e^{N_\sigma(s,t)\ln\mu}\Gamma(s)ds\right\}.
\end{aligned} \tag{3.34}$$

Under zero initial condition, we can obtain

$$E\left\{\int_{t_0}^{t} e^{N_\sigma(s,t)\ln\mu}\Gamma(s)ds\right\} < 0. \tag{3.35}$$

Moreover, we have

$$E\left\{\int_{t_0}^{t} e^{N_\sigma(s,t)\ln\mu}z^T(s)z(s)ds\right\} < \gamma^2\int_{t_0}^{t} e^{N_\sigma(s,t)\ln\mu}v^T(s)v(s)ds. \tag{3.36}$$

Multiplying both sides of (3.36) by $e^{-N_\sigma(t_0,t)\ln\mu}$ leads to

$$E\left\{\int_{t_0}^{t} e^{-N_\sigma(t_0,s)\ln\mu}z^T(s)z(s)ds\right\} < \gamma^2\int_{t_0}^{t} e^{-N_\sigma(t_0,s)\ln\mu}v^T(s)v(s)ds. \tag{3.37}$$

Noticing $N_\sigma(t_0,s) \leq ((s-t_0)/T_\alpha)$ and $T_\alpha \geq T_\alpha^* = \ln\mu/\alpha$, we have

$$E\left\{\int_{t_0}^{t} e^{-\alpha(s-t_0)}z^T(s)z(s)ds\right\} < \gamma^2\int_{t_0}^{t} v^T(s)v(s)ds. \tag{3.38}$$

When $t \to \infty$, it leads to

$$E\left\{\int_{t_0}^{\infty} e^{-\alpha(s-t_0)}z^T(s)z(s)ds\right\} < \gamma^2\int_{t_0}^{\infty} v^T(s)v(s)ds. \tag{3.39}$$

The proof is completed. □

Remark 3.5. When $dw(t) = 0$, system (3.21) is reduced to a switched delay system, which can be described as

$$\begin{aligned} \dot{x}(t) &= A_{\sigma(t)}x(t) + B_{\sigma(t)}x(t-h(t)) + G_{\sigma(t)}v(t), \\ x(t) &= \varphi(t), \qquad t \in [t_0-h, t_0]. \end{aligned} \tag{3.40}$$

Using the method proposed in Theorem 3.4, we can obtain the following conclusion.

Corollary 3.6. *Considering system* (3.40)*, for a given scalar* $\alpha > 0$*, if there exist symmetric positive definite matrices* P_i*,* $Q_i > 0$ *such that*

$$\begin{bmatrix} A_i^T P_i + P_i A_i + \alpha P_i + Q_i & P_i B_i & P_i G_i & M_i^T \\ * & -(1-h_d)Q_i & 0 & 0 \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -I \end{bmatrix} < 0 \tag{3.41}$$

hold for all $i \in \underline{N}$, *system* (3.40) *is said to have weighted* H_∞ *performance* γ *under arbitrary switching signal with the average dwell time scheme* (3.23).

3.3. Design of Robust H_∞ Controller

In this subsection, the following robust H_∞ controller

$$u(t) \ = K_{\sigma(t)} x(t) \tag{3.42}$$

will be designed for system (2.1). Then the corresponding closed-loop system can be described as

$$\begin{gathered} dx(t) = \left[\left(\widehat{A}_{\sigma(t)} + C_{\sigma(t)} K_{\sigma(t)} \right) x(t) + \widehat{B}_{\sigma(t)} x(t-h(t)) + G_{\sigma(t)} v(t) \right] dt + \widehat{D}_{\sigma(t)} x(t) dw(t), \\ x(t) = \varphi(t), \quad t \in [t_0 - h, t_0], \\ z(t) = M_{\sigma(t)} x(t). \end{gathered} \tag{3.43}$$

Theorem 3.7. *Considering system* (2.1), *for given scalars* α, $\varepsilon_i > 0$, $h_d < 1$, *if there exist matrix* Z_i, *symmetric positive definite matrices* X_i, $Y_i > 0$ *such that*

$$\begin{bmatrix} \Sigma_{11}^i & B_i X_i & G_i & X_i D_i^T & X_i M_i^T & X_i E_{1i}^T & X_i E_{3i}^T \\ * & \Sigma_{22}^i & 0 & 0 & 0 & X_i E_{2i}^T & 0 \\ * & * & -\gamma^2 I & 0 & 0 & 0 & 0 \\ * & * & * & \Sigma_{44}^i & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & -\varepsilon_i I & 0 \\ * & * & * & * & * & * & -\varepsilon_i I \end{bmatrix} < 0 \tag{3.44}$$

holds for all $i \in \underline{N}$, *with the average dwell time:*

$$T_\alpha \geq T_\alpha^* = \frac{\ln \mu}{\alpha}, \tag{3.45}$$

where $\mu \geq 1$ *satisfies*

$$X_j \leq \mu X_i, \quad X_i^{-1} Y_i X_i^{-1} \leq \mu X_j^{-1} Y_j X_j^{-1}, \quad \forall i, j \in \underline{N}, \tag{3.46}$$

then, there exists a robust H_∞ *controller:*

$$u(t) = K_{\sigma(t)} x(t), \qquad K_i = Z_i X_i^{-1}, \tag{3.47}$$

which can render the corresponding closed-loop system (3.43) *mean-square exponentially stable with weighted H_∞ performance γ, where*

$$\sum_{11}^{i} = X_i A_i^T + Z_i^T C_i^T + A_i X_i + C_i Z_i + \alpha X_i + Y_i + \varepsilon_i H_i H_i^T,$$
$$\sum_{22}^{i} = -(1-h_d) Y_i, \tag{3.48}$$
$$\sum_{44}^{i} = \varepsilon_i H_i H_i^T - X_i.$$

Proof. By Theorem 3.4, system (3.43) is mean-square exponentially stable with weighted H_∞ performance γ if the following inequalities are satisfied:

$$\begin{bmatrix} \Lambda_{11}^i & P_i \widehat{B}_i & P_i G_i & \widehat{D}_i^T P_i & M_i^T \\ * & -(1-h_d) Q_i & 0 & 0 & 0 \\ * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & -P_i & 0 \\ * & * & * & * & -I \end{bmatrix} < 0, \tag{3.49}$$

where $\Lambda_{11}^i = (\widehat{A}_i + C_i K_i)^T P_i + P_i(\widehat{A}_i + C_i K_i) + \alpha P_i + Q_i$.

Then using $\Lambda_{Pi} = \mathrm{diag}\{P_i^{-1}, P_i^{-1}, I, P_i^{-1}, I\}$ to pre- and postmultiply Λ^i, we have

$$\widehat{T}^i = \Lambda_{Pi} \Lambda^i \Lambda_{Pi} < 0. \tag{3.50}$$

Furthermore,

$$\widehat{T}^i = \begin{bmatrix} \widehat{T}_{11}^i & \widehat{B}_i X_i & G_i & X_i \widehat{D}_i^T & X_i M_i^T \\ * & \widehat{T}_{22}^i & 0 & 0 & 0 \\ * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & -X_i & 0 \\ * & * & * & * & -I \end{bmatrix} < 0, \tag{3.51}$$

where $\widehat{T}_{11}^i = X_i(\widehat{A}_i + C_i K_i)^T + (\widehat{A}_i + C_i K_i) X_i + \alpha X_i + Y_i$, $\widehat{T}_{22}^i = -(1-h_d) Y_i$, $X_i = P_i^{-1}$, and $Y_i = P_i^{-1} Q_i P_i^{-1}$.

Combining (3.51) with (2.4), we have

$$\widehat{T}^i = T^i + \Delta T^i, \tag{3.52}$$

where

$$\widehat{T}^i = \begin{bmatrix} T_{11}^i & B_iX_i & G_i & X_iD_i^T & X_iM_i^T \\ * & T_{22}^i & 0 & 0 & 0 \\ * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & -X_i & 0 \\ * & * & * & * & -I \end{bmatrix} < 0,$$

$$\Delta T^i = \begin{bmatrix} X_iE_{1i}^TF_i^TH_i^T + H_iF_iE_{1i}X_i & H_iF_iE_{2i}X_i & 0 & X_iE_{3i}^TF_i^TH_i^T & 0 \\ X_iE_{2i}^TF_i^TH_i^T & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ H_iF_iE_{3i}X_i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.53}$$

$$T_{11}^i = X_i(A_i + C_iK_i)^T + (A_i + C_iK_i)X_i + \alpha X_i + Y_i, \text{ and } T_{22}^i = -(1 - h_d)Y_i.$$

Moreover,

$$\Delta T^i = \begin{bmatrix} H_i & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & H_i \\ 0 & 0 \end{bmatrix} \begin{bmatrix} F_i & 0 \\ 0 & F_i \end{bmatrix} \begin{bmatrix} X_iE_{1i}^T & X_iE_{3i}^T \\ X_iE_{2i}^T & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}^T + \begin{bmatrix} X_iE_{1i}^T & X_iE_{3i}^T \\ X_iE_{2i}^T & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} F_i & 0 \\ 0 & F_i \end{bmatrix}^T \begin{bmatrix} H_i & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & H_i \\ 0 & 0 \end{bmatrix}^T. \tag{3.54}$$

According to Lemma 2.5, we have

$$\Delta T^i \leq \varepsilon_i \begin{bmatrix} H_i & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & H_i \\ 0 & 0 \end{bmatrix} \begin{bmatrix} H_i & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & H_i \\ 0 & 0 \end{bmatrix}^T + \varepsilon_i^{-1} \begin{bmatrix} X_iE_{1i}^T & X_iE_{3i}^T \\ X_iE_{2i}^T & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} X_iE_{1i}^T & X_iE_{3i}^T \\ X_iE_{2i}^T & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}^T. \tag{3.55}$$

Substituting (3.55) to (3.52), and using Schur complement, we can obtain that (3.52) is equivalent to (3.44). Denoting $X_i = P_i^{-1}$, and $Y_i = P_i^{-1}Q_iP_i^{-1}$, it is easy to get that (3.46) is equivalent to (3.24).

The proof is completed. □

Remark 3.8. Theorem 3.4 presents the sufficient conditions which could guarantee that the switched stochastic delay system is stable with H_∞ performance; when the robust H_∞ control problem is considered, we can solve the problem by substituting the closed-loop uncertain parameters $\widehat{A}_i + C_i K_i$, $\widehat{B}_i$, $\widehat{D}_i$ to Theorem 3.4.

4. Numerical Example

In this section, a numerical example is given to illustrate the effectiveness of the proposed approach. Consider system (2.1) with the following parameters:

$$
\begin{gathered}
A_1 = \begin{bmatrix} 2 & 0.9 \\ 1.4 & -2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -0.3 & 0 \\ 0.1 & -0.5 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}, \quad D_1 = \begin{bmatrix} -0.3 & 0 \\ 0.1 & 0.2 \end{bmatrix}, \\
G_1 = \begin{bmatrix} -0.8 & 0 \\ 0 & -0.1 \end{bmatrix}, \quad M_1 = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}, \quad E_{11} = \begin{bmatrix} 0.7 & 0 \\ 0.9 & 0 \end{bmatrix}, \quad E_{21} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.8 \end{bmatrix}, \\
E_{31} = \begin{bmatrix} 0.6 & 0 \\ 0 & 0 \end{bmatrix}, \quad H_1 = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.8 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -2 & 1 \\ 0 & -3 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -0.4 & 0.1 \\ 0 & -0.8 \end{bmatrix}, \\
C_2 = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}, \quad D_2 = \begin{bmatrix} -0.2 & 0.05 \\ 0 & -0.4 \end{bmatrix}, \quad G_2 = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.9 \end{bmatrix}, \quad M_2 = \begin{bmatrix} 1 & -2 \\ 3 & 1 \end{bmatrix}, \\
E_{12} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0 \end{bmatrix}, \quad E_{22} = \begin{bmatrix} 0.3 & 0 \\ 0.1 & 0 \end{bmatrix}, \quad E_{32} = \begin{bmatrix} 0 & 0.4 \\ 0 & 0.5 \end{bmatrix}, \quad H_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.3 \end{bmatrix}, \\
F_1 = \begin{bmatrix} \sin t & 0 \\ 0 & \sin t \end{bmatrix}, \quad F_2 = \begin{bmatrix} \cos t & 0 \\ 0 & \cos t \end{bmatrix},
\end{gathered}
\tag{4.1}
$$

the disturbance input $v(t) = \left[\, 50e^{-0.5t} \;\; 10e^{-0.5t} \,\right]^T$.

Let $\alpha = 0.6$, $h(t) = 1 + 0.5 \sin t$, $\varepsilon_1 = \varepsilon_2 = 1$, $\gamma = 1$; then solving the LMIs in Theorem 3.7, we have

$$
\begin{gathered}
X_1 = \begin{bmatrix} 1.0527 & 0.0392 \\ 0.0392 & 1.0410 \end{bmatrix}, \quad Y_1 = \begin{bmatrix} 6.0116 & 0.0659 \\ 0.0659 & 6.5879 \end{bmatrix}, \quad Z_1 = \begin{bmatrix} -17.0837 & 30.0731 \\ 13.0224 & -37.1398 \end{bmatrix}, \\
K_1 = \begin{bmatrix} -17.3295 & 29.5401 \\ 13.7192 & -36.1921 \end{bmatrix}, \quad X_2 = \begin{bmatrix} 0.9153 & -0.0358 \\ -0.0358 & 1.0266 \end{bmatrix}, \quad Y_2 = \begin{bmatrix} 6.0651 & 0.0640 \\ 0.0640 & 5.4975 \end{bmatrix}, \\
Z_2 = \begin{bmatrix} -26.1526 & 4.0064 \\ 13.4430 & -5.1025 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -28.4582 & 2.9099 \\ 14.5120 & -4.4641 \end{bmatrix}, \quad \mu = 8.3261.
\end{gathered}
\tag{4.2}
$$

Then we obtain that $T_\alpha^* = \ln \mu / \alpha = 3.5323$. Thus, under the average dwell time $T_\alpha > T_\alpha^*$, the designed controller can guarantee that the corresponding closed-loop system is mean-square exponentially stable with weighted H_∞ performance.

Simulation results are shown in Figures 1–3, where the initial state $x(t) = [0, 0]^T$, $t \in [-h, 0)$, and $x(0) = [2, -2]^T$. The switching signal with the average dwell time $T_\alpha = 4$ is shown in Figure 1, and Figures 2 and 3 show state x_1 and x_2 of the closed-loop system, respectively.

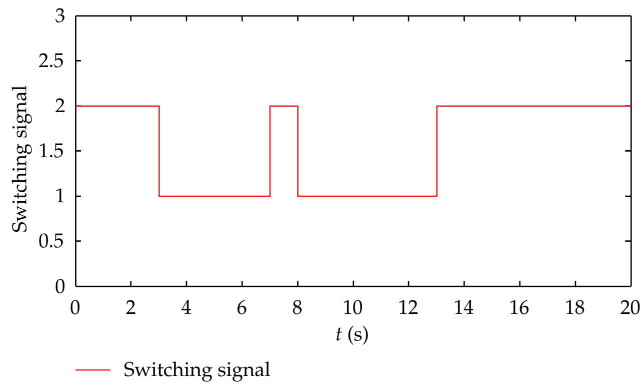

Figure 1: Switching signal.

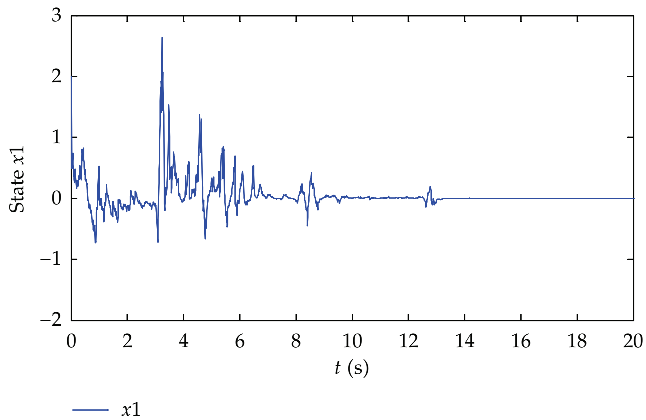

Figure 2: State x_1 of the closed-loop system.

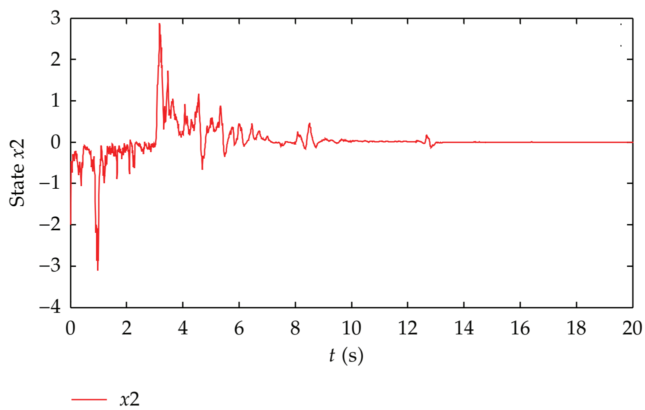

Figure 3: State x_2 of the closed-loop system.

5. Conclusions

In this paper, the exponential stability analysis and robust H_∞ control for switched stochastic time delay systems have been investigated. Based on the average dwell time method and Gronwall-Bellman inequality, a new mean-square exponential stability criteria and H_∞ performance analysis are presented. Furthermore, robust H_∞ controller is designed to guarantee that the corresponding closed-loop system is mean-square exponentially stable. Finally, a numerical example is given to illustrate the effectiveness of the proposed approach. The proposed method provides a powerful tool to solve many other problems such as controller design under asynchronous switching and actuator failures. These problems are the topics of the future research.

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